Stress index of 45 and 90 degree elbow subjected to in-plane and out-of-plane moment

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ABSTRACT: This paper describes a design method for the Class-1 welding elbow which has a stiffener in a shorter distance than its pipe radius. Elastic stress analysis has been carried out for a 45 degree and a 90 degree elbow subjected to an in-plane and an out-of-plane moment, respectively. A modified stress index was proposed for stress evaluation using flexibility analysis. This method has been investigated for adopting to the design standard for the Class-1 piping of the Japanese demonstration fast breeder reactor.

1 INTRODUCTION

ASME Sec.III NB defines condition for application of flexibility analysis to the Class-1 piping design. In ASME Code, the distance between a stiffener and an end of an elbow is required to be more than the pipe radius. The piping, however, should be routed within a small space in the design of a fast breeder reactor, FBR, in order to save the construction cost. Thus some stiffeners, for instance, nozzles, supports, and so on, might be placed near the end of an elbow in the design of future FBR plants. This study has been carried out to prepare a stress evaluation method for an elbow which has a stiffener in a shorter distance than its pipe radius.

In a paper presented at the 12th SMiRT conference, Kamishima et al introduced a stress magnification factor, \( m \), for the stress index, \( C_i \), in flexibility analysis for a 90 degree elbow subjected to an in-plane moment. In this paper, a further study has been made on the stress index for a 45 and a 90 degree elbow subjected to an in-plane and an out-of-plane moment, respectively. The analytical results show that the modified stress index, \( C_m \), which was proposed in the previous paper can be applied to both case.

2 STRESS ANALYSIS

Elastic stress analyses using the finite element method, FEM, have been carried out to confirm end effects of an elbow subjected to moment loading. Analyzed elbows are 45 and 90 degree elbows whose finite element models are shown in Fig. 1. The geometries of these elbows are shown in Fig. 2. The values of \( R/t \) and \( r/t \) of the analytical model are same as the primary coolant piping elbow used in Japanese
demonstration FBR plant. The analytical parameters are the length of the attached straight pipes, \( L_1 \) and \( L_2 \). The length of the straight pipe ranges from 0 to 5 times of the pipe diameter on condition that at least one leg of straight pipes is longer than its pipe radius, because it seems very seldom that there are stiffeners at both sides of the elbow. A taper junction, which has a length of 100 mm and a slope of 1 in 3, is placed between the straight pipe and the stiffener to prevent stress concentration due to the structural discontinuity. An out-of-plane moment is loaded to 90 degree elbow model and an in-plane moment is loaded to the 45 degree elbow model. Typical stress distributions are shown in Fig. 3. The maximum stresses appear near the center of the elbow when the attached straight pipes are long, while they appear at the elbow-stiffener joint when the straight pipes are short.

3 STRESS MAGNIFICATION FACTOR

3.1 90 DEGREE ELBOW SUBJECTED TO OUT-OF-PLANE MOMENT

When the elbow subjected to the out-of-plane moment, \( M_{beam} \), as shown in Fig. 2, bending moment and torque are loaded to the straight pipe A and the straight pipe C, respectively. Elbow B are subjected to combined loading of moment and torque. The flexibility factor, \( k_{beam} \), and the stress index, \( C_{beam} \), are defined in ASME Sec III NB.

\[
k_{beam} = \frac{1.65}{\left( \frac{t R}{r^2} \right)}
\]

\[
C_{beam} = \frac{1.95}{\left( \frac{t R}{r^2} \right)^{1/3}}
\]

where \( t \) is the wall thickness of the elbow, \( R \) is the elbow bend radius and \( r \) is the mid-wall radius of the elbow. The stress of the elbow, \( \sigma_{beam} \), and the rotating angle at the free end around \( Y \) axis, \( \theta_{beam} \), are given by the following equations.

\[
\sigma_{beam} = \frac{M_{beam}}{Z} \left( C_{beam} \cos \phi + \sin \phi \right)
\]

\[
\theta_{beam} = \frac{M_{beam}}{EI} \left( L_1 + L_2 \left( 1 + v \right) + \frac{\pi R}{4} \left( k_{beam} + 1 + v \right) \right)
\]

where \( Z \) is the modulus of section, \( I \) is the moment of inertia and \( v \) is the Poisson’s ratio. The stress is represented at the center of elbow, where \( \phi = 45 \) degree, in flexibility analysis.

On the other hand, the flexibility factor, \( k_{FEM} \), and the stress index, \( C_{FEM} \), with FEM analysis using three dimensional shell are evaluated by the following equations.

\[
k_{FEM} = \frac{E J}{M_{FEM}} \left( \frac{\theta_{FEM}}{L_1 + L_2 \left( 1 + v \right)} \right) \cdot \frac{4}{\pi R} - \left( 1 + v \right)
\]

\[
C_{FEM} = \frac{\left( \frac{\sigma_{FEM} - M_{FEM}}{2 Z \sin \phi} \right) Z}{M_{FEM} \cos \phi}
\]

where \( M_{FEM} \) is the out-of-plane moment at the free end of the straight pipe A. The rotating angle around \( Y \) axis, \( \theta_{FEM} \), and the stress of the elbow, \( \sigma_{FEM} \), could be calculated by Eq. (3) and (4) using \( k_{FEM} \) and \( C_{FEM} \) instead of \( k_{beam} \) and \( C_{beam} \). The stress magnification factor is expressed as a ratio of the stress analyzed with FEM to that calculated using flexibility analysis subjected to deformation controlled loading. The stress magnification factor is written in the form as follows.

