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A mechanical model for damageable elasto-plastic bars

Chimisso, F.E.G.,
Fundação Universidade de Rio Grande, Depto. de Materiais e Construção, Rio Grande, RS, Brazil
Costa Mattos, H.S.
Universidade Federal Fluminense, Depto. de Engenharia Mecânica, Niterói, RJ, Brazil

ABSTRACT: In the present work, a thermodynamically consistent mechanical model for damageable elasto-plastic materials is proposed. The governing equations are obtained within the framework of a microstructure theory since an scalar variable, β, (called cohesion variable and related with the links between material points) is introduced as an additional kinematic variable. The constitutive equations are developed within a thermodynamic framework, taking the cohesion variable β and also its gradient $\nabla\beta$, as state variables. This model leads to an adequate description of the softening behavior due to the degradation of the microstructure.

1 INTRODUCTION

Continuum Damage Mechanics is a promising tool for the failure prediction of structural components. Nevertheless, it is not a simple task to do a mathematically correct and physically realistic description of the strain-softening behavior due to the degradation of the microstructure. In the case of metallic bars, the cyclic plastic deformation induces a strain-hardening and also a degradation of the structure (fatigue damage). On the other hand, the degradation of the structure induces a softening behavior in the engineering stress-strain curves. Hence, it is very important to model the coupling between plasticity and damage in order to perform an adequate lifetime prevision.

In the present work, a new damage theory for elasto-plastic materials is proposed. A important feature of the proposed model is that the constitutive equations are developed within the framework of Thermodynamics of Irreversible Processes considering that the free energy is a function of the total strain, $\underline{\underline{\varepsilon}}$, of the plastic strain, $\underline{\underline{\varepsilon}}^p$, of two variables related respectively with the isotropic and kinematic hardening, of β, and also of $\nabla\beta$.

The effectiveness of the theory is checked by comparing numerical simulations of cyclic uniaxial tests in aluminium ASTM 6351 bars with experimental results.

2 PRELIMINARY DEFINITIONS

A damageable body is defined as a set of material points B which occupies a region Ω of the Euclidean space at the reference configuration. In this theory, besides the classical variables that characterize the kinematics of a continuum medium (displacements, velocities and accelerations of material points), an additional variable, $\beta \in [0,1]$, called cohesion variable, is introduced. This variable is related with the links between material points and can be interpreted as a measure of the local cohesion state of the material.

If $\beta = 1$, all the links are preserved. If $\beta = 0$, a local rupture is considered since all the links between material points have been broken. Since the degradation is an irreversible phenomenon, the cohesion variation rate $\dot{\beta} = \frac{D\beta}{Dt}$ must be negative or equal to zero.

For the sake of simplicity the hypothesis of small deformation will be assumed throughout this work. Hence, the density ρ is assumed to be constant in time and the conservation of mass principle is automatically satisfied.

The proposed principles may be regarded as a special case of the theories of microstructures (Minuin 1964, Toupin 1964, Goodman and Cowin 1972). In particular these governing principles are very close to those proposed in the theory of elastic materials with voids (Cowin and Nunziato 1983). Nevertheless, the definition and the physical interpretation of the additional kinematic variable and also the proposed constitutive equations make both theories very different. In the theory of elastic materials with voids the additional variable is related with the change in solid volume fraction. The present theory assumes that the degradation is related with microcracks and not with micro-voids (Fremond, Costa Mattos and Mamyia 1990, 1992) and hence the damaged body is not considered a porous medium and the cohesion variable is not directly related with a volume change (Costa Mattos and Sampaio 1993, Chimisso 1994).

Appropriate conservation laws that govern the evolution of a continuous damageable body which occupies a region Ω at the reference configuration was postulated. The evolution of a damageable body is then governed by balance relations in the local form (Chimisso and Costa Mattos 1994).

3 CONSTITUTIVE EQUATIONS

In order to simplify the theory we shall consider the following hypothesis: small deformations, isothermal and quasi-static processes. The fundamental principles are valid for any kind of damageable body. In the present section we restrict the study to elasto-plastic damageable materials.

3.1 State Variables

Under the hypothesis of small deformations, the local state of a elasto-plastic material is supposed to be a function of the total strain $\underline{\underline{\varepsilon}} = \frac{1}{2}(\nabla \underline{u} + (\nabla \underline{u})^T)$, of the plastic strain $\underline{\underline{\varepsilon}}^p$, of the cohesion variable β, of its gradient $\nabla\beta$, and also of a scalar variable p associated with the isotropic hardening, and of a second order tensor variable $\underline{\underline{c}}$ associated with the kinematic hardening .

3.2 Free Energy - State Laws

Following the classical assumption of the Thermodynamic of Irreversible Processes, the free energy is supposed to be a function of the state variables.

