Evaluation of sub-surface motion from specified free-field motion

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ABSTRACT: The 1-D wave propagation problem has been solved by Fourier Transform to obtain subsurface motion from free-field motion. Results of some case studies are presented in the paper.

1. INTRODUCTION

Seismic design basis motion is generally prescribed at free field. Evaluation of sub-surface motion may become necessary to assess the seismic response of embedded structures or structures with deep foundation. It may also be useful for various other geophysical studies. Earlier work of [Schnabel et al., 1972] on the evaluation of base rock motion from known ground motion is based on a Fourier series synthesis of the steady state solution assumed to be of the form of a sinusoid. This paper presents an analysis based on Fourier Transform approach and the results of various parametric studies on amplification/attenuation of the free field motion.

2. THEORY

The surface motion is assumed to be due to upward travelling shear waves from underlying rock formations. The subsurface medium is modelled as a combination of various viscoelastic layers (Fig.1). The Fourier transform of the equation of motion can be written as

\[
\left( G_j + i \eta_j \omega \right) \frac{d^2 \tilde{U}_j}{dz^2} + \rho_j \omega^2 \tilde{U}_j = 0; \quad i = \sqrt{-1} \tag{1}
\]

where \( \tilde{U}_j \) is the Fourier transform of the horizontal displacement \( u_j \) of the \( j \)th layer. The solution to equation (1) is

\[
\tilde{U}_j = E_j \exp(ik_j z) + F_j \exp(-ik_j z) \tag{2}
\]

The complex shear wave velocity, \( V_j \) is expressed as

\[
V_j = \left( G_j + i \eta_j \omega \right)/\rho_j = G_j (1 + 2i \beta_j)/\rho_j; \quad k_j = \omega/V_j \tag{3}
\]

where \( \beta_j \) is the fraction of critical damping for the \( j \)th layer.
At the interface of the jth and the (j+1)st layers, i.e., \( z=h \) (for the jth layer) and \( z=0 \) (for the (j+1)st layer) displacement and the shear stress and the corresponding Fourier transforms will be continuous. Thus,

\[
2 E_j = E_j (1+\alpha_j) \exp(ik_j h_j) + F_j (1-\alpha_j) \exp(-ik_j h_j) \tag{4}
\]

\[
2 F_j = E_j (1+\alpha_j) \exp(ik_j h_j) + F_j (1-\alpha_j) \exp(-ik_j h_j) \tag{5}
\]

where \( \alpha_j = \frac{\phi_j}{(F_{j+1} V_{j+1})} \)

Equations (4) and (5) permit recursive evaluation of the constants E and F from those of the previous layers.

Since, the top of the first layer is stress-free,

\[
E_1 = F_1 \tag{6}
\]

The Fourier transform of the acceleration in the jth layer is,

\[
\int_0^\infty d\omega \tilde{u}_j \exp(-i\omega t) dt = (i\omega)^2 \tilde{u}_j \tag{7}
\]

Then the ratio of the Fourier transforms of the acceleration on top of the (n+1)st layer and on the free-ground is the transfer function \( T_{\eta} \) for the acceleration at the former location.

\[
T_{\eta} = \tilde{u}_{\eta+1}(z=0) / \tilde{u}_1(z=0) = \left( E_{\eta+1} + F_{\eta+1} \right) / 2 E_1 ; \quad \therefore E_1 = F_1 \tag{8}
\]

\[
\tilde{u}_{\eta+1}(z=0) = T_{\eta} \tilde{u}_j \tag{9}
\]

\( \tilde{u}_j \) is computed from the known time-history, \( u_j(t) \)free-ground. Then the acceleration on top of the (n+1)st layer is obtained by inverting \( \tilde{u}_{\eta+1}(z=0) \). Subsequently, the pseudo-absolute acceleration response spectrum of \( u_{\eta+1} \) is evaluated.

3. NUMERICAL ANALYSIS

3.1 Response of a Four-Layer system

Fig.2 compares the variation of peak acceleration with frequency at two different values of depth for a two-layer and a four-layer system. The material properties considered are given in Table-1. Attenuation is feasible in a frequency band of about 5-12 rad/sec. It is also seen that the attenuation/multiplication is not monotonically varying with depth or frequency. The effect of layering seems to be marginal. Fig.3 shows a comparison of the variation of peak acceleration with depth for three different values of the shear modulus of the lowermost layer in a two-layer and a four-layer system. The results are for free-field acceleration equal to \( \cos \omega t \) and \( \omega = 1 \) rad/sec. The difference between the responses of the two-layer and the four-layer systems also reduces with increase in the value of the shear modulus of the lowermost layer.
3.2 Studies on a power plant site