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\[ m = \frac{C_{FEM} \cos \phi + \frac{\sin \phi}{2} \left( L_1 + L_2 (1 + v) + \frac{\pi R}{4} \left( k_{beam} + 1 + v \right) \right)}{C_{beam} + 0.5 \left( L_1 + L_2 (1 + v) + \frac{\pi R}{4} \left( k_{FEM} + 1 + v \right) \right)} \] (7)

3.2 45 DEGREE ELBOW SUBJECTED TO IN-PLANE MOMENT

When the 45 degree elbow subjected to the in-plane moment as shown in Fig. 2, the flexibility factor and the stress index are calculated with Eq. (1) and (2) as are done for the 90 degree elbow. The stress of the elbow and the rotating angle around Y axis using flexibility analysis are given by the following equations.

\[ \sigma_{beam} = C_{beam} \frac{M_{beam}}{Z} \] (8)

\[ \theta_{beam} = \frac{M_{beam}}{EI} \left( L_1 + L_2 + \frac{\pi R}{4} \right) \] (9)

The flexibility factor, \( k_{FEM} \), and the stress index, \( C_{FEM} \), which are calculated using the FEM analysis, are given by the following equations.

\[ k_{FEM} = \left( \frac{E I}{M_{FEM}} \theta_{FEM} - (l_1 + l_2) \right) \frac{4}{\pi R} \] (10)

\[ C_{FEM} = \frac{\sigma_{FEM} Z}{M_{FEM}} \] (11)

The rotating angle around Y axis and the stress of the elbow are calculated with Eq. (8) and (9) using the above \( k_{FEM} \) and \( C_{FEM} \). Equation (12) represents the stress magnification factor under the deformation controlled moment.

\[ m = \frac{C_{FEM} \left( l_1 + l_2 + k_{beam} \frac{\pi R}{4} \right)}{C_{beam} \left( l_1 + l_2 + k_{FEM} \frac{\pi R}{4} \right)} \] (12)

4 STRESS EVALUATION METHOD

The comparison of the load controlled stresses between the flexibility analysis and the FEM analysis is shown in Fig. 4. The stresses analyzed with FEM are smaller than those calculated by the flexibility analysis in all analytical cases. This is because the stiffener restricts the deformation of the elbow. So the flexibility analysis method gives conservative results for the case when the stiffeners placed within a pipe radius from the end of the elbow under a load controlled moment.

Figure 5 shows the comparison of stresses for a case where a rotating deflection is applied at the free end of the straight pipe. The ratios of \( \sigma_{FEM} \) to \( \sigma_{beam} \) are smaller than unity when the attached straight pipe is longer than the pipe radius. However, where the straight pipe is shorter than the pipe radius, the ratio of \( \sigma_{FEM} \) to \( \sigma_{beam} \) is over unity, and nearly 1.5 when the length of straight pipe is zero. This figure shows that it is necessary to correct the stress index for the case the stiffener placed at a distance shorter than the pipe radius. Thus a stress magnification factor shown in Fig. 6 has been introduced. The modified stress index \( (C_f) \), which is the product of
the stress magnification factor \( m \) and the stress index \( C_2 \) defined in ASME Code, can be applied to the flexibility analysis on the following condition.

\[
\begin{align*}
\text{Min. } [L_1, L_2] &< r \quad (11) \\
\text{Max. } [L_1, L_2] &\geq r \quad (12) \\
2r &\quad (13)
\end{align*}
\]

\[
m = 1.5 - \frac{\text{Min. } [L_1, L_2]}{2r} \quad (13)
\]

\[
C_2' = m \times C_2 \quad (14)
\]

The proposed evaluation method gives a conservative result compared with FEM.

5 CONCLUSIONS

For the case that the distance between the stiffener and the end of an elbow is shorter than its pipe radius, which is beyond ASME Code, a stress evaluation method is recommended. The recommended method is the following:

a. When an elbow is subjected to a load-controlled moment, the flexibility analysis method specified in ASME Code is applicable, because the stress can be evaluated conservatively.

b. When an elbow is subjected to a deformation-controlled moment, the stress magnification factor expressed by Eq. (11) to (14) should be applied for the stress analysis.

6 ACKNOWLEDGEMENT

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REFERENCES

(1) Y. Kamishima et al, The effect of stiff ends on stress index of welding elbow, E03/4 SMiRT-12.
(6) A. R. C. Marcel, Fatigue tests of piping components, Pressure Vessels and Piping Design and Analysis, p1148-1164.
Fig. 1 Finite element analytical model

<table>
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<th>Value</th>
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<td>Outer Diameter ($D_o$)</td>
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<td>Thickness ($t$)</td>
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<tr>
<td>Elbow Bend Radius ($R$)</td>
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<tr>
<td>Mid Radius ($r$)</td>
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</tr>
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<tr>
<td>$t/r$</td>
<td>29.9</td>
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<tr>
<td>Straight Pipe Length ($L_1, L_2$)</td>
<td>$0 \sim 5D_o$</td>
</tr>
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</table>

Fig. 2 Geometrical condition of analytical model

- 90 degree elbow (long straight pipe)
- 90 degree elbow (short straight pipe)
- 45 degree elbow (long straight pipe)
- 45 degree elbow (short straight pipe)

Fig. 3 Stress distribution of elbow subjected to moment loading ($M=1 \times 10^7$ kg-mm)

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Fig. 4  Comparison of stress between FEM and flexibility analysis under the load controlled moment

Fig. 5  Comparison of stress between FEM and flexibility analysis under the deformation controlled moment

Fig. 6  Design diagram for welding elbow which has stiffener in the distance of its pipe radius