Thus, the following expression is proposed for the free energy :

(1) $\psi(\underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^p, p, \underline{\underline{c}}, \beta, \nabla\beta) = \beta \left[\psi_e(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) + \psi_p(p) + \psi_c(\underline{\underline{c}})\right] + \frac{1}{2} C \, \nabla\beta . \nabla\beta$

with

(2) $$\psi_e = \frac{E}{2(1+\nu)} \left\{ \frac{\nu}{1-2\nu} \left[tr(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) \right]^2 + (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) : (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) \right\}$$

3) $$\psi_p = v_1 \left\{ p + \frac{e^{-v_2 p}}{v_2} \right\} + p\sigma_y$$

(4) $$\psi_c = \frac{1}{2} a \, (\underline{\underline{c}} : \underline{\underline{c}})$$

where ψ is the specific free energy, E is the Young Modulus, ν is the Lamé constant, C, v_1, v_2, a, σ_y are non negative constants and $(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) = \underline{\underline{\varepsilon}}^e$ is the elastic strain tensor.

The here called thermodynamic forces $(\underline{\underline{\sigma}}, \underline{\underline{x}}, y, G, \underline{H})$, related to the state variables $(\underline{\underline{\varepsilon}}^e, \underline{\underline{c}}, p, \beta, \nabla\beta)$ are defined from the free energy by the state laws:

(5) $$\underline{\underline{\sigma}} = \frac{\partial\psi}{\partial\underline{\underline{\varepsilon}}^e} = \frac{\beta E}{1+\nu} \left[\frac{\nu}{1-2\nu} tr(\underline{\underline{\varepsilon}}^e) \underline{\underline{1}} + \underline{\underline{\varepsilon}}^e \right]$$

(6) $$\underline{\underline{x}} = \frac{\partial\psi}{\partial\underline{\underline{c}}} = \beta \, (a\underline{\underline{c}})$$

(7) $$y = \frac{\partial\psi}{\partial p} = \beta \left[v_1 (1 - e^{-v_2 p}) + \sigma_y \right]$$

(8) $$G = \frac{\partial\psi}{\partial\beta} = \psi_e + \psi_p + \psi_c$$

(9) $$\underline{H} = \frac{\partial\psi}{\partial\nabla\beta} = C \, \nabla\beta$$

where $\underline{\underline{\sigma}}$ is the stress tensor and $\underline{H}$ a microscopic stress vector associated with $\nabla\beta$.

3.3 Plastic Potential - Evolution Laws

To complete the constitutive equations additional informations about the dissipative behavior must be given. These informations can be obtained from a plastic potential F and are called evolution laws. The potential F is supposed to have the following form :

(10) $$F(\underline{\underline{\sigma}}, \underline{\underline{x}}, y, G ; \underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^p, p, \underline{\underline{c}}, \beta) \leq 0 ,$$ where

(11) $$F = J(\underline{\underline{\sigma}} - \underline{\underline{x}}) - y + g(\underline{\underline{x}}, G ; \underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^p, p, \underline{\underline{c}}, \beta)$$

with $J(\underline{\underline{\sigma}} - \underline{\underline{x}})$ being the Von Mises equivalent stress, and $$g = \frac{b}{2a}\left(\underline{\underline{x}} : \underline{\underline{x}}\right) - \frac{ab}{2}\left(\beta\, \underline{\underline{c}} : \underline{\underline{c}}\right) + \frac{G^2}{2S_0} - \frac{1}{2S_0}\left[\frac{\psi_e + \psi_p + \psi_c}{\beta}\right]^2$$

Besides the plastic potential F, another potential $\hat{F}(\dot{\beta}) = \dot{\beta}$, is used to take into account the restriction $\dot{\beta} \leq 0$. Hence the following evolution laws are postulated:

(12) $$\dot{\underline{\underline{\varepsilon}}}^p = \lambda \frac{\partial F}{\partial \underline{\underline{\sigma}}} = \lambda \frac{\frac{3}{2}\left(\underline{\underline{\sigma}} - \underline{\underline{x}}\right)_{dev}}{J\left(\underline{\underline{\sigma}} - \underline{\underline{x}}\right)}$$

(13) $$\dot{\underline{\underline{c}}} = -\lambda \frac{\partial F}{\partial \underline{\underline{x}}} = \dot{\underline{\underline{\varepsilon}}}^p - \frac{b}{a}\underline{\underline{x}}\lambda$$

(14) $$\dot{p} = -\lambda \frac{\partial F}{\partial y} = \lambda$$ and

(15) $$\dot{\beta} = M - \lambda \frac{\partial F}{\partial G} - \hat{\lambda}\frac{\partial \hat{F}}{\partial \dot{\alpha}} = M - \frac{\lambda\left(\psi_e + \psi_p + \psi_c\right)}{S_0} - \hat{\lambda}$$

where $\lambda \geq 0$, $F \leq 0$, $\lambda F = 0$ and $\hat{\lambda} \geq 0$, $\hat{F} \leq 0$, $\hat{\lambda}\hat{F} = 0$.

M is a microscopic internal force associated with β , λ is the Lagrange multiplier related with the restriction $F \leq 0$, and $\hat{\lambda}$ is the Lagrange multiplier related with the restriction $\hat{F} \leq 0$.

It is possible to prove that the state laws (5) - (9) and the evolution laws (12) - (15) define a complete set of thermodynamically admissible constitutive equations.

