Transactions of the **13th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 13), Escola de Engenharia - Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil, August 13-18, 1995

A simplified inelastic analysis method of structures applicated in the case of large variations of temperature

Lejeail, Y., Gatt, J.M., Cabrillat, M.T.
CEA, Cadarache, Saint-Paul-Lez-Durance, France

SUMMARY : An attempt has been made to apply the limit inelastic analysis of structures developped by J. ZARKA when the materials are submitted to high fluctuations of temperature, leading to variations of Young's modulus, yield stress and thermal expansion coefficient.

The principle of the method is presented and some 1D analytical and numerical tests are compared with kinematic hardening plasticity results for a better understanding.

Afterwards much complex cases are shown, like local heating/cooling of a pipe or a plate.

Then, the limits and the application field are discussed together with the possible extensions of the method.

1 INTRODUCTION

The mechanical engineers carrying out analysis of structures often use finite element codes with plasticity hardening models representing non linear behaviour of materials in case of very high temperature fields (near the melting point), involving very important variations of mechanical characteristics such that Young's modulus, yield stress, hardening slope and thermal expansion coefficient. This is particularly the case for the simulation of welding, quenching,... or calculation in the domain of severe accidents.

When residual stress determination is required, it is partly possible to avoid finite element analysis with non linear behaviour which are very expensives and difficult to achieve.

So, an attempt has been made at CEA Cadarache to test a method based on the inelastic analysis of structures developed by J. ZARKA ([1] to [5]), extended to the case of very large temperature variations. This could be applied when parametric finite element calculations is necessary, for instance to study the effects of the choice :

- of mechanical characteristics which are not well defined at high temperatures because of creep effects,
- of the temperature field, not well known for real structures,
- of boundary conditions,
- ...

on the residual stress field.

Moreover, the method could also be applied when very complicated modelling are required :

- to take in account the propagation of a heating source,
- to perform a 3D analysis,

- to carry out a multipass welding simulation,
- ...

We present in this paper the principle of the basic method, the application to 1D problems and then to 2D geometries submitted to local heating and cooling. The obtained stress results are compared to plasticity simulations. Finally, the domain of validity and the possibilities of extensions are discussed.

2 PRINCIPLE OF THE METHOD AND 1D NUMERICAL TESTS

The real total strain in any point of the structure is the sum of three tensorial terms in case of material elastoplastic behaviour. In an incremental formulation, it can be written :

$$(d\varepsilon_t)_{real} = d\varepsilon_e + d\varepsilon_p + d\varepsilon_I$$

where $d\varepsilon_e$ and $d\varepsilon_p$ are respectively the elastic and plastic parts and $d\varepsilon_I$ is an imposed strain (such that pure thermal strain $d(\alpha I \Delta T)$). The elastic strain $d\varepsilon_e$ is equal to $d(M\sigma)$, M being the Hooke matrix and σ the true stress.

Now, assuming that the behaviour is elastic, the elastically calculated total strain in any point is equal to the sum of two terms :

$$(d\varepsilon_t)_{él} = d\varepsilon_{él} + d\varepsilon_I$$

where $d\varepsilon_{él}$ is equal to $d(M\sigma_{él})$, $\sigma_{él}$ being the elastically calculated stresses.

By difference of the two equations, we obtain :

$$(d\varepsilon_t)_{inel} = d\varepsilon_e - d\varepsilon_{él} + d\varepsilon_p = d(M(\sigma - \sigma_{él})) + d\varepsilon_p$$

We know that $d\varepsilon_p = C^{-1}\, dX$ when the hardening slope of the linear kinematic model, $H = 3C/2$, is not dependent of the temperature (X is the well known internal kinematic variable). In addition, we can define at this point $\rho = \sigma - \sigma_{él}$ and $dY = dX - \text{dév}\, d\rho$, using the deviatoric operator "dév", so that :

$$\begin{cases} (d\varepsilon_t)_{inel} = d(M\rho), \text{ when } dX = d\varepsilon_p = 0 \text{ (thus } dY = -\text{dév}\, d\rho) \\ \text{and} \\ (d\varepsilon_t)_{inel} = d\left[(M + C^{-1}\,\text{dév})\,\rho\right] + C^{-1}\, dY, \text{ when } d\varepsilon_p \neq 0 \text{ (thus } dX = dY + \text{dév}\, d\rho) \end{cases}$$

The initial non linear problem is then changed in a linear one with a new elastic matrix, $(M + C^{-1}\,\text{dév})$, and an initial strain $C^{-1}\, dY$. At any time increment, the Von Mises plasticity criteria $J(\sigma_{él} - Y) < \sigma_y$ is checked.

Here, the simplified hypothesis of the basic method is introduced, illustrated on figure 1. It corresponds to a limit analysis with a great amount of strain at any point.

