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Piping elastic calculations with a method allowing for section ovalization

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ABSTRACT : This paper briefly presents the simplified method and associated references, then describes its implementation in finite element code (CASTEM 2000 : reference /8/). Several significant results are compared with "shell" and "beam" type calculations.

1 INTRODUCTION

The bending of thin-walled pipes results in a deformation of sections, which is propagated to neighbouring straight portions. The term of order 2 of this deformation (ovalization) is preponderant and is the source of additional bending in elbows. This phenomenon is accounted for in traditional theories by applying a flexibility factor.

This deformation, which is contrary to the assumption of non-deformability of sections in the beam theory is, however, taken into account in a simplified manner in classical piping designs based on this theory, by the utilisation of flexibility factors and stress intensification factors. However, as a result of this, stresses are generally overestimated (reference /3/). These classical coefficients, which result from work on "infinite" elbows (references. /1/ and /2/), (i.e. elbows with no adjacent straight portions and with uniform loading) processed by shell-type calculations, do not take into account the propagation of ovalization along the piping, and in particular, in the straight portions in the vicinity of the elbows.

In reference /4/, an analytical solution to this problem was proposed for straight portions, and the problem of elbows was processed numerically. This study showed that it was essential to take the warping of the section into account.

In references /6/ and /7/, the basic theory of ovalization propagation is extended to the general case of an elbow or a straight portion subjected to any load. This allows the simplified method to be redefined in order to deal with all load and geometry situations. Comparisons with finite element calculations, firstly of thin shell type elements, and secondly with classic beam type elements, allow assessment of this method's performances.

2 NOTATIONS

O ovalization matrix

K_A stifness matrix (ASME flexibility in elbows)
K stifness matrix (elbows processed as straight beams)
U displacement matrix
F load matrix
B generalized displacement - strain relation matrix (curvatures)
D Hooke's matrix
M generalized stresses matrix (forces and moments)
C curvature flexibility-ovalization relation matrix (defined in references/6/and /7/)
P bending moment-ovalization relation matrix : relation between ovalization and bending moments in elbows of all pipe line (defined in references /6/ and /7/)
a iterative calculation relaxation factor
ε ovalization precision at convergence

3 REMINDER ON THE FORMULATION OF THE SIMPLIFIED METHOD

Classic finite element pipe calculations are reduced to the resolution of:

(1) $K_A U = F$

Our method introduces elbow flexibility which depends on section ovalization.

The equations to be resolved then becomes :

(2) $KU - B^T DCO = F$

The first term describes the flexibility of straight beams, whereas the second term corresponds to the additional flexibility of elbows compared to straight beams of identical cross-section (directly associated with ovalization).

Pipe ovalization is calculated based on line elbows moments, with :

(3) $O = PM$

with M expressed as a function of U:

(4) $M = D(BU - CO)$

we obtain :

(5) $PDBU - (PDC + 1)O = 0$

The details of matrixes P and C are given in references /6/ and /7/.

Instead of equation (1), the system comprised of equations (2) and (5) will be resolved. If O is eliminated, this system can be reduced to :

(6) $K_V U = F$

with $K_V = K - B^T DC(PDC + 1)^{-1} PDB$

Equation (6) could be directly resolved. However, in order to allow for the extension of plasticity, we deliberately chose an iterative resolution method using equations (2), (3) and (4).

Stresses calculations are based on forces, moments and ovalization values.

4 IMPLEMENTATION OF THE SIMPLIFIED METHOD UNDER ITERATIVE FORM IN "BEAM" TYPE FINITE ELEMENT CODE

CASTEM 2000 (reference /8/) programming was performed under the following iterative form:

Initialisation

$O_0 = O$

$i = O$

→ Calculation of U_i by resolving:

- if $i = 0$ $\quad K_A U_0 = F$
- if $i > 0$ $\quad K U_i = B^T D C O_i + F$

$M_i = D (B U_i - CO_i)$

$i = i + 1$ $\quad O = P M_i$

$O_{i+1} = (1 - a) O + a O_i$

convergence test:

no ← $|O_i - O_{i+1}| - \varepsilon | O_{i+1} | < 0$

yes ↓

Calculation of stresses

5 PRESENTATION OF SEVERAL SIGNIFICANT RESULTS

Many comparisons of shell and beam type calculations with our simplified "VICTUS" method are presented in references /6/ and /7/.

The implementation of our method in finite element code permitted more extensive comparisons.

It seemed interesting to make flexibility comparisons in several simple loading situations which are presented in the following table.

Results on a VICTUS elbow :

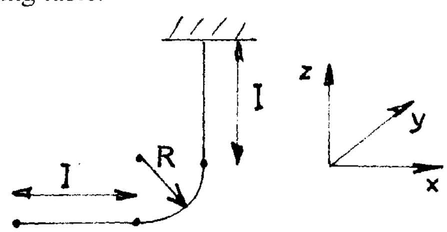
$\lambda = eR / r^2 = .224$

$R = 305$ mm

$I = 1\ 200$ mm

$e = 8.18$ mm

$r = 105.5$ mm



Displacements in mm at the free end

		Beam	Shell	VICTUS
In plane bending My = 10^6 daN x mm	dz dx	10.89 - 1.83	11.44 -2.11	11.51 -2.37
Out of plane bending and torsion Mz = 10^6 daN x mm	dy	8.13	9.28	9.31
Pure torsion in elbow Mx 10^6 daN x mm	dy	5.28	7.86	7.54
In plane bending : moment inversion in elbow	dz dx	.887 -.887	1.266 -1.266	1.271 -1.271

The three calculations are very close in the case of in plane bending. In other cases, due to a more precise evaluation of flexibility, our method is very close to shell calculations, whereas beam calculations deviate from the latter.

