Study on a concrete filled steel structure for nuclear power plants (part 4). Analytical method to estimate shear strength

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ABSTRACT: A macroscopic model for calculating the maximum strength of concrete-filled steel shear walls with H-shaped horizontal cross sections are developed. According to the lower bound theory of limit analysis, this model assumes that both a truss and an arch mechanism are coexisting. When applied to the experiment specimen in this study (Part 3), a concrete-filled shear wall with an H-shaped cross section, the maximum strength could be predicted successfully.

1. INTRODUCTION

Analytical studies include those by Hiroyuki Suzuki et.al[1] and the authors'[2] regarding models for calculating maximum strength. The Suzuki's model concerns SC walls with rectangular horizontal cross sections, and assumes an arch mechanism which allows for compression in the concrete and tension in the steel plates. The authors' model also concerns SC walls with rectangular cross-sections, but is for calculating shear strength in the case when flexural reinforcement is infinite, and assumes both an arch mechanism and a truss mechanism are coexisting.

In actual SC walls, lateral reinforcement is finite. Also, not only rectangular cross-sections, but also the use of those with columns at either end or of perpendicular wall members is thought to be required. Therefore, this study gives a model for calculating...
the maximum strength of SC walls with H-shaped cross-sections, with allowance for the yield of lateral reinforcement.

2. ANALYSIS SUBJECT

The subject of the analysis is an SC wall with an "H" shaped horizontal cross-section, as shown in Figure 1. A rigid loading beam and a base are attached to the upper and lower portions, adding vertical axial force and horizontal force to the loading beam.

3. ANALYTICAL MODEL

The model is based on the lower bound theory of limit analysis and is assumed that both a truss mechanism and an arch mechanism are coexisting. The following suppositions are made concerning the concrete and the steel plates.

1. Since tensile stress causes cracks to form in the concrete, it is supposed that it only bears compressive stress.

2. Wrinkles form in the steel plate at web owing to compressive stress. It may be possible that at a later stage a certain amount of compressive stress may be borne, but, for simplicity's sake, it is imagined that only tensile stress is supported.

3.1 Truss Mechanism

Figure 2 depicts the assumed force distribution in the truss mechanism. This structure is composed of diagonal concrete compression struts and steel web tension struts perpendicular to these, and these two forces are balanced with the vertical force of the steel flanges ("flange plate-1" and "flange plate-0"). The conditions for equilibrium are as given below. For an explanation of symbols, refer to the end of this report. For the segmentation and labeling of the steel plates, refer to Figure 1.

The supported shear stress \( \tau Q \) and the axial force \( \tau N \) are as in the following expressions:

\[
\begin{align*}
\tau Q &= t_x D_u p_o \sigma_\phi \cot \phi_\omega + 2t_f D_f p_f \sigma_f \cot \phi_f \\
\tau N &= t_x D_u p_o \sigma_\phi (\cot^2 \phi_\omega - 1) + 2t_f D_f p_f \sigma_f (\cot^2 \phi_f - 1)
\end{align*}
\]  

(1)  

(2)

The equilibrium of vertical force in flange plates- o and i is given by the following:

Fig.2 Truss Mechanism
\[ t_f H p_f \sigma_f (\cot^2 \phi_f + 1) \sin \phi_f \cos \phi_f = (1 - \kappa) S_u \]
\[ t_u H p_u \sigma_u (\cot^2 \phi_u + 1) \sin \phi_u \cos \phi_u - t_f H p_f \sigma_f (\cot^2 \phi_f + 1) \sin \phi_f \cos \phi_f = (1 - \kappa) S_i \]  

Here the bond between the concrete and the flange plates is assumed to be sufficiently strong.

The following stress limitations owing to material strength are given for the concrete, the web plates-\( f \) and \( w \), and the flange plates-\( o \) and \( i \):

\[ \varepsilon \sigma_u = p_f \sigma_f \cot^2 \phi_f \leq f_c = \nu \sigma_u \]  
\[ \varepsilon \sigma_w = p_w \sigma_w \cot^2 \phi_w \leq f_c = \nu \sigma_u \]  
\[ p_f \sigma_f \leq p_f \sigma_f \]  
\[ p_w \sigma_w \leq p_w \sigma_w \]  
\[ S_u \leq A_f \sigma_f \]  
\[ S_i \leq A_i \sigma_i \]  

3.2 Arch Mechanism

An arch mechanism as given in Figure 3 is assumed to be formed from the concrete and steel plates remaining from the truss mechanism. It is taken into consideration that, regarding the concrete compression strut, the direction of the force changes at the boundary between the web and the flanges [3].

