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Elastic-plastic wave propagation in engineering structures of power plants

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ABSTRACT: The study highlights the effect of plastic deformations and dry friction in structural supports on the magnitude of the wave propagating in power plants. An upper limit for the deformation amplitude is observed. This limit does not depend on the external load power but is a function of the spatial distance from the load and the mechanical characteristics. Non-anchored support was found out to have a maximum distance in which the waves propagates. These nonlinear properties are useful in estimating the size and mechanical characteristics of the non-anchored supports in power plants to prevent spread of waves into the interior to the sensitive components.

1 INTRODUCTION

The review of the literature in structural dynamics shows that the existing analytical methods of the wave propagation in complex structures are too idealised and do not adequately reflect the inherent complexity of structures. Complex structures are supposed to consist of a few structural members of well-studied shapes (e.g. shells, beams etc.), otherwise lumped-mass models are applied. The secondary systems of complex structures are seldom considered in the present literature. This ignorance is surprising since the secondary systems actually comprise the major portion of the structural members of a unit. The effect of contacting surfaces of the structural elements is also underestimated. Actually any nuclear power plant consists of a primary structure and secondary systems attached to the primary structure. Beyond the region of the global resonances the vibrations localise within each substructure. This phenomenon is known as vibration localisation or normal mode localisation, cf. a review paper by Hodges and Woodhouse (1986) and it should be taken into account as well.

2 GOVERNING EQUATIONS AND GENERALISED JENKINS MODEL

The field of vibration in power plants e.g. caused by aircraft crash is three-dimensional, however high-frequency waves propagate along the waveguides as uniaxial longitudinal waves. It allows one to perform a one-dimensional analysis. Due to the vibration localisation within the substructures the absolute displacement $v_n(x,t)$ within a substructure L_n can be sought in the form of the expansion in terms of the substructure's normal modes $v_{nk}(x)$

$$x \in L_n \qquad v_n(x,t) = \sum_{k=1}^{\infty} v_{nk}(x) q_{nk}(t) + V(x,t) \qquad (1)$$

where $q_{nk}(t)$ is the generalised coordinate. The normal modes are specified so that they vanish at the cross-sections x_n separating the substructures, i.e. $v_{nk}(x_n)=0$. The function V(x,t) is supposed to be spatially smooth within the whole structure. In this case V(x,t) may be called the displacement of the framework since it is smooth and coincides with the actual displacement of the substructures´ boundaries. The infinite sum in eq. (1) describes the local vibration of the structural members.

Kinetic and potential energy and the work of the external loads due to eq.(1) are given by (for details see Belyaev 1991)

$$T=\frac{1}{2}\sum_{n=1}^{N}\int_{L_n}\mu\dot{v}_n^2dx=\frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{\infty}\left\{\dot{q}_{nk}^2+2\langle\mu\rangle_n L_n\langle v_{nk}\rangle\dot{q}_{nk}\dot{V}\right\}+\frac{1}{2}\int_L\langle\mu\rangle\dot{V}^2dx$$

$$\Pi=\frac{1}{2}\sum_{n=1}^{N}\int_{L_n}EA\left(v_n'\right)^2dx=\frac{1}{2}\int_L\langle EA\rangle(V')^2dx+\frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{\infty}\omega_{nk}^2q_{nk}^2\,;\;\; W=F_0V(0)+F_LV(L)\ . \quad (2)$$

Here E is Young´s modulus, A is the cross-sectional area, ω_{nk} is the k-th eigenfrequency of the substructure n, $\langle\mu\rangle$ and $\langle EA\rangle$ are averaged density and axial rigidity, respectively. Equation (2) is derived under the assumption that the normal modes of the substructures are orthonormal within each substructure and $V(x,t)$ is a spatially smooth function compared with normal modes of the substructures, elastic moduli and density of power plant. The resulting boundary value problem is then obtained by employing Hamilton´s variational principle

$$x\in L_n:\quad \langle EA\rangle\frac{\partial^2V}{\partial x^2}-\langle\mu\rangle\left[\ddot{V}+\sum_{k=1}^{\infty}\langle v_{nk}\rangle\ddot{q}_{nk}\right]=0\,,\;\; \ddot{q}_{nk}+\omega_{nk}^2q_{nk}=-\langle\mu\rangle_n\langle v_{nk}\rangle\ddot{V} \quad (3)$$

$$x=0:\quad \langle EA\rangle V'=-F_0(t)\,;\qquad x=L:\quad \langle EA\rangle V'=F_L(t)\quad . \quad (4)$$