The calculation of a complete pipe line evidences even more obviously the superiority of our method compared to the classical "beam" calculation method, the "shell" calculation method being considered as the reference.

The calculation of a three-dimensional pipe line submitted to imposed displacement is presented in Figures 1 to 5.

Figure 1 shows the geometrical characteristics of this pipe (VICTUS loop No. 3).

Figure 2 shows the meshing for the shell calculation under imposed displacement at pipe end (dx = 27.9 mm). The deformed structure (calculated with the VICTUS method) is given by figure 3 (displacement amplification: 50).

Figure 4 shows the excellent "shell-VICTUS" correlation for ovalization (the "beam" method does not calculate this).

Figure 5 illustrates the "shell-beam-VICTUS" stresses calculation comparison. As already evidenced in references /6/ and /7/ examples, the traditional "beam" method overestimates stresses, in particular in elbows submitted to complex loads, whereas our simplified method gives satisfactory results. It also provides more realistic results at straight portion-elbow junctions, which are a particularly delicate zone in pipe design since they usually include welds.

6 CONCLUSIONS

The implementation of the simplified VICTUS pipe calculation method in finite element code (CASTEM 2000 : reference /8/) now permits calculation of industrial pipe lines. Tests performed confirm results obtained on the elbows presented in references /6/ and /7/, the results of which were far better than classical "beam" calculations and very close to shell calculations.

Work currently under way on this simplified method involves the study of plasticity (references /9/ to /13/). A global plasticity value adapted to specific pipe loads is determined on the basis of elasticity described herein and in references /6/ and /7/.

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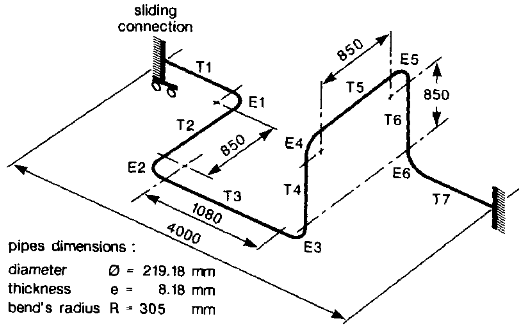

Figure 1 : VICTUS – TEST SECTION 3

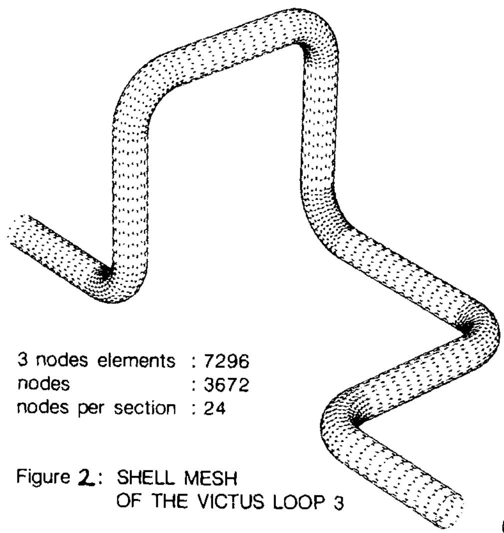

Figure 2: SHELL MESH
OF THE VICTUS LOOP 3

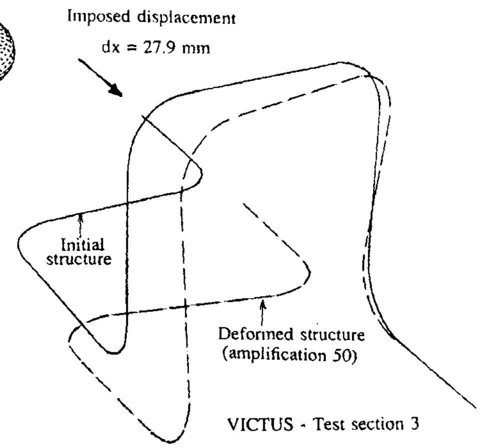

Figure 3 : Beam deformed structure (VICTUS meth

Figure 4 : Shell-VICTUS comparison
Ovalization

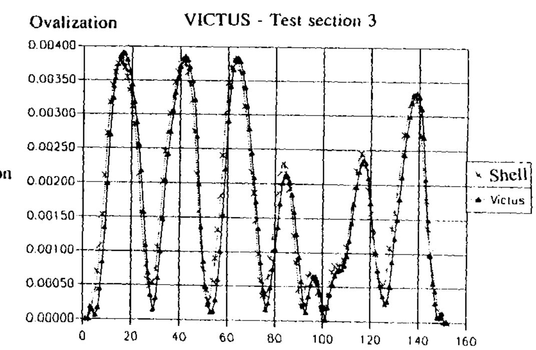

Figure 5 : Shell-beam-VICTUS comparison
Maximum stress