In the bottom horizontal section, axial force, shear stress, and bending moment are balanced as given by the following expressions:

\[ T_s - C_s + \gamma_a t_f f_c x \sin^2 \theta_s - \gamma_a t_f f_c D_f \sin^2 \theta_2 + N_s \cos \theta_s + N_t = 0 \]  
\[ A Q - A_t f_c x \cos \theta_s - \gamma_a t_f f_c D_f \sin \theta_2 \cos \theta_2 - N_s \sin \theta_s = 0 \]  
\[ A Q H_f - \frac{D}{2} T_s - \frac{D + D_f}{2} P_s - A_t f_c x \frac{D - D_f - x}{2} \sin^2 \theta_s - \gamma_a t_f f_c D_f \frac{D}{2} \sin^2 \theta_2 \]  
\[ -N_s \frac{H}{2} \sin \theta_s = 0 \]  

The equilibrium between the axial force and the shear stress in the top horizontal section is given by the following expressions:

\[ \text{Fig.3 Arch Mechanism} \]
\[ T_i - C_s + P_i - \Delta t_s f_c x' \sin^2 \theta_1 - \gamma' \Delta t_s f_c D_f \sin^2 \theta_3 + N_c \cos \theta_4 + \Delta N = 0 \]  
(14)

\[ \Delta Q - \Delta t_s f_c x' \sin \theta_1 \cos \theta_1 - \gamma' \Delta t_s f_c D_f \sin \theta_3 \cos \theta_4 - N_c \sin \theta_4 = 0 \]  
(15)

The equilibrium of the bending moment in the top horizontal section is satisfied automatically, if the equations (11) through (15) are satisfied.

The equilibrium of the vertical and horizontal forces at the boundary between the tensioned web and flange is

\[ P_i = \gamma' \Delta t_s f_c y' \sin \theta_1 \cos \theta_3 - \Delta t_s f_c y' \sin \theta_1 \cos \theta_1 \]  
(16)

\[ \gamma' \Delta t_s f_c y' \cos^2 \theta_3 = \Delta t_s f_c y' \cos^2 \theta_1 \]  
(17)

The equilibrium of the vertical and horizontal forces at the boundary between the compressed web and flange is

\[ P_i = \gamma' \Delta t_s f_c y \sin \theta_1 \cos \theta_2 - \Delta t_s f_c y \sin \theta_1 \cos \theta_1 \]  
(18)

\[ \gamma' \Delta t_s f_c y \cos^2 \theta_2 = \Delta t_s f_c y \cos^2 \theta_1 \]  
(19)

At the top section, the flange plate-i sustains the component \( T_{i'} \) of force \( T_i \), as well as force \( P_i \) that changes the direction of the strut. Flange plate-o sustains component \( T_{o'} \) of force \( T_o \). For these values there exists a limit in yield strength.

\[ P_i + T_{i'} \leq S_p - \kappa S_i \]  
(20)

\[ T_{o'} \leq S_o - \kappa S_o \]  
(21)

Here, \( T_i + T_{o'} = T_i \cdot T_{o'} = (S_p - \kappa S_i)(S_o - \kappa S_o) \)  
(22)

Similarly, the conditions in the bottom section are as follows:

\[- S_p + S_o \leq P_o - C_{o'} \leq S_p - S_i \]  
(23)

\[ C_{o'} \leq S_o - S_o \]  
(24)

Here, \( C_{o'} = C_s + C_{o'} \cdot C_{o'} = (S_p - S_i)(S_o - S_o) \)  
(25)

3.3 Strength

Strength \( Q \) is a combination of the shear stress \( \gamma Q \) supported by the truss mechanism and the shear stress \( \alpha Q \) supported by the arch mechanism.

4. SOLUTION

The number of combinations of solutions that satisfy the above conditions to achieve equilibrium is unlimited. However, as the basis is the lower bound theory, the solution giving the largest strength is the correct solution.

The number of unknowns is large compared to the number of equilibrium equations, and thus several values have to be assumed for a solution. In this instance, a total of five unknowns are assumed, namely, in the truss mechanism, the angles \( \phi_f \) and \( \phi_w \) describing the concrete force, and stresses \( \sigma_f \) and \( \sigma_w \) in the web plates, and in the arch mechanism, the steel plate's strut angle \( \theta_4 \).

Limitations of 0° through 90° exist for \( \phi_f \) and \( \phi_w \), 0 through yield strength for \( \sigma_f \) and \( \sigma_w \), and 0° through tan-1(H/(Dw+2Df)) exist for \( \theta_4 \).
The computational process utilized was to take values for these parameters and to vary them little by little within the limitations to find the maximum strength.

