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A finite strain elastoplastic constitutive model

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ABSTRACT: A finite strain elasto-plastic model is presented, whose main features are: multiplicative kinematics, and a hyperelastic constitutive law for describing the elastic response. The hypothesis of small elastic strains is invoked a posteriori, in this way the model is conveniently simplified. An elastic predictor-plastic corrector type algorithm is used in order to integrate the constitutive law.

1. INTRODUCTION

Large strain plasticity is a topic frequently discussed for researchers in Computational Mechanics. Constitutive models based on hypoelasticity are mainly used in practice.

In the last few years, since the works of Simo and Ortiz (1985), large strain elastoplastic models based on the notion of hyperelasticity have been considered too.

This work proposes a large strain elastoplastic model based on a multiplicative kinematics, hyperelastic constitutive law, that is derived in the framework of the above cited work of Simo and Ortiz. The paper begins with a brief discussion of the kinematics of the problem, and a description of the constitutive law is done. A particularization of the model for metals is proposed taking into account the well known hypothesis of small elastic strains for this case.

The numerical implementation of the model is done by mean of an elastic-predictor plastic-corrector scheme. From this basis the complete algorithm is shown. Finally the proposed ideas are validated solving a classical application problem.

2. THEORETICAL FOUNDATIONS

In this point the proposed constitutive model is briefly discussed. An in depth discussion can be found in other works of the author and coworkers (García Garino & Oliver, 1992; García Garino, 1993).

2.1 *Kinematics*

The kinematics of the problem is defined by introducing an intermediate unstressed configuration that leads to the classical multiplicative decomposition of the deformation gradient tensor $\boldsymbol{F}$ in its elastic and plastic counterparts (Lee, 1969):

$$\boldsymbol{F} \;=\; \boldsymbol{F}^e\;\boldsymbol{F}^p \tag{1}$$

From this decomposition the *additive* relationships of Table 1 can be derived (Simo and Ortiz, 1985; García Garino, 1993).

$^0\Omega$	$^t\Omega^e$	$^t\Omega$
$\boldsymbol{E} = \boldsymbol{E}^e + \boldsymbol{E}^p$	$\overline{\boldsymbol{E}} = \overline{\boldsymbol{E}}^e + \overline{\boldsymbol{E}}^p$	$\boldsymbol{e} = \boldsymbol{e}^e + \boldsymbol{e}^p$
$\dot{\boldsymbol{E}} = \dot{\boldsymbol{E}}^e + \dot{\boldsymbol{E}}^p$	$\overline{\boldsymbol{D}} = \overline{\boldsymbol{D}}^e + \overline{\boldsymbol{D}}^p$	$\boldsymbol{d} = \boldsymbol{d}^e + \boldsymbol{d}^p$

Table 1 : Kinematics of elastoplastic continuum

where $\boldsymbol{E}$, $\boldsymbol{E}^e$ and $\boldsymbol{E}^p$ are respectively the total, elastic, and plastic Green-Lagrange Tensors defined in the initial configuration $^0\Omega$, $\overline{\boldsymbol{E}}$, $\overline{\boldsymbol{E}}^e$ and $\overline{\boldsymbol{E}}^p$ are the corresponding tensors in the intermediate configuration $^t\Omega^e$, and $\boldsymbol{e}$,$\boldsymbol{e}^e$,$\boldsymbol{e}^p$, are respectively the total, elastic and plastic Almansi tensors in the current configuration $^t\Omega$. $\overline{\boldsymbol{D}}$ and $\overline{\boldsymbol{d}}$ are the rate of deformation tensors in $^t\Omega^e$ and $^t\Omega$, respectively.

2.2 *Constitutive Law*

Plasticity effects are taken into account by mean of internal variables theory and the model is fully consistent with thermodynamics of irreversible solids. A detailed discussion of the model can be found in another work of the author (García Garino, 1993), for the present paper it is enough to define the model in the current configuration.

$$\begin{aligned}
&\boldsymbol{e} = \boldsymbol{e}^e + \boldsymbol{e}^p \\
&\boldsymbol{d} = \boldsymbol{d}^e + \boldsymbol{d}^p \\
&\boldsymbol{\tau} = \frac{\partial \psi^e(\boldsymbol{e}^e, \boldsymbol{b}^{e-1})}{\partial \boldsymbol{e}^e} \\
&\dot{\gamma} \geq 0 \quad f \leq 0 \quad \dot{\gamma} f = 0 \\
&\boldsymbol{d}^p = \dot{\gamma}\frac{\partial g}{\partial \boldsymbol{\tau}} \\
&\mathcal{D}^p \;= \boldsymbol{\tau} : \boldsymbol{d}^p + \boldsymbol{p} : \dot{\boldsymbol{\alpha}} \geq 0
\end{aligned}$$