The initial non linear problem of n increments is solved by 2n elastic simulations, i.e. n classical calculations to determine the elastic stresses $\sigma_{éli}$ with the real boundary conditions, and n elastic calculations to estimate ρ_i stresses with zero boundary conditions and the choosen transformed parameters dYi.

This method has been tested by finite elements on a bar fixed at the extremities, the material being assumed temperature dependent as in the table 1. The comparison has been carried out with the code CASTEM 2000 developed by CEA [6].

First we show the results of a bar initially at 1220°C without stresses cooled to 20°C, on figure 2. The exact solution is found in one calculation step.

Then, we present the results of the same bar alternatively heated and cooled between 20 and 1220°C on figure 3. There is always a good agreement on stresses and strains in 1D calculations. So, there is a need of more complex problems to test further the method.

3 SIMULATION OF LOCAL HEATING AND COOLING OF A PIPE OR A PLATE

3.1 *Heating and cooling of a plate*

The mesh of the plate (90 mm length, 100 mm width) is composed of 100 eight nodes rectangle elements. During the calculations, performed in stress plane conditions, the displacements of the symmetry axis are blocked as shown on figure 4. The material properties are assumed linearly temperature dependent (table 1).

First, the plate, initially without stresses, is cooled to 20°C (figure 4). The simplified method gives a good approximation of longitudinal and transversal stresses (figure 5).

Another comparison is obtained after heating and cooling of the plate center (figure 4). Figure 6 illustrates the results of longitudinal and transversal stresses.

3.2 *Heating and cooling of a pipe*

The mesh of the pipe is made of 1057 elements (6 nodes triangles) in axisymmetrical representation. The internal and external diameters are respectively of 180 mm and 220 mm, and the pipe is 400 mm length (figure 7). Axial displacements of the symmetry axis are blocked during the simulations. The material properties are those of the table 1.

- The center of the pipe, initially at very high temperature without stresses, is cooled to 20°C (figure 7). It can be seen that the final axial and tangential stresses obtained with the simplified method are in good agreement with the plastic ones (figure 8).
- Otherwise, after heating to a maximum temperature of 1220°C and cooling to 20°C (figure 7), we obtain the results presented on figure 9.

4 DISCUSSION

The basic method that we have presented the principles gives correct agreements with plasticity results, as it has been shown in chapters 2 and 3. Much more complex calculations are now in progress, such that "simulation of a multipass weldment" and "simulation of friction welding", and seems to confirm that correct agreement on stresses.

In fact, the basic method is intended to give an approximation of true stresses, and not the exact values. The explanation is that the choosen transformed parameters Y are only limit parameters (it is not possible to know before the exact variables). An other reason is that the basic method doesn't permit to impose $dX = 0$ in the elastic areas, so, the plastic strains are not good. That's why it would not be applied when a very accurate knowledge of the stresses is required.

The scope of application is rather parametric studies in view of an estimation of the influence on the final results, or very complex cases, in fact all that is not possible to carry out with incremental models which are complementary of the simplified method.

Also, we recall that the hardening is assumed to be kinematic linear and that the uniaxial hardening slope H is supposed to remain constant at any temperature (the room temperature value has been choosen in our calculations).

It seems it is not interesting to improve the model by imposing $dX = 0$ in the elastic zones because it needs a lot of iterations decreasing the gains of time calculation. However, it could be more interesting to improve the basic method by a more accurate plastic model identification, taking in account the fact that a fixed bar heated to high temperatures remains always in plasticity domain (that was not the case in example of chapter 2). Although this improvement should not be on the simplified method, this one would be much more efficient to reach the exact values of stresses and strains on this 1D problem, and to approach the stresses in more complex cases.

5 CONCLUSION

A simplified analysis method of structures, based on the J. ZARKA method principle, has been presented in this paper. It concerns the domain of very high temperatures inducing drastic variations of material properties.

Some examples of finite element applications have been compared with plasticity results, giving an approximation of stress results in relation to an important gain of calculation time.

The scope of application has been defined and it appears that the method would be used when extensive parametric studies are needed, for instance to estimate the influence of the choice of parameters which are not well known, and when very complex cases should be studied (3D cases...), because it's not possible (or easy, or cheap,...) to apply incremental plasticity algorithms in these cases.

The basic method could be indirectly improved by a more accurate elastoplastic identification on the fixed and heated bar problem, because the results would be reached in less steps than today.

Acknowledgement : This work was sponsored by EDF and FRAMATOME that we thank for financial and technical support.