The site has been modelled as a combination of thirteen layers overlying an elastic half space having the same properties as that of the thirteenth layer. This problem was earlier analysed by Banerjee (1992) by using the well-known computer program SHAKE. The present study analyses the sub-surface motion at this site and attempts to evaluate the influence of the change in material properties under dynamic loading. The strain dependent dynamic properties of the layers are given in Table 2. Figs. 4 and 5 show the peak acceleration at various depths for free-field acceleration corresponding to $S00E$ and $S90W$ components of the motion recorded at El Centro during the Imperial Valley earthquake of 18.05. 1940 normalised with respect to the peak acceleration on free-field. The attenuation factor computed with the static material properties are generally found to be higher than that computed with the dynamic material properties. Figs. 6 and 7 present the 5% damping normalised response spectra at various depths. It is seen that in each case, for periods between 0.5 second and 1 second the spectral ordinate reduces with increasing depth, and beyond a period of 1 second there is no discernible change in the spectral ordinate. For periods below 0.5 second the response spectra at various depths intersect. In general, the spectral ordinate is higher when static material properties are considered.

Subsequent calculations have been carried out using the dynamic material properties and artificially generated free-field time-histories consistent with the 5% damping response spectrum of USNRC RG1.60 (1978). As a typical example, Fig. 8 shows a comparison between the 5% damping response spectra depths calculated by the method presented here and by the program SHAKE.

4. CONCLUSION

For realistic earthquake motions there exists a possibility of obtaining a lower value of peak acceleration below the free surface as compared to that in the free-field. The spectral ordinates too show a reduction with depth in certain ranges of frequency.

REFERENCES


### TABLE 1

**INPUT DATA FOR STUDIES ON A SINGLE LAYER SYSTEM**

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Layer h (m)</th>
<th>( \rho ) (kg/m³)</th>
<th>( G ) (N/m²)</th>
<th>( \beta )</th>
<th>Layer No.</th>
<th>Layer h (m)</th>
<th>( \rho ) (kg/m³)</th>
<th>( G ) (N/m²)</th>
<th>( \beta )</th>
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<td>1800</td>
<td>2.943</td>
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<td>W</td>
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<td>( \times 10^7 )</td>
<td></td>
<td></td>
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<td>O</td>
<td>( \times 10^7 )</td>
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<tr>
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<td>4.905</td>
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<td></td>
<td>Y</td>
<td>( \times 10^8 )</td>
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</tr>
</tbody>
</table>

**Note:** \( G = 4.905 \times 10^7 \) for TLS and \( FFA = \cos (10t) \) in Fig.1

**TLS:** Two layer system

### TABLE 2

**INPUT DATA FOR STUDIES ON A POWER PLANT SITE**

<table>
<thead>
<tr>
<th>Layer No. (i)</th>
<th>Layer thickness (m)</th>
<th>( \rho ) (Kg/m³)</th>
<th>( G ) (N/m²)</th>
<th>( \beta ) static</th>
<th>( \beta ) dynamic</th>
</tr>
</thead>
<tbody>
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<td>1800</td>
<td>( \times 10^7 )</td>
<td>8.476 ( \times 10^7 )</td>
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<td>( \times 10^7 )</td>
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<td>( \times 10^7 )</td>
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<td>5</td>
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<td>( \times 10^7 )</td>
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<td>6.865 ( \times 10^7 )</td>
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<td>( \times 10^7 )</td>
<td>1.623 ( \times 10^7 )</td>
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<td>( \times 10^7 )</td>
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<td>1800</td>
<td>( \times 10^7 )</td>
<td>2.328 ( \times 10^7 )</td>
<td>1.038 ( \times 10^7 )</td>
</tr>
</tbody>
</table>
**Fig. 1. Subsurface Model**

**Fig. 2. Variation of Peak Acceleration with Frequency: Effect of Depth and Number of Layers.**
TLS = Two Layer System, FLS = Four Layer System.

**Fig. 3. Variation of Peak Acceleration with Depth: Effect of Shear Modulus and Number of Layers.**
TLS = Two Layer System, FLS = Four Layer System.

**Fig. 4. Normalised Peak Acceleration Variation with Depth: Based on the Static Material Properties.**
FIG. 5: NORMALISED PEAK ACCELERATION VARIATION WITH DEPTH BASED ON THE DYNAMIC MATERIAL PROPERTIES.

FIG. 6: 5% DAMPING NORMALISED RESPONSE SPECTRA OF ACCELEROMETERS AT VARIOUS DEPTHS: FREEFIELD MOTION- SOOE COMPONENT OF EL CENTRO, 1940; BASED ON STATIC PROPERTIES.

FIG. 7: 5% DAMPING NORMALISED RESPONSE SPECTRA OF ACCELEROMETERS AT VARIOUS DEPTHS FREE FIELD MOTION- SOOE COMPONENT OF EL CENTRO: 1940, BASED ON DYNAMIC PROPERTIES.

FIG. 8: 5% DAMPING NORMALISED RESPONSE SPECTRA AT A DEPTH OF 3.05 M BY PRESENT STUDY AND BY SHAKE [4]; FREE-FIELD ACCELERATION COMPATIBLE WITH RG 160 SPECTRUM.