Box 1 Constitutive Model in the current configuration

The Kirchhoff stress tensor is computed from the hyperelastic potential written in terms of elastic strains $\boldsymbol{e}^e$. The elastic Finger $\boldsymbol{b}^{e-1}$ has to be included as an argument of the free energy function in order to satisfy objectivity. The plastic component of rate of deformation tensor $\boldsymbol{d}^p$ accounts for the flow rule. The yield and plastic potential functions are denoted by f and g respectively and γ is the plastic multiplier. The internal variables are denoted by $\boldsymbol{\alpha}$, and $\boldsymbol{p}$ are their conjugate forces.

For the case of metals the elastic strains are negligible, then tensor $\boldsymbol{F}^e$ approaches to the Identity, and consequently tensor $\boldsymbol{b}^{e-1}$ tends to the spatial metric tensor $\boldsymbol{g}$. On the other hand, because elastic strain are small, it is possible to write the elastic component of free energy function as follows (García Garino, 1993):

$$\psi^e = \frac{1}{\rho_o}\left[\frac{1}{2}\lambda\ tr(\boldsymbol{e}^e)^2\ +\ \mu\ (\boldsymbol{e}^e : \boldsymbol{e}^e)\right] \tag{2}$$

in terms of elastic Almansi strain $\boldsymbol{e}^e$ and material constants λ and μ. This model has been used previously by the author instead of neohookean models proposed by another authors (Simo and Ortiz, 1985).

Plasticity is taken into account by mean of an associative flow rule. The yield function is the Von Mises or J2 model. In this case only isotropic hardening is considered. The hardening law can be linear or non linear, and it is written in terms of effective plastic strain $\bar{e}^p$:

$$\psi^p = \psi^p(\bar{e}^p) \tag{3}$$

3. NUMERICAL IMPLEMENTATION

In the numerical implementation of the model an elastic predictor – plastic corrector scheme is used, where the elastic Finger tensor plays the role of an internal variable. For the predictor problems the trial elastic Finger tensor results (García garino, 1993):

$${}^{t+\Delta t}\boldsymbol{b}^{e-1(trial)} = \boldsymbol{f}^{-T}\cdot{}^{t}\boldsymbol{b}^{e-1}\cdot\boldsymbol{f}^{-1} \tag{4}$$

where $\boldsymbol{f}^{-1}$ is the inverse of the incremental deformation gradient tensor written in terms of current coordinates and incremental displacements as: $\boldsymbol{f}^{-1} = \boldsymbol{I} - \nabla\boldsymbol{u}$, with $\nabla\boldsymbol{u} = \frac{\partial\boldsymbol{u}}{\partial^{t+\Delta t}\boldsymbol{x}}$. The corrector problem leads to the final expression for the elastic Finger tensor as (García Garino, 1993):

$${}^{t+\Delta t}\boldsymbol{b}^{e-1} = {}^{t+\Delta t}\boldsymbol{b}^{e-1(trial)}\ +\ 2\ \gamma\ {}^{t+\Delta t}\boldsymbol{n} \tag{5}$$

The term $2\gamma\ {}^{t+\Delta t}\boldsymbol{n}$ is computed by mean of the radial return algorithm. The vector $\boldsymbol{n}$ is the normal to the yield surface.

Given the displacements ${}^{t+\Delta t}\boldsymbol{u}$ and internal variables ${}^{t}\boldsymbol{b}^{e-1}$, y ${}^{t}\boldsymbol{\alpha}$ stored in the data base for a converged time step t:
i Update geometry and compute tensor $\boldsymbol{f}$:
ii Update the elastic Finger tensor:

$$ {}^{t+\Delta t}\boldsymbol{b}^{e-1TR} = \boldsymbol{f}^{-T}\ {}^{t}\boldsymbol{b}^{e-1}\ \boldsymbol{f}^{-1} $$

iii Compute the elastic Almansi strain and Cauchy stresses:

$$ {}^{t+\Delta t}\boldsymbol{e}^{e} = \frac{1}{2}(\ {}^{t+\Delta t}\boldsymbol{g}\ -\ {}^{t+\Delta t}\boldsymbol{b}^{e-1}) \quad ; \quad {}^{t+\Delta t}\boldsymbol{\sigma} = \rho \frac{\partial \psi({}^{t+\Delta t}\boldsymbol{e}^{e})}{\partial^{t+\Delta t}\boldsymbol{e}^{e}} $$

iv Check the yield criteria and compute the plastic corrector if necessary:

$$ f({}^{t+\Delta t}\boldsymbol{\sigma}, {}^{t+\Delta t}\boldsymbol{\alpha}) \leq 0 $$

v Correct elastic Finger tensor by mean of radial return algorithm.

$$ {}^{t+\Delta t}\boldsymbol{b}^{e-1} = {}^{t+\Delta t}\boldsymbol{b}^{e-1TR} + 2\ \lambda\ {}^{t+\Delta t}\boldsymbol{n} $$

vi Correct elastic Almansi strain and Cauchy stresses and store in the database.

Box 7: Numerical scheme of hyperelastic model

An important advantage of this scheme is the fact that it is not necessary to compute explicitly the multiplicative decomposition of the deformation gradient, then important operations are saved at gauss point level.

It is worthwhile to note that it is enough to store in the code database, for a given time step ${}^{t}\ t$, the global displacement ${}^{t}\boldsymbol{u}$, plus the free and internal variables ${}^{t}\boldsymbol{b}^{e-1}$ and ${}^{t}\boldsymbol{\alpha}$, respectively. The storage requirements are similar to the small strain problems.

4. NUMERICAL APPLICATION

In this problem, known as Taylor's bar, the impact of a cylindrical copper bar with initial velocity of 227 m/seg against a rigid wall is studied. The initial length and radius are 32.4 y 3.2 mm respectively. The material properties are the ones of the copper: E= 117 GPa, $\nu = 0.35$, $\sigma_y = 0.4$ GPa, and H= 0.1 GPa. The bar has been discretized using 216 Q1/P0 elements.

The mesh pattern is based on six elements in radial sense and 36 elements in longitudinal sense. The boundary conditions in the rigid wall are simulated using sliding supports, i.e., let to the bar move freely in the radial direction. The studied transient occurs in 80 μs. At this time the bar departs from the rigid wall. In Figure 1 in the same scale, the original geometry and different deformed shapes during the transient are shown.

In order to modelate the transient about of 10000 adjustable size time steps have been used. The scheme for adjusting the size of time steps is based on Courant's