Introducing equations (9) and (15) in the equation of the balance of microscopic forces (Chimisso and Costa mattos, 1994) and using the change of variables: $\beta = 1\text{-}D$, the following balance equation is obtained:

(16) $$\left\langle C\,\Delta D + \frac{\lambda(\psi_e + \psi_p + \psi_c)}{S_0}\right\rangle = \dot{D}$$

where $< a > = \max\{0, a\}$.

It is interesting to use D instead of β because this variable is closer to the definition of the state variable usually adopted in the traditional works of Continuum Damage Mechanics.

4 EXAMPLES

In this section the study is restricted to a one dimensional example. The equation that governs the evolution of the damage variable D is then reduced to

$$\left\langle C\frac{\partial^2 D}{\partial z^2} - \lambda\frac{\partial F}{\partial G}\right\rangle = \dot{D} \qquad (17)$$

It is considered an ASTM 6351 aluminium alloy bar with length L submitted to a reversal prescribed cyclic axial displacement : $u(z = 0, t) = 0$, $u(z = L, t) = u_L(t)$.
The material constants are:

$\sigma y = 264$ MPa, $E = 70$ GPa, $a = 29{,}28$ GPa, $b = 328$,
$v_1 = v_2 = 0$, $C = 0{,}01$ and $S_0 = 15$ GPa.
The following boundary conditions are adopted for the damage variable D :
$D(z = 0, t) = D(z = L, t) = 0$.
These boundary conditions are reasonably adequate to simulate cyclic uniaxial tests in a test specimen with a gauge length L (see fig. 1).

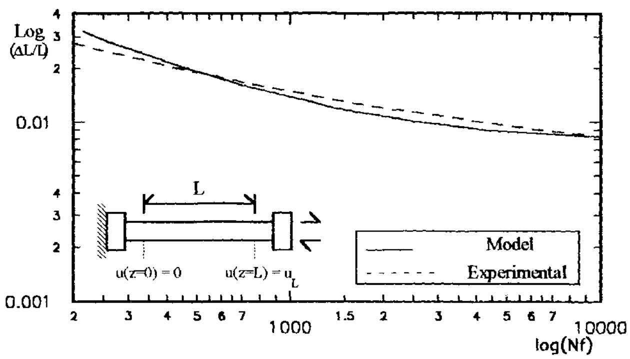

Fig.1 : ASTM 6351 Aluminium : log(ΔL/L) x log(Nf) curves.

In figure 1 its is shown the estimated lifes N_f for different displacement amplitudes (ΔL/L). The model prevision is in good agreement with experimental data. This model allows an adequate description of the strain localization phenomenon due to the damage. It is show in figures 2, 3 and 4:

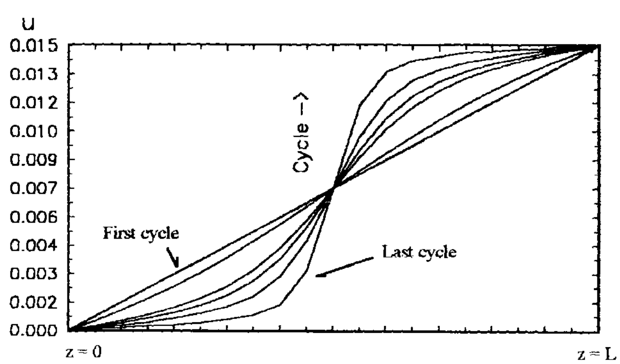

Fig.2 : Displacement u vs. z at different instants.

The cyclic stress softening is shown in figure 2 for cycles vs. displacement diagram: symmetric positions related to de middle of the bar.

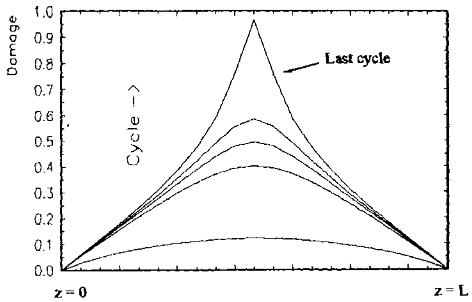

Fig.3 : Evolution of the damage distribution along the bar.

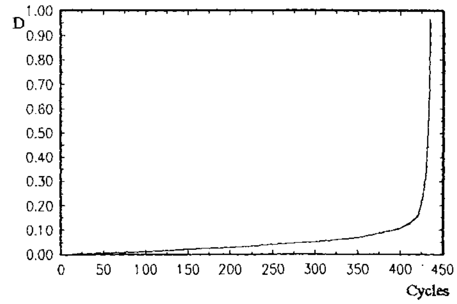

Fig. 4 : Damage vs. Life curve. z = 0.5L.

Figure 4 show, for the model, the damage evolution (D) along the life (cicles) at the middle of the bar. Simulation: $\Delta L = \pm 0.01$ L. The damage increasing slowly for 3/4 of the life and increasing very fast at the remaining life.

5 CONCLUSIONS

This paper is a first step to propose a consistent framework in which to model the fatigue of elasto-plastic materials. The theory allows an adequate description of the strain localization phenomenon induced by the damage. The numerical results show that this theory is a promising tool in the analysis of low cycle fatigue in metallic structures.

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