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Structural analysis of non-uniformly perforated plates

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ABSTRACT : This paper summarizes a research in the field of structural analysis of non-uniformly perforated plates. Tube support plates of steam generators in Belgian nuclear unit of Doel 4 are perforated by two sets of holes of different diameters, that make their pattern a non uniform square pattern, which is not covered by classical theories of perforated plates. The methodology proposed by these theories was applied and adapted for finite element calculations, showing the influence of ligament efficiency and thickness on equivalent plate properties.

1. Introduction

During the 1990 plant outage of Doel 4, OD axial Stress Corrosion Cracking(SCC) has been detected on some Steam Generator (SG) tubes. The cracks are located at the top level of Tube Support Plates (TSP) 2, 3 and 4.

During normal operating conditions of the plant, the cracks are confined in the TSP and, as a consequence, there is no associated risk for the tube to burst. While in accidental conditions (Feed Water or Main Steam Line Breaks), the TSPs deflect due to the resultant pressure differential which develops in the SG, and cause the axial cracks to be partially or even totally disengaged. The calculated TSPs deflection was one of the factors taken into account in the establishment of the plugging criterion for the affected tubes[4].

The TSPs in Doel 4 are perforated by circular holes of different diameters, leading to a non-uniform pattern with flexure properties very different from those of a solid plate.

2. Classical patterns & theories

O'Donnell [1] is one of the firsts to present formulations as well for thin perforated plates than for thick perforated plates, as well for square than for triangular patterns. These results are still used in the concept of the equivalent solid plate.

For thin plates, he considered the problem of perforated plates under plane stress conditions. It can be observed that perforated plates with triangular patterns are isotropic and then can be described by the equivalent properties E^*, ν^* and G^*, where $G^* = E^*/2(1+\nu^*)$. Square patterns exhibit an orthotropic behaviour with two preferred directions : the *Pitch* (P) direction and the *Diagonal* (D) direction. O'Donnell showed that such plate can be described by four equivalent characteristics : E_d^*, ν_d^*, E_p^*, ν_p^* related by :

$$(1-\nu_p^*)/E_p^* = (1-\nu_d^*)/E_d^* \qquad (1)$$

It was shown that, for a given ligament efficiency, equivalent E^* depends on E but not on ν and that equivalent ν^* depends on ν but not on E.

For tubesheets and pressure vessel heads, where the thickness of the plate is thick relative to the diameter of the perforations, O'Donnell introduced the concept of 'perforated plates under generalized plane strains conditions' for plates subjected to in-plane loading. He assumed that the effective elastic constants are the same for in-plane loading, bending or torsion.

Classical theories cover thus only the range of uniform square or triangular holes patterns, especially for thick plates like the tubesheet ([1],[2],[6]). They are not applicable for non-uniform patterns and a research has been undertaken in TEE to evaluate the mechanical properties of an equivalent solid support plate by applying the methodology of classical theories through membrane behaviour calculations. A special 3D-bending calculation was also developed to evaluate its bending properties. Finally, results for support plate in Doel 4 were integrated into a large spatial plate model of the steam generator coupled with dynamic simulation of a Steam Line Break. An estimation of the maximal displacements of support plates was finally found.

3. Membrane tests

TSPs in Doel 4 are perforated by two series of holes arranged in a double square pattern (fig.1). The first series consists of 9832 tubes holes (ϕ =19.45mm) and the second series contains 9910 through-water holes. These holes are smaller (ϕ =12.70mm), making the pattern a *non-uniform double square pattern*. The smallest ligament (equal to 3.32mm) is located between one big and one small hole

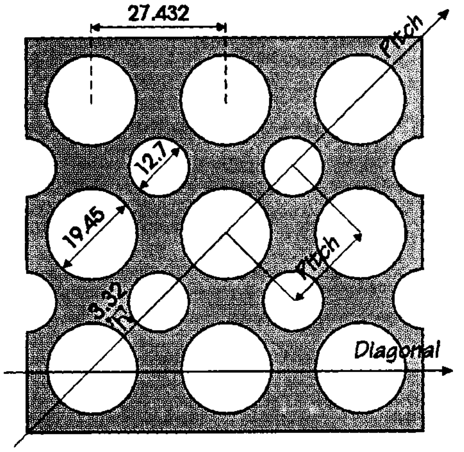

figure 1 : real plate of Doel 4 with non-uniform double square pattern

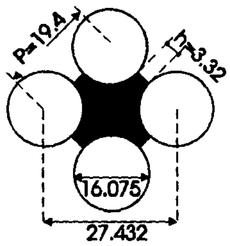

fig.2a : 'most equivalent' uniform square pattern

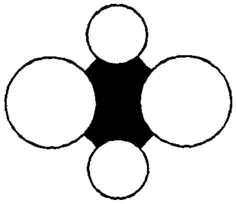

fig.2b : pattern in pitch direction

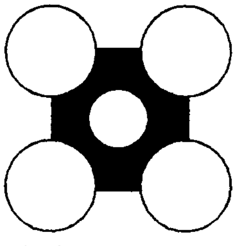

fig.2c : pattern in diagonal direction

- In order to have a preliminary idea of the elastic characteristics of the equivalent plate, the pattern can be slightly modified to go back to classical theories of uniform square patterns. By looking pattern of figure 1 with a angle of 45° and by letting small holes growing and big holes shrinking, we then obtain holes of equal diameter ϕ=16.075mm with a pitch P of 19.4mm and a ligament equal to 3.32mm (fig.2a), leading to ligament efficiency η=0.17. Effective elastic properties given in [1] are interpolated in table 1 for what is called the *'most equivalent uniform square pattern'*.