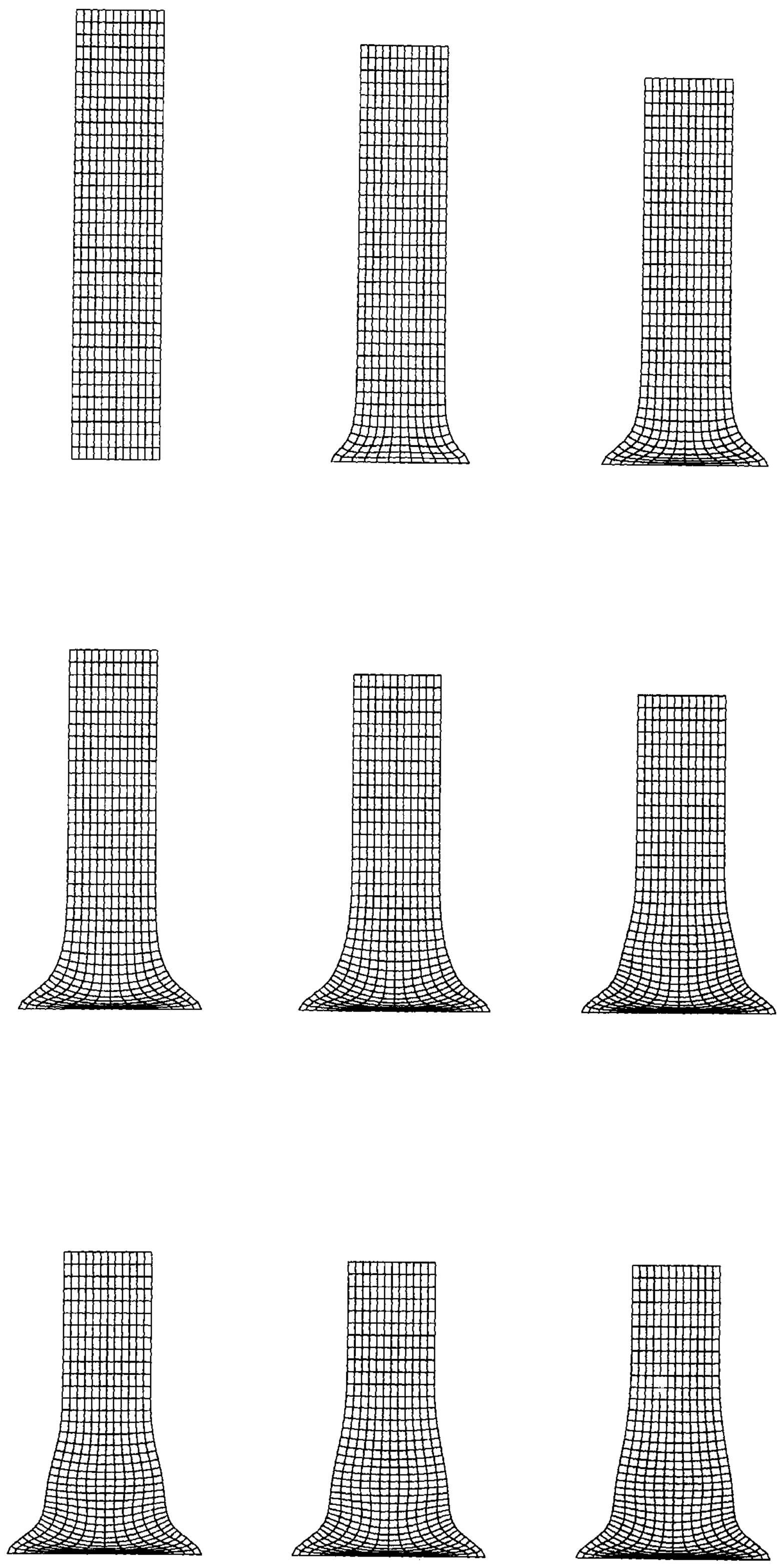

Figure 1. Impact of a bar. Original and deformed shapes

number as usual.

The obtained results are in an excellent agreement with the available ones, as can be seen in Table 2, where final length and radius, as well as maximum effective plastic strain are given.

CODE	Final Radius	Final Length	Max. $\bar{e}_p$
This work	7.11	21.47	3.09
NIKE2D (Hallquist, 1986)	7.07	21.47	2.97
DYNA2D (Hallquist, 1982)	7.13	21.47	3.05

Table 2 Results comparison

5. CONCLUSIONS

A constitutive model for the analysis of elastoplastic solids undergoing large strains has been presented. The main elements for the numerical implementation of the model have been given, and the concept of hyperelasticity is fully exploited. One important novelty is that the usual assumption of small elastic strains, suitable for metals, is taken into account a posteriori.

The numerical algorithm derived is very simple and easy to code in any nonlinear finite element program. The validation test choiced in the paper, the classical problem of Taylor's, shows an excellent agreement with other results available in the literature.

REFERENCES

C.García Garino 1993 *A Numerical Model for the analysis of large strain elastoplastic solids*, Ph.D. Thesis (in spanish), UPC, Barcelona.

García Garino C. & Oliver J. 1992. A numerical model for elastoplastic large strain problems. Fundamentals and applications, in R. Owen et al (eds.), Computational Plasticity III, Pineridge Press-CIMNE, 117-129.

Hallquist, J.O. 1982. User's Manual for DYNA2D. Report UCID-18756, Rev. 1. Lawrence Livermore National Laboratory.

Hallquist, J.O. 1986. NIKE2D, Report UCID-19677, Rev 1. Lawrence Livermore National Laboratory.

Lee, E.H, 1969. Elastic-Plastic deformation at finite strains. J. Appl. Mech., 36:1-6.

Marsden J.E. & Hughes T.J.R., 1983. *Mathematical Foundations of Elasticity.* Englewood Cliffs: Prentice Hall.

Simo, J.C. & Ortiz M., 1985. A unified approach to finite deformation elastoplastic analysis based on the use of hyperelastic constitutive equations, Comp. Meth. Appl. Mech. Engng. 49:221-245.