To take into account the inherent material damping and the dissipation due to dry friction between the structural members in power plants, we use the generalised Jenkins model. Its rheological model is composed of an infinite number of Jenkins elements, cf. Fig.1. Each Jenkins element consists of an elastic element Edh in series with a Coulomb (or dry friction) damper which has a maximum allowable load $Ehdh$, where h is the dimensionless yield strength ($0<h<\infty$). Being a universal model, the generalised Jenkins rheological model is valid for the description of nonlinear stress-strain behaviour, which results in the amplitude-dependent internal friction. The model, obviously, can also be used to describe friction in supports or other parts of the structure during relative motion, e.g. non-anchored coupling of structural members, cf. Krutzik 1988, Belyaev 1993 and Belyaev and Krutzik 1994.

The following equations are valid for a single Jenkins element

$$d\sigma_h=E(\varepsilon-\varepsilon_h)dh=Ehdh\ \mathrm{sign}\dot{\varepsilon}_h\qquad , \quad (5)$$

where $d\sigma_h$ and ε_h are the stress and the deformation of the element having yield strength h. The summation over all elements gives the state equation for the entire model

$$\sigma=\int_0^{\infty}d\sigma_h dh=E\left[\varepsilon-\int_0^{\infty}\varepsilon_h R(h)dh\right]\;;\;\; \varepsilon=h\ \mathrm{sign}\dot{\varepsilon}_h+\varepsilon_h\quad , \quad (6)$$

where σ and ε are the stress and the deformation of the material described by the generalised Jenkins model, $R(h)$ is the density of the yield strength distribution which has the following property $\int_0^{\infty}R(h)dh=1$. In the case of harmonic vibrations with a frequency ω, the essentially nonlinear dependence $\mathrm{sign}\dot{\varepsilon}_h$ in eq. (6) can be removed by means of equivalent linearization using the describing function method, i.e. $\mathrm{sign}\dot{\varepsilon}_h=4[\pi\omega a_h]^{-1}\dot{\varepsilon}_h$, where a_h is the amplitude of ε_h. The second equation in (6) is then rewritten as

$$\varepsilon_h=\varepsilon\left[1+i4h(\pi a_h)^{-1}\right]^{-1}\quad \text{or}\quad a_h=\sqrt{a^2-\left(4\pi^{-1}h\right)^2}\quad . \quad (7)$$

It is clear from the latter formula that $0<h<0.25\pi a$. The case $h\geq 0.25\pi a$ corresponds to the absence of plastic deformation, i.e. $a_h=0$. Substituting eq.(7) into eq.(6) yields the complex Young's modulus (Palmov 1976)

$$\sigma = \hat{E}\,\varepsilon\,; \quad \hat{E} = E\,(1 + i\chi)^2\,; \quad \chi(a)=\int_0^1 \tfrac{1}{2}\eta\,\sqrt{1-\eta^2}\,R\!\left(\frac{\pi a\eta}{4}\right)\frac{\pi a}{4}\,d\eta \tag{8}$$

3 GOVERNING EQUATIONS IN FREQUENCY DOMAIN

The equations obtained are not restricted to the case of a single harmonic but they are also suitable for analysis of nonstationary dynamic processes. In frequency domain the boundary value problem, eqs. (2) and (3), is given by

$$x \in L_n\,; \quad (1+i\chi)^2\langle EA\rangle V'' + \langle\mu\rangle\omega^2\left(V + \sum_{k=1}^{\infty}\langle v_{nk}\rangle q_{nk}\right) = 0\,; \quad q_{nk} = \frac{\omega^2\langle\mu\rangle_n\langle v_{nk}\rangle V}{-\omega^2 + (1+i\chi)^2\omega_{nk}^2} \tag{9}$$

$$x = 0: \quad (1+i\chi)^2\langle EA\rangle V' = -F_0(\omega)\,; \qquad x = L: \quad (1+i\chi)^2\langle EA\rangle V' = F_L(\omega) \tag{10}$$

