Strain Estimation for Low Cycle Dynamic Loads

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ABSTRACT

The most effective way to avoid the overestimation of low cycle dynamic load cases is to shift the design basis from limitation of stress to the limitation of strain. As the high effort needed for step-by-step numerical integration of non-linear structural response is prohibitive methods for approximate strain evaluation are inevitable.

The fundamentals of a strain estimation method based on the concept of energy dissipation and structural hardening as functions of strain range are described. The need for non-linear information is reduced either to a limited number of quasistatic non-linear computations or to a certain knowledge of the yield behaviour of structural members.

1 INTRODUCTION

With regard to approximate strain evaluation methods proposed in the past we may distinguish between two main approaches. The first approach is due to NEWMARK/HALL / 1 / and uses elastic modal analysis in connection with reduced response spectra. The reduction is controlled by displacement ductility. Uncritical use of the method neglects localized yielding. The second approach is based essentially on the paper of CAUGHEY / 2 / and its generalizations e.g. by TANSIRIKONGKOL/PECKNOLD / 3 /. These methods involve iterative processes to develop equivalent stiffness and damping matrices. It is not clear if these processes converge unequivocally. Actually it is proven in / 2 / that energy dissipation by material yielding is not equivalent to viscous damping.

Due to the limited space available only the fundamentals of a strain estimation method based upon the concepts of energy dissipation and structural hardening as functions of strain range can be described. Proofs are omitted as well as the results of parametric studies supporting certain hypotheses. The regress from overall structural data to non-linear characteristics of structural members in the sense of / 3 / can only be touched. There is one example added which shows how the method works in principle.

2 NOMENCLATURE

For the sake of simplicity and without too great a loss of generality elastic-plastic material characterized by YOUNG's modulus E, yield stress σ_y or yield strain ε_y and POISSON's ratio ν is assumed. Exclusion of failure by limitation of strain means to prove the strain tensor G(t) as a function of time to be allowable. For ε a one dimensional equivalent strain G(t) can be judged by the set of strain ranges $\varepsilon(t_1, t_2)$:

$$\varepsilon(t_1, t_2) = \int_{t_1}^{t_2} |\dot{\varepsilon}(t)|dt; \quad \dot{\varepsilon}(t_1) = \dot{\varepsilon}(t_2) = 0, \quad \dot{\varepsilon}(t) \neq 0 \text{ für } t_1 < t < t_2$$

For a given structure let ω_i, ϕ_i and γ_i be the elastically computed circular frequencies, mode shapes and participation factors. Let θ_i and σ_i be the force and stress distribution resulting for the deformation ϕ_i and fictously linear material behaviour.

For ε greater ε_y let $\alpha_i(\varepsilon)$ be the factor so that the equivalent strain ε is reached within the structure by the qusistatic process $0 \rightarrow \alpha_i(\varepsilon)\theta_i$. Let $g_i(\varepsilon)$ and $h_i(\varepsilon)$ be the deformation reached and energy dissipated by this process.

2.1 DEFINITION

Modal Volume Coefficient $v_i(\varepsilon)$:

$$v_i(\varepsilon) = \frac{h_i(\varepsilon)}{(\frac{\varepsilon}{\varepsilon_y} - 1)\ \alpha_i^2(\varepsilon_y)\omega_i^2} \tag{1}$$

Modal Hardening Coefficient $s_i(\varepsilon)$:

$$s_i(\varepsilon) = \frac{\frac{\alpha_i(\varepsilon)}{\alpha_i(\varepsilon_y)} - 1}{\frac{\varepsilon}{\varepsilon_y} - 1} \tag{2}$$

Modal Load Coefficient ρ_i:

$$\rho_i = \max_p \frac{\omega_i^2}{\gamma_i \sigma_i(p)} \text{ ; p structural locations} \tag{3}$$

Structural Load Coefficient β:

$$\beta = \max_p (\sum_i (\frac{\gamma_i \sigma_i(p)}{\omega_i^2})^2)^{-1} \tag{4}$$

