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Simulation of cyclic stress-strain relation under nonproportional loading

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ABSTRACT: A series of cyclic constitutive experiments have been conducted on 42CrMo steel on MTS809 machine under tension-torsional loading. Thin-walled tube specimen were used. Two kinds of cruciform strain path have been investigated. The paper suggests a simple method for the calculation of stable cyclic stress and strain values based on a modified endochronic constitutive theory(MECT) by redefined intrinsic time scale.

1 INTRODUCTION

Recent results show that the cyclic deformation behaviour is different when under proportional loading is compared with nonproportional loading. It has been hypothesized that the rotation of the principal axes during each cycle of the loading causes additional cyclic hardening in the material, which results in increased damage leading to shorter fatigue life(Socie 1987 and Chen et al 1994). As a consequence, several constitutive theory has been developed to describe cyclic deformation behaviours under nonproportional loading.

The endochronic constitutive theory(ECT) proposed by Valanis (1980) has received widespread attention as it is capable of accurately describing the constitutive behaviour of materials under certain types of loading conditions. The theory is primarily formulated on the basis of the hypothesis that the current stress state of material can be represented in terms of a function concerning the entire deformation history defined by the "intrinsic time scale", implying material memory. But endochronic constitutive theory is incapable of accurately describing the constitutive behaviours of material under nonproportional loading although it does provide good predictions for the plastic behaviour corresponding to some specific deformation path(Ning and Chen 1991). It is therefore, in general, necessary to have the endochronic constitutive theory modified to describe the constitutive behaviour of material under nonproportional loading.

Strain controlled cyclic constitutive experiment have been conducted on 42CrMo steel on MTS809 machine under tension-torsional loading. Thin-walled tube specimen were used. Two kinds of cruciform strain path have been investigated and is shown in Fig.1. The paper suggests a simple method for the calculation of stable cyclic stress and strain values based on a modified endochronic constitutive theory(MECT) by redefined intrinsic time scale considering loading path effects.

2 EXPERIMENTS

The 42CrMo high-strength steel material was tested with chemical composition: C:0.43, Si:0.34, Mn:0.64, S:0.011, P:0.022, Cr:1.08, Mo:0.22 in weight percentage. Thin-walled tubes with outside and inside diameter of 25mm and 21mm were used. Experiments were conducted on a computer-controlled testing system comprising a DEC PDP/11 processor, a MTS809 servo-controlled electro-hydraulic testing machine and a data acquision system. Strain components in the specimen were evaluated using MTS extensometer with a gage length of 25mm and diameter of 25mm. Fully reversed tensile strain range and shear strain range of trinanglar waves were used. Equivalent strain rate was maintained at $0.004s^{-1}$. Equivalent strain range were 0.47% for two kinds of cruciform path.

The definition of the axial torsional subspace follows as a subspace of Ilyushin's five-dimensional deviatoric vector space. Define the stress vector as

$$\boldsymbol{\sigma}=\sigma_1\boldsymbol{n}_1+\sigma_3\boldsymbol{n}_3 \tag{1}$$

where $\sigma_1=\sigma$, $\sigma_3=\sqrt{3}\tau$ is the effective shear stress, and σ and τ are components of the axial stress and shear stress, respectively, while $\boldsymbol{n}_1$ and $\boldsymbol{n}_3$ are orthonormal base vectors in the stress space.

The strain vector is defined as

$$\boldsymbol{\epsilon}=\epsilon_1\boldsymbol{n}_1+\epsilon_3\boldsymbol{n}_3 \tag{2}$$

where $\epsilon_1=\epsilon$, $\epsilon_3=\gamma/\sqrt{3}$ is the effective shear strain, and ϵ and γ are components of axial strain and engineering shear strain, respectively.

The Von Mises equivalent stress may be expressed as

$$\sigma_e=|\boldsymbol{\sigma}|=[\sigma_1^2+\sigma_3^2]^{1/2} \tag{3}$$

The equivalent strain is expressed as

$$\epsilon_e=|\boldsymbol{\epsilon}|=[\epsilon_1^2+\epsilon_3^2]^{1/2} \tag{4}$$

3 MODIFIED ENDOCHRONIC CONSTITUTIVE THEORY

The endochronic constitutive theory(ECT) has been formulated on the basis of the irreversible thermodynamics of internal variable. The theory does not have to rely upon the yield surface concept, and the material memory is defined in terms of an intrinsic time scale, a material property at hand.

In deviatoric vector space, the basic equation of endochronic theory is

$$\sigma=\int_0^z \rho(z-z')\frac{\partial e^p}{\partial z'}dz' \tag{5}$$

$$dz=d\xi/f(\xi) \tag{6}$$

$$d\xi=[(d\epsilon_1^p)^2+(d\epsilon_3^p)^2]^{1/2} \tag{7}$$

where $d\epsilon_1^p$ and $d\epsilon_3^p$ being components of plastic strain increments.