Substituting q_{nk}, eq. (9), into the first equation in (9) yields the differential equation for the primary structure

$$(1+i\chi)^2\langle EA\rangle V'' + \omega^2 M(\omega) V = 0\,; \quad M(\omega) = \langle\mu\rangle\left[1+\langle\mu\rangle_n\omega^2\sum_{k=1}^{\infty}\frac{\langle v_{nk}\rangle^2}{-\omega^2+\omega_{nk}^2(1+i\chi)^2}\right]\,. \tag{11}$$

The parameter $M(\omega)$ occupies the place of mass in the vibration equation, hence it may be called a generalised mass. This parameter is crucial for further analysis since it reflects the inertial and spectral properties of the power plant. As seen from eq. (11), $M(\omega)$ is formed by an infinite number of resonance curves, each corresponding to a resonance curve of a SDOF-system. The width of each resonance curve is $2\chi\omega_{nk}$ at the "half-power" level. If the resonance curves are located so densely that

$$\Delta\omega_{nk} = \left|\omega_{nk+1} - \omega_{nk}\right| \leq \chi\left|\omega_{nk+1} + \omega_{nk}\right| \qquad \text{or} \qquad \frac{\Delta\omega_{nk}}{\omega_{nk}} \leq 2\chi \tag{12}$$

(i.e. large modal overlap) then the resonance curves in eq. (12) merge, forming a smooth frequency function. Therefore, the sum in eq. (12) can be replaced by an integral with a locally smooth distribution function of the eigenfrequencies $\Phi(\alpha)$, i.e.

$$M(\omega) = \langle\mu\rangle\left[1+\omega^2\int_0^{\infty}\frac{\Phi(\alpha)d\alpha}{-\omega^2+\alpha^2(1+i\chi)^2}\right]\,. \tag{13}$$

As follows from eq. (13), in the high frequency domain the power plant has actually a continuous spectrum of eigenfrequencies, which is confirmed by means of numerical computations in Krutzik 1985. Equation (13) can be now rewritten as follows

$$M(\omega) = \langle\mu\rangle\,[1-i\kappa(\omega)]^2\,; \quad \kappa(\omega) = \chi\omega^2\int_0^{\infty}\frac{\alpha^2\Phi(\alpha)d\alpha}{\left[-\omega^2+\alpha^2(1-\chi^2)\right]^2+4\alpha^4\chi^2}\,, \tag{14}$$

where κ is the non-dimensional absorption of high frequency energy in the power plant. By assuming that χ is small ($\chi<<1$) and $\Phi(\alpha)$ is smooth, the integral in eq. (14) can be estimated by the methods of the theory of random vibrations, to give $\kappa(\omega) = \pi\omega\Phi(\omega)/2$. Therefore, despite the vanishing material damping, the spatial absorption κ is still present and it is determined by the function of the eigenfrequency distribution $\Phi(\alpha)$. It implies that the vibration absorption is partly of a resonant character and the structural members act as dynamic absorbers with respect to the primary structures.

4 THE BASIC TENDENCIES OF ELASTIC-PLASTIC WAVE PROPAGATION

Equation (17) is nonlinear and it can be solved by means of certain asymptotic methods, e.g. WKB method, Heading 1962. We consider the case $x \geq 0$, i.e. the waves propagate in the positive direction. The deformation amplitude a is supposed to be a slowly changing function of x. A new variable y and a new unknown function $\Psi(y)$ are introduced as follows

$$y(x) = \int_0^x \sqrt{M(\omega)/\langle\hat{E}(x)A\rangle}\,dx \ , \ \operatorname{Im} y < 0 \ ; \ V(x) = \langle\hat{E}(x)A\rangle^{-1/4}\Psi(y) \ . \tag{15}$$

Equation (11) then takes the form

$$\frac{d^2\Psi}{dy^2} + \left[\omega^2 - \langle\hat{E}A\rangle^{-1/4}\frac{d^2}{dy^2}\langle\hat{E}A\rangle^{1/4}\right]\Psi = 0 \ . \tag{16}$$

At high frequencies the second term in the square brackets, being a slowly varying function of y, may be neglected in comparison with the first one. Hence, the displacement $V(x)$, which asymptotically satisfies the boundary condition at $x=0$ is given by

$$V(x,\omega) = B\,\langle\hat{E}A\rangle^{-1/4}\cos\omega y(x) \ , \tag{17}$$

where B is still to be determined from the boundary conditions (10).

