New developments in coupled seismic analysis of equipment and piping

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1 INTRODUCTION

Two computer programs, CREST and CREST-IRS, were developed at Center for Nuclear Power Plant Structures, Equipment and Piping to perform accurate coupled response spectrum analysis of secondary systems such as piping. CREST performs coupled response spectrum analysis by a modal synthesis approach (Gupta and Jaw 1985). CREST-IRS gives the floor spectra and the required correlations between various floors while taking into account the mass interaction effect (Gupta and Jaw 1991). CREST-IRS approximates the analysis performed by CREST.

The computer program CREST, as originally developed, needed the uncoupled modal properties of the primary and secondary systems for all the modes. This is not practical for systems with large DOF. In the past and in many cases presently, only some of the modes for both the systems are calculated and rest ignored. This concept of mode truncation is well understood in the analysis of individual systems and does not result in any significant error in many simple structures. However in the analysis of complex systems, higher modes may contribute significantly to the total response. Techniques have been developed to account for the higher modes in the uncoupled analysis in terms of “residual rigid response” or “missing mass effect” (Powell 1979; Vashi 1981; Gupta and Jaw 1984) but none that can be used in a coupled analysis. We developed new formulations to include the effect of high frequency rigid modes of a multiply connected piping system in a coupled analysis and incorporated them in the computer program CREST. We have also made changes in the CREST-IRS program to exactly evaluated the instructure response spectra for zero mass ratio.

2 COUPLED RESPONSE USING CREST

For an N-DOF nonclassically damped coupled system, there exist N-pairs of complex eigenvalues and eigenvectors. The complex eigenvalue pairs give the coupled modal frequency \( \omega \) and damping ratio \( \zeta \). Each complex eigenvector pair gives two real vectors \( \Psi_i^P \) and \( \Psi_i^T \) as described in (Gupta 1992). Using modal superposition, the displacement vector of the coupled system relative to the base of primary system
can be written as

\[ U = \sum_{i=1}^{N} U_i = \sum_{i=1}^{N} U_i^d - U_i^v = \sum_{i=1}^{N} \Psi_i^d y_i - \Psi_i^v y_i \]

in which \( y_i \) is the relative displacement of an equivalent SDOF system. In the response spectrum method of analysis maximum values of \( U_i^d \) and \( U_i^v \) are calculated separately. The terms \( y_i \) and \( \tilde{y}_i \) are replaced by spectral quantities \( \tilde{S}_D^d \) and \( \tilde{S}_V \). The effect of high frequency modes can be included through the use of the residual modal vectors. The primary system residual modal vector is obtained using the procedure given in (Megahed and Gupta 1992). for the primary systems that are singly connected at its base. The residual modal vector is orthogonal to all the non-rigid modes of the primary system and is directly included as an additional primary system mode in the coupled analysis.

To incorporate the effect of high frequency rigid modes of the secondary system, the secondary system DOF are expressed relative to the primary system connecting DOF using the transformation given in (Gupta and Gupta 1994). The equation of motion of secondary system after transformation can be written as

\[ M_s \ddot{U}_s + C_s \dot{U}_s + K_s U_s = -M_s U_{sc} \ddot{U}_c \]

in which subscript \( s \) represents a secondary system property and the matrix \( U_{sc} \) contains one secondary system vector for each connecting DOF. Each such vector represents the static deformation shape of the secondary system when the corresponding connecting DOF undergoes a unit displacement. The \( \ddot{U}_c \) vector consists of total accelerations at the connecting DOF. The residual response in high frequency modes is pseudo-static and is evaluated by using the following equation.

\[ K_{sc} \ddot{U}_{so} = -M_s \sum_{a=1}^{n_s} U_{soa} \ddot{u}_c \]