According to Ning and Chen(1991), endochronic constitutive theory cannot accurately describe the hardening behavior and plastic flow properties of material under nonproportional loading. This indicates that the intrinsic time scale defined in current endochronic constitutive theory is inadequate. It cannot reflect the effects of hardening mechanism and plastic flow properties of materials under nonproportional loading. For nonproportional cyclic loading the intrinsic time scale has to be redefined(Ning 1991).

The intrinsic time scale is redefined as

$$dz=\frac{d\xi}{f(\xi)\,g(\xi)} \tag{8}$$

where $f(\xi)$, $g(\xi)$ is hardening function and shape function respectively, and for stable cyclic condition $f(\xi)=1$. One propose a linear function simply as

$$g(\xi)=1+c_1\phi \tag{9}$$

where ϕ is nonproportionality corresponding to loading path shape.

$$\phi=\sin^2\theta=1-\cos^2\theta \tag{10}$$

$$\cos\theta=\frac{\dot{\epsilon}_{ij}^p\epsilon_{ij}^p}{|\dot{\epsilon}_{ij}^p|\cdot|\epsilon_{ij}^p|} \tag{11}$$

When $\phi=0$, for proportional loading, and $0<\phi\leq 1$, for nonproportional loading.

In the case where the strain history is prescribed according to Murakami and Read(1987), one has

$$\begin{aligned}&\left[\left(\frac{Q_1}{R+E}\right)^2+\left(\frac{Q_3}{R+3G}\right)^2-f^2(\xi)\,g^2(\xi)\right]dz^2+\\&+2\left[\frac{d\epsilon_1Q_1E}{(R+E)^2}+\frac{3d\epsilon_3Q_3G}{(R+3G)^2}\right]dz+\left[\frac{d\epsilon_1E}{R+E}\right]^2+\left[\frac{3d\epsilon_3G}{R+3G}\right]^2=0\end{aligned} \tag{12}$$

where

$$d\sigma = R d\epsilon - Q dz \tag{13}$$

$$Q = Q_1 n_1 + Q_3 n_3 = \sum_{r=1}^{3} \alpha_r (Q_{r1} n_1 + Q_{r3} n_3) = \sum_{r=1}^{3} \alpha_r Q_r \tag{14}$$

$$\frac{dQ_r}{dz} + \alpha_r Q_r = R_r \frac{d\epsilon^p}{dz} \tag{15}$$

$$R = \sum_{r=1}^{3} R_r \tag{16}$$

$$d\epsilon_1^p = d\epsilon_1 - d\sigma_1 / E \tag{17}$$

$$d\epsilon_3^p = d\epsilon_3 - d\sigma_3 / 3G \tag{18}$$

where G and E being the material shear modulus and young's modulus, respectively. The model parameters α_r and R_r are available from material uniaxial cyclic stable stress-strain relations as

$(\alpha_1, \alpha_2, \alpha_3) = (18779, 743, 35)$

$(R_1, R_2, R_3) = (6967, 143, 19)$ GPa

E=210 GPa, G=82 GPa

The model parameter c_1 are available from a material nonproportional cyclic experiment. For 42CrMo steel, $c_1=0.25$.

Fig.2 and Fig.3 show the hysteresis loops simulated by modified endochronic constitutive theory under two kinds of cruciform strain path. One note that different loading order produce obvious differene strain response although they have same path shape.

4 CONCLUSIONS

The experimental and theoretical investigation of cyclic behavior under nonproportional loading show that the modified endochronic constitutive theory(MECT) can describe the cyclic constitutive behaviours of materials under nonproportional cyclic loading. The modification involves the redefinition of the intrinsic time scale accounting for loading path dependence.