Equation (17) is not the final solution since the complex Young's modulus $\hat{E}$ and the new variable y depend on the deformation amplitude a and the latter is to be found through the displacement V. Therefore, at the most, eq. (17) may be considered as the equation that determines a. Obtaining a^2 from eq. (17) we get asymptotically

$$a^2(x) = \frac{1}{4}\,\omega^2|B^2|\langle\hat{E}\{a(x)\}A\rangle^{-3/2}|M(\omega)|\exp\left[2\omega \operatorname{Im}\int_0^x \sqrt{M(\omega)/\hat{E}\{a(x)\}A}\;dx\right] \ . \tag{18}$$

Taking the logarithmic derivative from this integral equation yields

$$\frac{a'}{a} = -\frac{\omega}{g}[\kappa(\omega)+\chi(a)] = -\frac{\omega}{g}\left[\kappa(\omega) + \frac{1}{2}\int_0^1 \eta\sqrt{1-\eta^2}\,R\!\left(\frac{\pi a\eta}{4}\right)\frac{\pi a}{4}d\eta\right]; \ \ g = \sqrt{\frac{\langle EA\rangle}{\langle\mu\rangle}} \ . \tag{19}$$

where g is the group velocity. For the distribution function of the theory of internal material damping (i.e. $R(h) = \beta\tilde{R}h^{\beta-1}$ ($H>0$, $\beta>0$), Palmov 1976) one obtains

$$a(x) = \left[\left(a(0)^{-\beta} + \frac{\psi}{\kappa}\right)\exp\left(\frac{\beta\omega\kappa}{g}x\right) - \frac{\psi}{\kappa}\right]^{-1/\beta} ; \quad \psi = \frac{\beta\tilde{R}}{2}\left(\frac{\pi}{4}\right)^{\beta}B\!\left(\frac{\beta+1}{2},\frac{3}{2}\right) \ . \tag{20}$$

where $B(\ ;\ )$ is the Eulerian beta-function.

If the loss of the mechanical energy from plastic deformations is negligible compared with the resonance absorption ($\chi(a) \leq \chi[a(0)] \ll \kappa$) and for large values of x, one gets

$$a(x) = a(0)\exp\{-\omega\kappa x/g\} \ . \tag{21}$$

Since $\kappa(\omega)$ is given through the function of the substructural eigenfrequencies $\Phi(\omega)$ the internal degrees of freedom of each substructure correspond to a set of dynamic absorbers with respect to the primary structure, thus providing considerable spatial absorption of the energy of propagating waves in the whole high-frequency domain.

If the material damping, the frictional and plastic effects are considerable, the structure demonstrates evident nonlinear properties. For example, for any amplitude of the external force the following inequality holds

$$a(x) < \left\{\frac{\psi}{\kappa}\left[\exp\left(\frac{\beta\omega\kappa x}{g}\right) - 1\right]\right\}^{-1/\beta} = a_m(x) \ . \tag{22}$$

The latter formula indicates an upper limit of the deformation amplitude at any point of structure, even if the power of the external excitation (e.g. aircraft crash) is unbounded. This upper limit does not depend on the power of the source, but it is a function of the spatial distance from the source and of the mechanical characteristics of the structure. It implies that for any building one can plot a universal curve which is a majorant of the curves of the amplitude of the actual deformation. The existence of such an upper limit of the field of deformation means that the structure can conduct only a limited amount of the mechanical energy. This phenomenon of vibration saturation in complex structures can be explained only by means of nonlinear analysis.

The following parameters: κ=0.2 (Palmov 1976), $\omega=2\pi\cdot 500\ s^{-1}$, $g=2\cdot 10^3$m/s are chosen for numerical example. The yield strength h is assumed to be uniformly distributed (β=1), hence $\chi=\psi a$. We take ψ=5 that corresponds to the energy loss parameter χ=0.05 while the deformation amplitude is a=0.01. The result of computation for $a(x)$, eq. (20), and the majorant $a_m(x)$, eq. (22) are presented in Fig. 2.