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TABLE 1

Material properties used for the one-dimensional tests

	T_f (20°C)	Tc (1220°C)
E (MPa)	192000	10230
σ_y (MPa)	260	14.5
α (°C^{-1})	15.9 x 10^{-6}	20.2 x 10^{-6}
H (MPa) for the bar	6579	6579
H (MPa) for plate and pipe	3000	3000

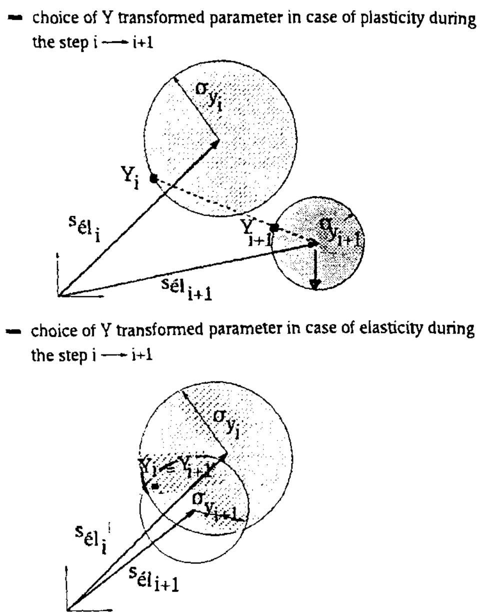

Figure 1 : choice of Y transformed parameters in the basic method presented in this paper. The choice is made every step of calculation at any point in the sructure.

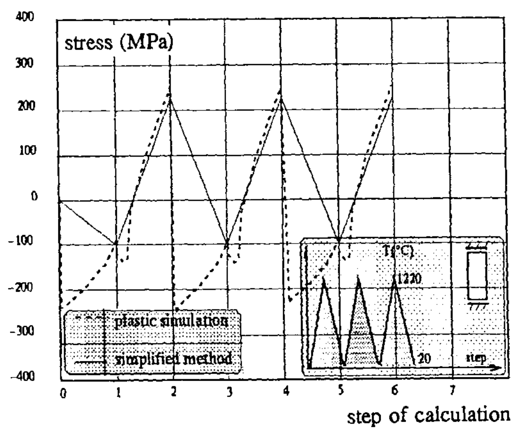

Figure 3 : comparison between plastic simulation and the simplified method, on the problem of a fixed bar initially at room temperature without stress, and then submitted to 3 cycles of heating and cooling.

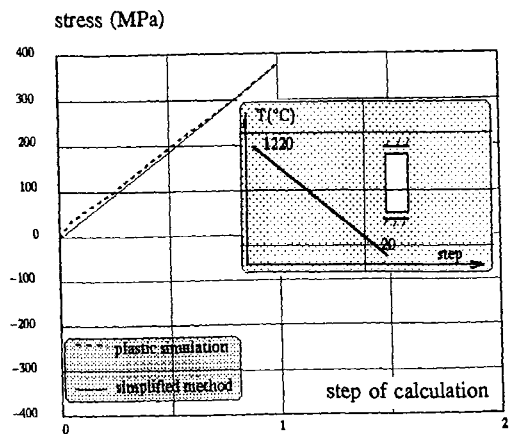

Figure 2 : comparison between plastic simulation and the simplified method, on the problem of a fixed bar initially at high temperature without stress, and then cooled.

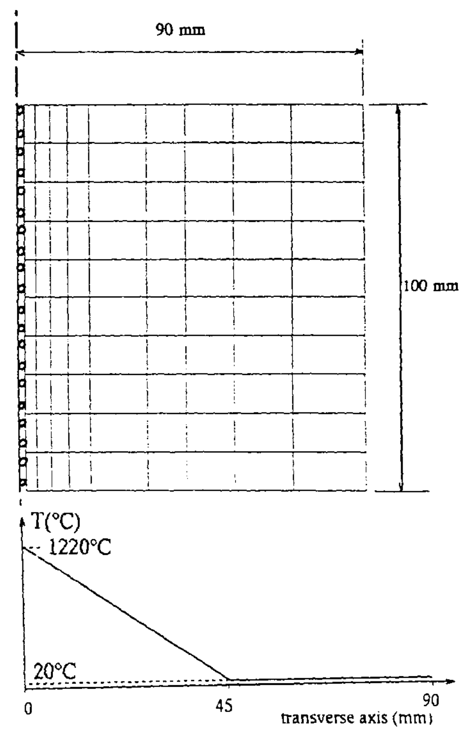

Figure 4 : mesh and initial/final fictitious temperature fields used for the calculations of the plate :
1/ initial high temperature, plate cooled to 20 °C.
2/ initial temperature is 20 °C, plate heated to maximum temperature and then cooled.