Note that, in principle, the force \( T_r \) in the tensioned flange is assumed as yield strength, and it may be that no solution exists if the amount of flange plates is large. In this case, the computation is repeated, and \( T_r \) is gradually lowered until the solution is found.

As it is impossible to solve the equilibrium equations by means of variables, the iterative calculation technique is employed.

5. APPLICATION CASE

The analytical model was applied, using the SC wall member with an H-shaped cross section reported in this study (Part 3) as an experimental specimen.

Within the confines of the analytical model the direction of the concrete struts in the truss mechanism may be free within the range of 0° through 90° as given above, but in reality, owing to limitations in the capacity of shear transfer at cracks and in the bond between the flange and the concrete, it is expected to be subject to added limitations. In the experiment, the value was indicated to be 45°. Hence the analytical results for when the angle is fixed at 45° are given in Table 1. The maximum strength does not change largely, but it is noticeable that the ratio of burden-sharing between the truss mechanism and the arch mechanism differs.

6. CONCLUSIONS

This report has been concerning a macroscopic model for calculating the maximum strength of SC wall members with H-shaped horizontal cross sections. Matters such as the direction of the concrete struts in the truss mechanism remain, but it was possible to successfully predict the strength of an experimental specimen with an H-shaped cross section.

<table>
<thead>
<tr>
<th>Table 1 Comparison between Experimental and Analytical Results</th>
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<tbody>
<tr>
<td><strong>Experiment</strong></td>
</tr>
<tr>
<td>**</td>
</tr>
<tr>
<td>H1OT10</td>
</tr>
<tr>
<td>H1OT10V</td>
</tr>
<tr>
<td>H5OT10</td>
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<tr>
<td>H5OT10S</td>
</tr>
<tr>
<td>H1OT15</td>
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<tr>
<td>H07T10</td>
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</tbody>
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SYMBOLS INDEX

Dimensions in the direction of the load are classified as "length," and as "width" in the direction perpendicular to the load. Therefore, note that the "length" and "width" of the flanges are expressed in reverse to custom.

Concerning dimensions
\( D_w \): web length, \( D_f \): flange length, \( D \): central flange distance, \( H \): clear height, \( H_f \): load point height of horizontal force, \( \kappa : (H_f - H) / H_f \), \( t_\text{w} \): concrete thickness at web, \( t_f \): concrete thickness at flange, \( t_\text{w} \): thickness of (one) web \( w \), \( p_w : 2, t_\text{w} / t_\text{w} \).
Concerning the truss mechanism

\(\tau Q\): supported shear stress, \(\tau N\): supported axial force, \(\phi_u\): strut angle from the vertical at web concrete, \(\phi_f\): strut's angle from the vertical at flange concrete, \(\varepsilon \sigma_f\): compressive stress of concrete at flange, \(\varepsilon \sigma_{w}\): compressive stress of concrete at web, \(\sigma_{u}\): unit tensile stress in web plate, \(\sigma_{f}\): unit tensile stress in web plate, \(S_o\): supported axial force of flange plate-o, \(S_i\): supported axial force of flange plate-i

Concerning the arch mechanism

\(\Delta Q\): supported shear stress, \(\Delta N\): supported axial force or \(N=N_{u}\), \(T_{b}\): bottom section tension on tensile flange, \(C_{s}\): bottom section compression in compressed flange, \(p_{b}\): force changing the concrete strut direction at flange and web border on compressed side, \(\Delta t_{u}\): equalized thickness of web concrete, or \(t_{u}(1-\varepsilon \sigma_{u}/f_{c})\), \(\Delta t_{f}\): equalized thickness of flange concrete, or \(t_{f}(1-\varepsilon \sigma_{f}/f_{c})\), \(x, x', y, y'\), \(\theta_1, \theta_2, \theta_3\): dimensions of the concrete strut (see Figures), \(\theta_{A}\): angle of tensile force with vertical in web, \(N_{w}\): tensile force in web plates, or \(t_{u}(D_{w}-H \tan \theta_{1})(p_{w} \sigma_{w} - p_{u} \sigma_{u}) + 2t_{f}D_{f}(P_{f} \sigma_{f} - p_{f} \sigma_{f})\), \(\gamma t_{f}\): valid width of compressed flange, \(T_{s}\): top section tensile force in tensioned flange, \(C_{s}\): top section compressive force in compressed flange, \(p_{i}\): force changing direction of concrete strut at flange and web border on tensioned side, \(\gamma ' t_{f}\): valid width of flange on tensioned side

REFERENCES