- For our *real pattern* (fig.2b-in Pitch dir.), a finite element calculation on a real plate model (fig.3) with SYSTUS code gives a more accurate evaluation of real membrane properties. By applying a uniform tension σ_x at one edge of the model, with the opposite edge constrained (Ux=0), elastic properties can be determined if displacements Ux et Uy are measured with use of following relations :

$$\varepsilon_x = \frac{U_x}{X} = \frac{\sigma_x}{E_p^*} \quad \textit{and} \quad \varepsilon_y = \frac{U_y}{Y} = \frac{-\nu_p^* . \sigma_x}{E_p^*} \tag{2}$$

The same method can be applied in the diagonal direction(fig.2c). It can readily observed in table 1 that elastic properties in pitch direction are weaker than those obtained with artificial holes of equal diameter, while they are comparable in diagonal direction. It has been verified that those elastic orthotropic properties, whenever applied to a solid equivalent plane, give the same results than results obtained with real plate models. Stiffness matrices of solid plates are found by applying following relations in Pitch direction (dual formulas in Diagonal direction) :

$$\begin{aligned} \sigma_x &= \frac{E_p^*}{(1-\nu_p^{*2})}(\varepsilon_x + \nu_p^* . \varepsilon_y) \\ \sigma_y &= \frac{E_p^*}{(1-\nu_p^{*2})}(\varepsilon_y + \nu_p^* . \varepsilon_x) \\ \tau_{xy} &= \frac{E_d^*}{2(1+\nu_d^*)} . \gamma_{xy} \end{aligned} \tag{3}$$

- Displacements obtained in solid plates are in good agreement with displacements in real plates but it is still possible to improve their adequacy by modifying elastic properties to get exactly the same results. Moreover, it was observed that there was still a small difference between displacements obtained with pitch properties introduced at 0° with those obtained with diagonal properties. These differences come from the fact that we don't have strict equality (1). From these observations, we finally get equivalent properties for '*real pattern adjusted*'.

Table 1 : Membrane approaches for non-uniform square pattern of Doel 4

Case	η	E_p^*/E	E_d^*/E	ν_p^*	ν_d^*
most uniform pattern	0.17	0.27	0.09	0.13	0.7
real pattern	0.17	0.166	0.11	0.45	0.55
real pattern adjusted	0.17	0.157	0.12	0.49	0.61

- With the same methodology, calculations are undertaken on a series of 2D-models with different non-uniform patterns. For each of them, the size of small holes is kept constant (ϕ=12.7mm) while big holes diameter is taken as parameter, fixing the ligament efficiency. For the special case where holes have the same diameter (ϕ=12.7mm), the pattern is uniform with η=0.35. Effective elastic constants for that ligament efficiency are given by Meijers[3]. These values are superposed in figure 4 with non-filled symbols. The concordance is excellent. In the range of ligament efficiencies examined, the behaviour of effective elastic constants looks globally like the one of a uniform pattern though ν_p^* contrasts from uniform patterns[1].

4. Bending tests

A classical plate behaviour yields relations between principal bending and torsion moments with principal curvatures :

$$\begin{bmatrix} M_{xx} \\ M_{xy} \\ M_{yy} \end{bmatrix} = \begin{bmatrix} D_{11} & 0 & D_{13} \\ 0 & D_{22} & 0 \\ D_{13} & 0 & D_{11} \end{bmatrix} . \begin{bmatrix} \chi_{xx} \\ \chi_{xy} \\ \chi_{yy} \end{bmatrix} \tag{4}$$

Literature[2] suggest that bending stiffnesses are related to properties in membrane with formulas similar to those of an isotropic material. In Pitch direction, formulas are (dual formulas for Diagonal direction) :

$$D_{11} = \frac{E_p^* . h^3}{12(1 - \nu_p^{*2})} \; ; \quad D_{22} = \frac{E_d^* . h^3}{12(1 + \nu_d^*)} \; ; \; D_{13} = \nu_p^* . D_{11} \tag{5}$$

A methodology has been developed on a fine 3D-model of only one pitch of the pattern (fig.5a pitch, 5b diagonal). To avoid the problem of proximity of loading forces to the nodes where the displacement is measured, a rotation is imposed to one face while the opposite one is constrained. This permits also to simulate an infinite plate with only one pitch of the pattern. By measuring reaction moments on constrained faces, it is possible to get matrix **D** terms and deduce equivalent properties E_d^*, ν_d^*, E_p^* and ν_p^* for comparisons from (5).