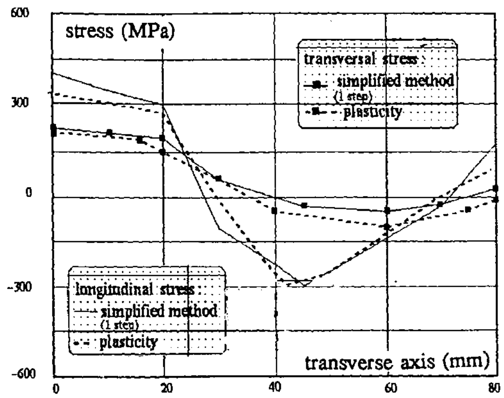

Figure 5 : comparison of transversal and longitudinal stresses obtained with the simplified method and plasticity algorithm, on a plate initially at high temperature without stresses, and then cooled.

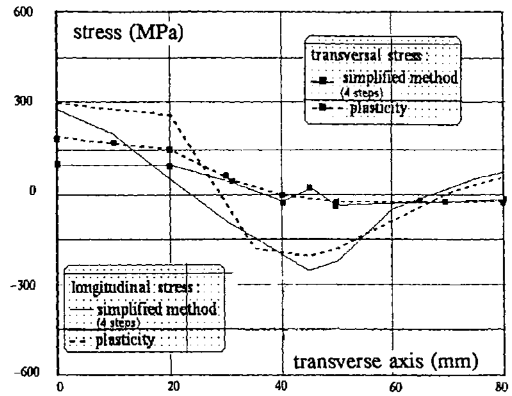

Figure 6 : comparison of transversal and longitudinal stresses obtained with the simplified method and plasticity algorithm, on a plate initially at room temperature without stresses, and then heated and cooled.

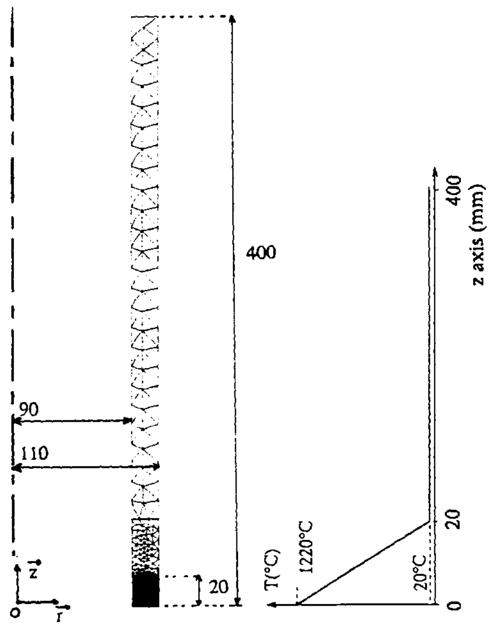

Figure 7 : mesh and initial/final fictitious temperature fields used for the calculations of the pipe :
1/ initial high temperature, pipe cooled to 20 °C.
2/ initial temperature is 20 °C, pipe heated to maximum temperature and then cooled.

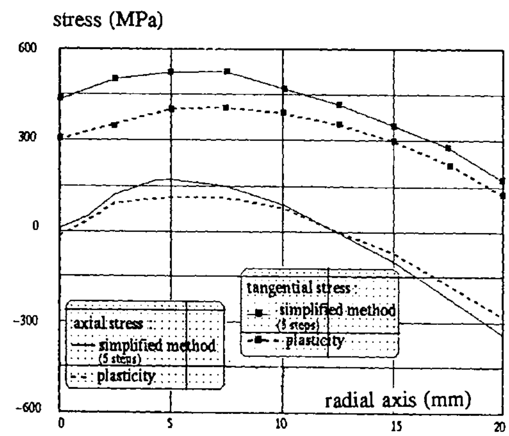

Figure 8 : comparison of tangential and axial stresses obtained with the simplified method and plasticity algorithm, on a pipe initially at high temperature without stresses, and then cooled.

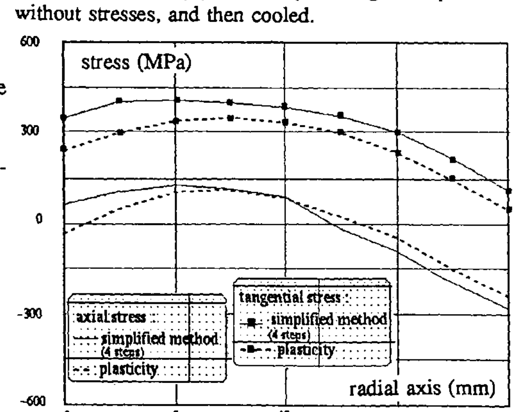

Figure 9 : comparison of tangential and axial stresses obtained with the simplified method and plasticity algorithm, on a pipe initially at room temperature without stresses, and then heated and cooled.