3 A SPECIAL CLASS OF 1-DOF-SYSTEMS

The subject we deal with in this chapter are structures consisting of a special configuration of three truss elements with areas A and lengths l and a rigid mass m according fig. 1. It is pressupposed that truss 3 does not reach σ_y while the structure is excited by a displacement function Z(t). Obviously the following magnitudes contain all information needed with regard to displacement X(t) and strain $\varepsilon(t)$:

$$l_1,\ \mu_1 = \frac{A_1}{m},\ \kappa_2 = \frac{1}{m}\frac{A_2 A_1}{A_1 l_2 + A_2 l_1},\ \kappa_3 = \frac{1}{m}\frac{A_3}{l_3}$$

3.1 THEOREM

$$\omega^2 = E(\kappa_2 + \kappa_3) \tag{1}$$

$$\beta = \frac{\omega^2}{E} \cdot \frac{\mu_1}{\kappa_2} = \frac{\kappa_2 + \kappa_3}{\kappa_2} \mu_1 \tag{2}$$

$$s(\varepsilon) = \frac{\kappa_3}{\beta} l_1 \quad \text{for} \quad \varepsilon \geq \varepsilon_y \tag{3}$$

$$v(\varepsilon) = \frac{\kappa_2}{\beta} l_1 \quad \text{for} \quad \varepsilon \geq \varepsilon_y \tag{4}$$

According 3.1, (1) to (4) we may refer to this class of 1-DOF-systems as the four parametric set R(ω, β, s, v).

3.2 THEOREM

Let X_y be the maximum displacement to be done elastically. For the strain ε resulting from a displacement $\beta \cdot X_y$ with $\beta > 1$ results:

$$\varepsilon = \varepsilon_y + (\beta - 1) \frac{1}{s + v} \cdot \varepsilon_y$$

4 FORMULATION OF A STRAIN ESTIMATION METHOD

Constituting for the method is the following hypothesis (for the verification see e.g. / 4 / in connection with theorem 3.2):

4.1 HYPOTHESIS

For a given ω and a given excitation $Z(t)$ the magnitude of strain ranges increases with decreasing s, decreasing v and decreasing ρ.

Let now ε^* be the allowable strain range. We refer to $R(\omega_i, \rho, s_i(\varepsilon^*), v_i(\varepsilon^*))$ as the modal representatives with respect to ε^*. Together with the normative character of ρ actually describing the acceleration level able to induce yielding hypothesis 4.1 motivates the following strain estimation:

4.2 STRAIN ESTIMATION METHOD

If $\tilde{\varepsilon}$ is the maximum strain range experienced by the modal representatives and if $\varepsilon^* \geq \tilde{\varepsilon}$ than ε^* is an upper limit for the strain range experienced by the structure.

So far the moethod replaces step-by-step non-linear numerical integration of the structure in question by quasistatic non-linear computations to evaluate $s_i(\varepsilon^*)$ and $v_i(\varepsilon^*)$. Modes with $\rho_i << \rho$ may be neglected. Step-by-step integrations are necessary for the $R(\omega, \rho, s, v)$. These computations can be done in advance delivering strain-tange-response spectra as functions of ω, ρ and s. Strain estimations can then be carried out using theorem 3.2. A further regress is possible by using analogously defined volume and hardening coefficients for structural members (e.g. elbows, tees, straight pipes in the case of pipe work). In principle every mode shape delivers an "order of yielding" for structural members which is then corrected by the hardening coefficients to deliver the strain magnitude. The corrected order allows estimations for s_i and v_i. A more complete description is beyond the scope of this paper.