- This method has been verified for same diameter holes pattern. Bending stiffnesses D_{11} and D_{13} (for E=210000MPa, ν=0.3) are compared with expected values from formulas (5). The differences between both approaches are sufficiently weak (<10%) to get confidence in the methodology proposed (see table 2).

Table 2 : Bending approaches for uniform square patterns

dir		D_{11} [Nm]	D_{13} [Nm]
pitch	formulas (5)	43806	61333
pitch	3D test	45072	59490
diagonal	formulas (5)	30870	20065
diagonal	3D test	34424	20310

Table 3 : Bending approaches for NON uniform square patterns

dir		D_{11} [Nm]	D_{13} [Nm]
pitch	formulas (5)	33484	16407
pitch	3D -fig.5a	39288	7857
diagonal	formulas (5)	30973	21371
diagonal	3D- fig.5b	28233	19199

- Finally, this methodology is applied to our real pattern and results are expressed again in terms of D_{11} and D_{13}. This time, differences between 3D models and theoretical formulas are a little larger, especially for term D_{13} in pitch direction (see table 3).

- With the same method, calculations are undertaken on 3D-models with different non uniform patterns. For each of them, the size of small holes is kept constant (ϕ =12.7mm) while big holes diameter is taken as parameter, fixing the ligament efficiency. In 3D-bending tests, different thickness ratios are also examined. Results are presented graphically in figure 6. It is observed, as shown for uniform patterns [2,5,6], that (a) effective elastic constants vary a lot with thickness in diagonal direction, (b) they don't vary very much with thickness in pitch direction , (c) they are quasi constant above thickness ratio t/P=2, (d) effective elastic properties are relatively good uncoupled vis-à-vis the influence of t/P and h/P.

For the special case where holes have the same diameter ($\phi=12.7$mm), the pattern is uniform with $\eta=0.35$. Effective elastic constants for that ligament efficiency found in membrane in chapter 3 are superposed on figure 6. It is seen that these values obtained in *membrane* correspond quite well with elastic constants obtained with 3D-models in bending in the domain of *thick* plates.

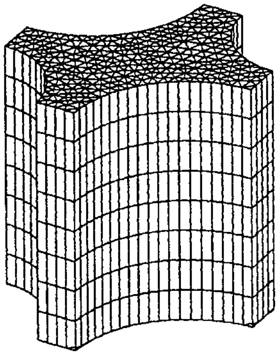

fig.5a : 3D model
- pitch dir.
- bending test

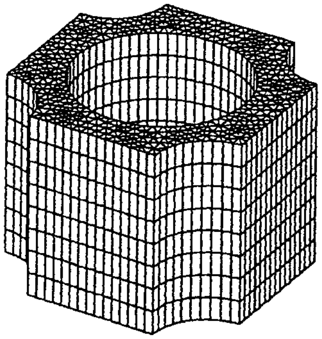

fig.5b : 3D model
- diagonal dir.
- bending test

5. Conclusions

A structural analysis has been undertaken to evaluate effective elastic properties of *non-uniformly* perforated plates with various thicknesses and ligament efficiencies. A methodology has been developed and applied to evaluate these properties in membrane and in bending. Main conclusions are :

(a) perforated plates with a non uniform square pattern behave qualitatively like these with a uniform pattern but
(b) they exhibit some quantitative differences, especially in bending for pitch direction.
(c) for thick plates (t/P>2), elastic properties in bending are quasi constant and can be obtained more easily by membrane tests.
(d) for thin plates (t/P<0.25), results in bending need future investigations.
(e) in the transition zone, no rules can be applied for uniform and non uniform square pattern; elastic characteristics must be calculated by finite elements for each specific case.
(f) in membrane, the evolution of Poisson's ratio ν_p^* with ligament efficiency differs from uniform pattern.

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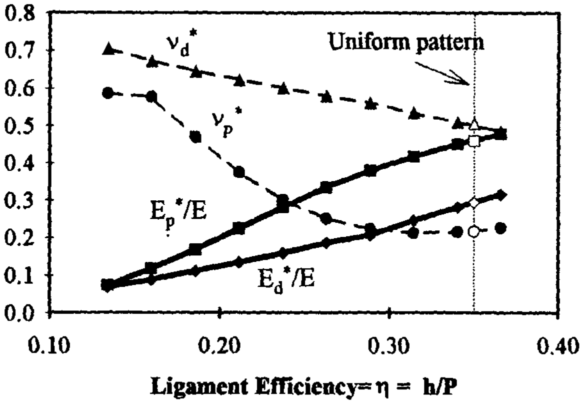
fig.4 : effective elastic constants for NON uniform pattern - membrane tests



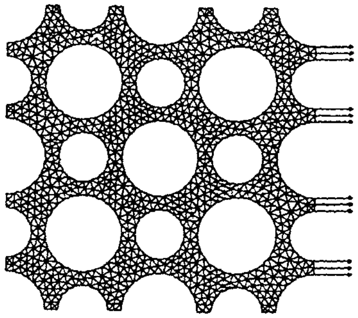


fig.3 : real 2D plate (pitch direction)

fig.6 : effective elastic constants in for NON uniform square patterns - bending tests