in which \( \phi_{so} \) and \( \gamma_{ca} \) are the normalized mode shape and the corresponding participation factor associated with the \( c \) connecting DOF of the \( \alpha \)th uncoupled secondary system mode, respectively, and \( n_s \) the number of uncoupled secondary system modes having frequencies less than the rigid frequency. The term \( \ddot{u}_c \) is a scalar and can be scaled out. Eq.3 would then give one residual modal vector corresponding to each connecting DOF. The residual vectors are normalized such that for each residual mode vector, \( \phi_{R}^T M \phi_{R} = 1 \). Each residual mode shape vector, \( \phi_{R} \), is orthogonal with respect to the given \( n_s \) modes of the secondary system but not with respect to the other residual mode shape vectors. Therefore, the secondary system residual modal vectors cannot be included directly in the coupled analysis. All residual modal vectors are defined by the residual modal matrix \( \Phi_{R} \). The residual response of the secondary system \( \ddot{U}_{so} \), in the \( i \)th coupled mode can also be expressed in terms of two real vectors \( \ddot{U}_{soi}^d \) and \( \ddot{U}_{soi}^v \)

\[ \ddot{U}_{soi}^d = \tilde{\Psi}_{Ri}^d y_i \quad \ddot{U}_{soi}^v = \tilde{\Psi}_{Ri}^v y_i \]

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(6) \[ \tilde{K}_R = \Phi_R^T K_s \Phi_R \]

Vectors \( \Phi_{c,i}^d \) and \( \Phi_{c,i}^s \) are responses at the primary system connecting DOF \( c \) in the coupled mode \( i \) (Gupta and Gupta 1994).

3 COUPLED RESPONSE USING CREST-IRS

In the CREST-IRS computer program the secondary system is a SDOF oscillator. For a coupled system with \( np \) significant primary system modes, there are \( np + 1 \) complex eigenvectors and eigenvalues, corresponding to each of the primary system modes and the secondary system oscillator. The complex eigenvectors of the coupled system can be represented in terms of the normal coordinates \( x_i \) and \( x_s \). It is assumed that the coupled frequencies and the damping ratios of the modes corresponding to the primary system modes remain practically the same as those of the uncoupled system, except for the uncoupled primary system mode \( 'I' \) whose frequency is closest to the oscillator frequency. Two sets of equations are used to calculate the coupled eigenvectors. First,

(7) \[ x_i = 1 \quad x_j = 0 \quad j \neq i \quad x_s = \frac{r_i^{1/2} (\omega_i^2 + 2\omega_s \zeta_s \lambda)}{\omega_s^2 + 2\omega_s \zeta_s \lambda + \lambda^2} \]

The second set of equations, for all \( i, s \), is:

(8) \[ x_s = 1 \quad x_i = \frac{r_i^{1/2} (\omega_i^2 + 2\omega_s \zeta_s \lambda)}{\omega_i^2 + r_i \omega_i^2 + 2(\omega_p \zeta_p + r_i \omega_s \zeta_s) \lambda + \lambda^2} \]

where, \( \lambda \) is the coupled eigenvalue and \( r_i \) the mass ratio. Subscripts \( p \) and \( s \) represent the primary and secondary system properties, respectively. When the CREST-IRS was originally developed, Eq.7 was used for evaluating the coupled eigenvectors corresponding to all the primary system modes, including that for \( i = I \); and Eq.8 was used to evaluate the coupled eigenvector corresponding to the SDOF oscillator. This process resulted in accurate response for zero mass ratio. However, for nonzero mass ratios, there exists no rational way of assigning the coupled eigenvalues that correspond to the SDOF oscillator mode and to the primary system mode \( I \).

To overcome the problems discussed above, a modification to the program was made before the version 0.0 of CREST-IRS was released. Eq.7 was used for all the coupled eigenvectors except that corresponding to \( i = I \), and Eq.8 was used for the eigenvectors corresponding to the \( I^{th} \) primary system mode and the SDOF oscillator. This version of the program gives accurate response values for nonzero mass ratios. A very small value (on the order of \( 10^{-3} \zeta^2 \)) is input when the mass ratio is zero, to avoid the overflow problem. This procedure gave less accurate spectral values for resonant oscillator frequencies than those given by the original (Pre 0.0) version of the program.

In the new version 1.0 of the program, a hybrid approach is taken. The algorithm of the original (Pre 0.0) version of the program is used for zero mass ratio. For nonzero mass ratios, the algorithm is chosen depending upon the tuning of the coupled primary mode with the secondary oscillator. Algorithm of version 0.0 is used if the primary system mode \( I \) and the secondary oscillator are tuned and the
algorithm of the original (Pre 0.0) version of the program is used when they are detuned. A decision regarding the tuning of two modes is made in version 1.0 by using the simplified approximate formulations for the response of a SDOF primary - SDOF secondary system presented in (Gupta and Gupta 1993).