5 WAVE PROPAGATION THROUGH NON-ANCHORED COUPLING

Local principle in structural dynamics, e.g. Belyaev 1993 and Belyaev and Krutzik 1994, allows one to describe various parts of structures by means of different approaches. Wave propagation along the primary structures may be described in integral form while the local effects of the wave transition through structural joints may be described by conventional methods of structural dynamics. Non-anchored coupling of structural member, Krutzik 1988, is known to be an effective measure against the transition of intensive external loads (earthquake, heavy-weight drop accident) into the building interior. This type of coupling has a rheological model depicted in Fig.3 and is described by the following state equation

$$F = C\varepsilon + H \,\mathrm{sign}\, \dot{\varepsilon} \quad , \tag{23}$$

where F is the coupling force. Equation (23) may be linearised by means of the methods of Part 3 which results in the following dynamic equation for the deformation

$$\frac{\partial^2}{\partial x^2}\left(C\varepsilon + \frac{H_*}{\omega}\frac{\partial \varepsilon}{\partial t}\right) - \mu\frac{\partial^2 \varepsilon}{\partial t^2} = 0\,; \quad H_* = \frac{4}{\pi}H \tag{24}$$

The solution of this equation is given by (Palmov 1976)

$$\varepsilon(x,t) = \begin{cases} (\varepsilon_0 - \varepsilon_1 x)\cos(\omega t - \lambda x - \varphi)\,, & 0<x<x_{max} \\ 0 & , \quad x \geq x_{max} \quad , \end{cases} \tag{25}$$

where

$$\varepsilon_0 = \frac{1}{c}\sqrt{Z^2 - H_*^2}\ ; \quad \varepsilon_1 = \frac{\lambda H_*}{2C}\,; \quad \lambda = \frac{\omega\sqrt{\mu}}{\sqrt{C}}\,; \quad x_{max} = \frac{2}{\omega}\sqrt{\frac{C}{\mu}}\,\sqrt{(Z/H_*)^2 - 1} \quad . \tag{26}$$

Here Z is the amplitude of the force applied at the support in question. The value for Z may be obtained using the previous formalism. As follows from eq. (25) there exists a maximum distance x_{max} in which the waves propagates. If this maximum distance is exceeded, the wave motion is arrested. This result may be useful in estimating the size and mechanical characteristics of the non-anchored supports in power plants to prevent spread of the waves into the interior to the sensitive components of the plant.

6 CONCLUSIONS

Propagation of elastic-plastic waves along the waveguides in engineering structures was considered. A generalised Jenkins elastic-plastic model was chosen to model the inherent material damping and the dissipation due to dry friction between the structural members in power plants. The solution obtained reveals a considerable spatial decay of the propagating waves in power plants caused by the resonant absorption of vibrations by secondary systems. Because of the nonlinear nature of the energy dissipation, an upper limit of vibration at any point of the structure is observed (saturation of vibration). This limit does not depend on the power of the external excitation, but it is a function only of integral mechanical characteristics of the structure and the spatial distance from the external load. A maximum distance in which the wave propagates was obtained for non-anchored supports. If this maximum distance is exceeded, the wave motion is arrested. The results obtained help to estimate the size and mechanical characteristics of the non-anchored supports in power plants to prevent spread of the waves into the interior to the sensitive components of the plant.

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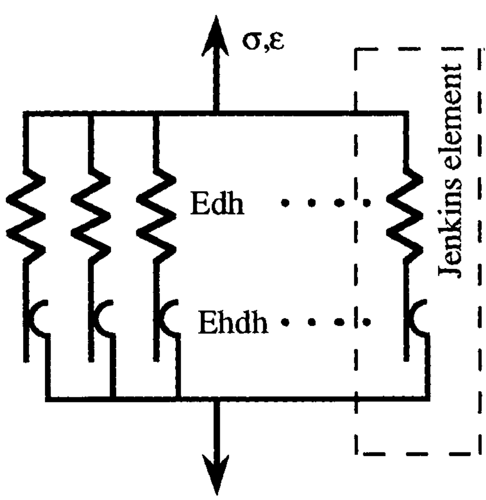

Fig.1 The generalized Jenkins model

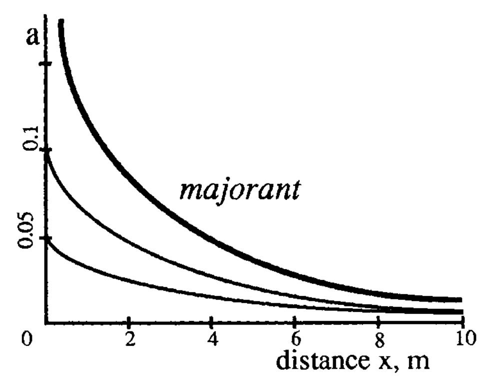

Fig.2 The deformation field and its majorant

Fig.3 Non-anchored support and its rheological model