5 A SIMPLE EXAMPLE

The structures of this chapter are defined as the combination of two truss elements of length l and areas 2A and A respectively supplied with two rigid masses m (fig. 2). With $\Omega^2 = \frac{AE}{lm}$ we get the following equations:

$$\omega_{1,2} = \Omega \sqrt{2 \mp \sqrt{2}} \qquad \rho_{1,2} = \frac{A}{m} 2 (\sqrt{2} \mp 1)$$

$$\alpha_{1,2}(\varepsilon) = l\varepsilon_y \sqrt{m (2 \pm \sqrt{2}} \text{ for } \varepsilon \geq \varepsilon_y \qquad \rho = \frac{A}{m} \sqrt{\frac{2}{3}}$$

$$s_{1,2}(\varepsilon) = 0 \text{ for } \varepsilon \geq \varepsilon_y \qquad v_{1,2}(\varepsilon) = \frac{A \cdot l \cdot \sigma_y \cdot \varepsilon_y}{\alpha^2_{1,2}(\varepsilon_y) \cdot \omega^2_{1,2}} = 0{,}5 \text{ for } \varepsilon \geq \varepsilon_y$$

As a numerical case has been chosen:

$$E = 2{,}1 \cdot 10^{11} \ / \ N/m^2 \ /, \ \sigma_y = 2{,}4 \cdot 10^8 \ / \ N/m^2 \ /, \ \rho = 1{,}67 \cdot 10^{-8} \ / \ m^2/kg \ /$$

$$\omega_1 = 2 \cdot \pi \cdot 3 \ / \ 1/s^2 \ /, \ \omega_2 = 2\pi \cdot 7{,}24 \ / \ 1/s^2 \ /$$

The value of ρ means that yielding occurs at about an acceleration of $b_y = \rho \cdot \sigma_y = 4{,}0 \ / \ m/s^2 \ /$.

As excitation the acceleration time history $\ddot{Z}(t)$ according fig. 3 has been chosen. The acceleration response spectrum for a damping of 2 % is shown on fig. 4. The results:

	Max. Strain Range
Structure	$7{,}0 \cdot 10^{-3}$
$R(\omega_1, \beta, s, v)$	$7{,}9 \cdot 10^{-3}$
$R(\omega_2, \beta, s, v)$	$4{,}1 \cdot 10^{-3}$

6 REFERENCES

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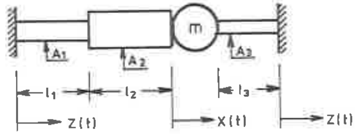
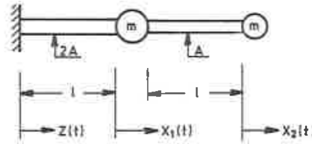


$z(t)$ = excitation
$x(t)$ = displacement response
$A_2 > A_1$

Fig. 1



$z(t)$ = excitation
$x_1(t)$ = displacement response truss 1
$x_2(t)$ = displacement response truss 2

Fig. 2

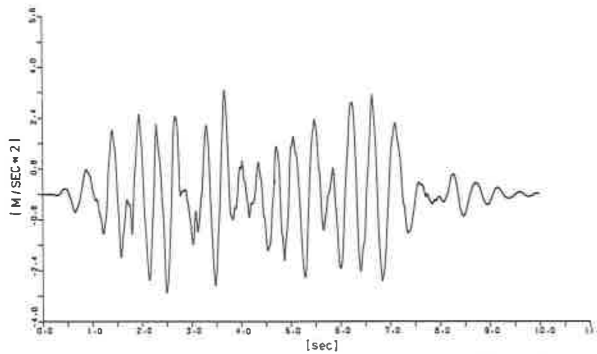


Fig. 3

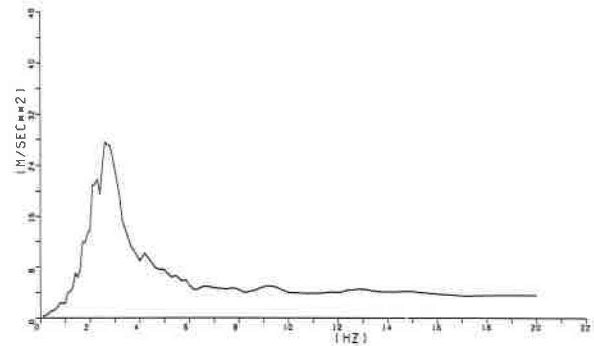


Fig. 4