The above algorithms breakdown for the primary - secondary systems that have equal damping values (classical damping case), have zero mass ratio, and have modes that are perfectly tuned; i.e. when the frequency of $I^{th}$ primary system mode and secondary system mode are identical. This problem is avoided by evaluating the limit of the coupled response for zero mass ratio (Gupta and Gupta 1993).

4 EXAMPLES AND VALIDATION

Five different coupled systems were taken to study the effect of high frequency modes of the secondary system in the coupled analysis. For all the five cases, coupled analysis was performed by considering all the modes of the secondary system; and by truncating the two highest secondary system modes while ignoring the effect of truncated modes or accounting for them as rigid modes. The five cases are described in (Gupta and Gupta 1994). One of the five cases is described here. Fig.1 shows the coupled system. The primary system frequencies varies from 2.101 Hz. to 16.93 Hz. and that of the secondary system from 9.83 Hz. to 30.27 Hz. The damping in the two systems is 7% and 2% respectively. Tables 1 - 2 compare the results from three sets of analyses. As shown in these tables and for other cases in (Gupta and Gupta 1994), the error in the coupled response can be very high when the high frequency modes of the secondary system are not included in the analysis. However, the responses are accurate when the effect of high frequency modes of the secondary system is included in the analysis by treating these modes as rigid.

Validation of the program CREST-IRS, version 1.0, was carried out in two parts. First, a total of fifty three instruction response spectra (IRS) generated by the program were compared with those obtained from the corresponding time history analyses. Second, an equivalent coupled analysis was performed for a set of nine
Table 1: Comparison of Nodal Displacements (inches) for Secondary System DOF

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Including All Secondary System Modes</th>
<th>Truncating Two Modes of Secondary System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not Including Residual Modes</td>
<td>Percent Error</td>
</tr>
<tr>
<td>1</td>
<td>1.281</td>
<td>0.8599</td>
</tr>
<tr>
<td>2</td>
<td>1.498</td>
<td>1.6330</td>
</tr>
<tr>
<td>3</td>
<td>1.688</td>
<td>1.9310</td>
</tr>
<tr>
<td>4</td>
<td>1.849</td>
<td>1.3430</td>
</tr>
</tbody>
</table>

Table 2: Comparison of Spring Forces (kips) in Secondary System

<table>
<thead>
<tr>
<th>Element No.</th>
<th>Including All Secondary System Modes</th>
<th>Truncating Two Modes of Secondary System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not Including Residual Modes</td>
<td>Percent Error</td>
</tr>
<tr>
<td>1</td>
<td>240.4</td>
<td>139.1</td>
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<tr>
<td>2</td>
<td>217.6</td>
<td>772.9</td>
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<td>3</td>
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<td>299.0</td>
</tr>
<tr>
<td>4</td>
<td>161.3</td>
<td>588.4</td>
</tr>
<tr>
<td>5</td>
<td>128.9</td>
<td>564.5</td>
</tr>
</tbody>
</table>

different coupled systems. Nodal displacements and spring forces in the secondary system, for each of the nine cases were compared with those from the corresponding time history analyses and also with results from the original (Pre 0.0) version (Gupta and Gupta 1993).

5 CONCLUSIONS

Mode truncation can result in significant error in the seismic response of complex systems such as piping. The different techniques available for incorporating the residual rigid response are applicable for an uncoupled analysis only. New formulations are presented to evaluate the response for multiply connected secondary systems by introducing residual modal vectors (one for each connecting DOF) to account for the truncated high frequency modes. Numerical validation of the formulations is carried out by performing coupled analysis for five different coupled structural systems using CREST. Results show that very large errors can be introduced in the secondary system response when the effect of the truncated modes is ignored and that the errors are practically eliminated when the effect is included in terms of residual rigid modes. In the computer program CREST-IRS, modifications have been made in the evaluation of coupled eigenvectors to enable accurate estimation of response in two different cases of zero mass ratio and nonzero mass ratio. If the dampings of the primary system and the secondary system are same, then the algorithm to evaluate
the coupled eigenvectors for zero mass ratio case breaks down in perfectly tuned 
modes. Formulations have been developed to obtain the response of such systems 
as limiting solution for zero mass ratio in the perfectly tuned modes.

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