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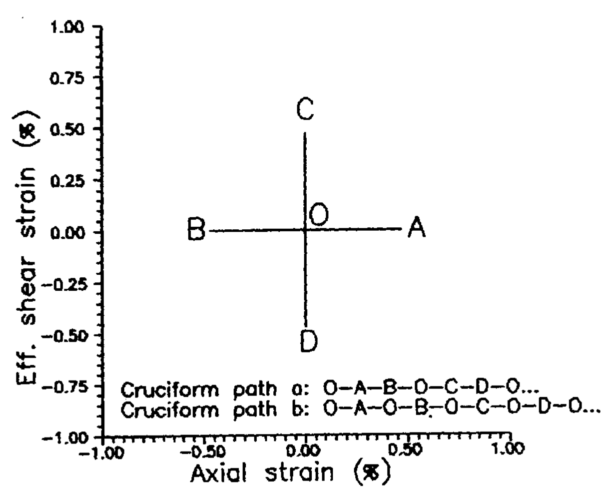
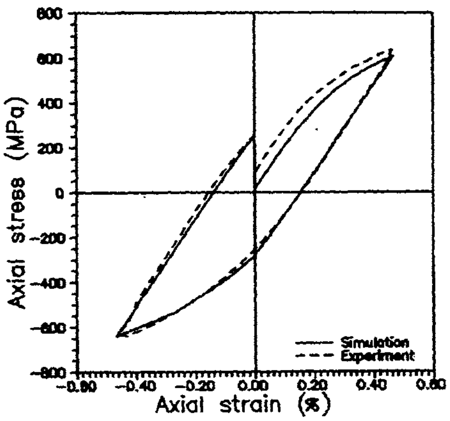
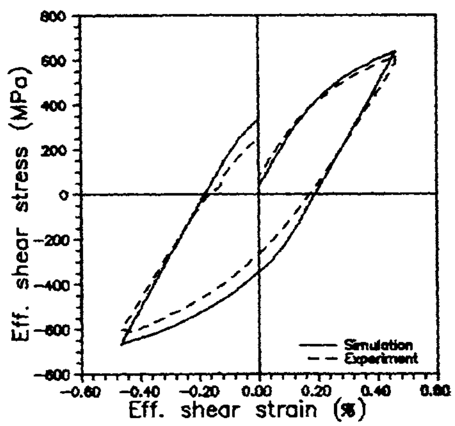


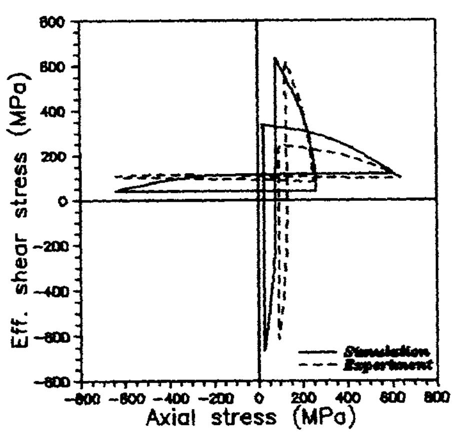
Fig.1 Two kinds of cruciform strain paths



(a) Axial

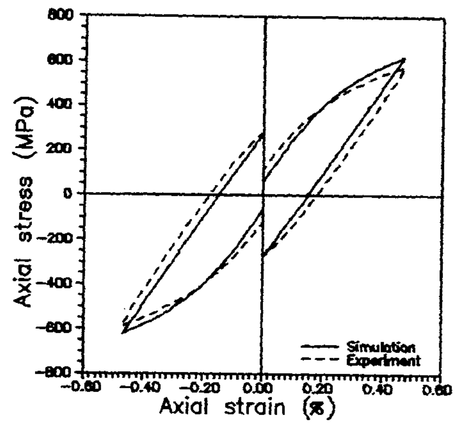


(b) Torsional

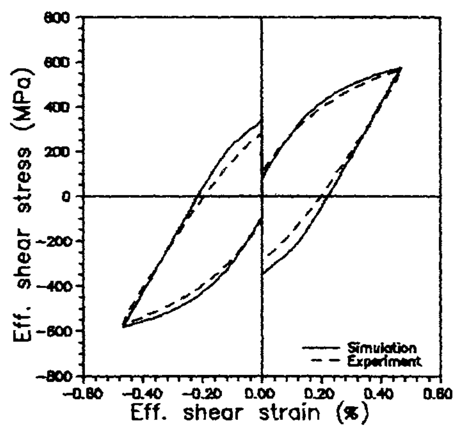


(c) Stress response

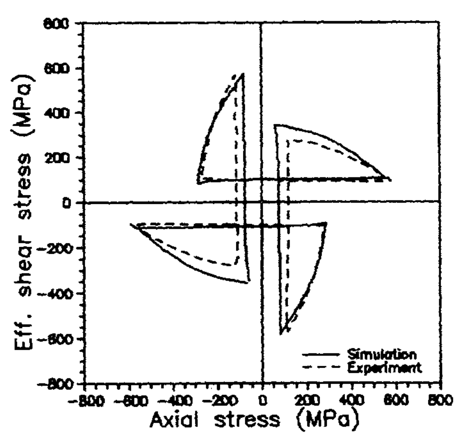
Fig.2 Comparision of simulation with experiment under cruciform path a



(a)Axial



(b) Torsional



(c) Stress response

Fig.3 Comparision of simulation with experiment under cruciform path b