Three dimensional elasto-plastic constitutive law for the description of ratchetting of 316 stainless steel

Frelat, J.¹, Taberi, S.¹, Geyer, Ph.², Gendrot, S.J.², Proix, J. M.³
1) EDF, Clamart, France
2) EDF, Moret sur Loing, France
3) EDF-SEPTEN, Villeurbanne, France

ABSTRACT: A constitutive elastic-plastic law with a discrete memory variable, plastic strain at the last unloading and a ratchetting stress as a material parameter is proposed. The essential feature of this law is that it contains no non proportional parameter. However with only uniaxial identification acceptable results are obtained for ratchetting in uniaxial and non proportional cases. Different results are presented for homogenous isothermal situations or non anisothermal and non homogenous situations. We compare experimental data, and simulations obtained by the present model, using either uniaxial or non proportionnal identification and with some other models.

1 INTRODUCTION

One of the main problems for the elastic-plastic constitutive laws is their none ability for a good representation of ratchetting phenomena. In fact the real problem is that with a uniaxial identification it is not possible to obtain a good multiaxial response. That is why some authors separate 1D and 2D ratchetting [1]. This means that an identification on some type of loading may give bad responses on other types of loading, so an identification on multiaxial loading may also give bad responses for in service loadings. We have proposed in [1, 2, 3] a law with a discrete memory variable, the plastic strain at the last unloading, and a ratchetting stress. This law gives acceptable results in multiaxial case where only uniaxial data is used for identification. It is able to represente the following properties: cyclic hardening in a push-pull test, cyclic softening after overloading, elastic shakedown, plastic shakedown and ratchetting (constant increment of strain). On the other hand the choice of all macroscopic variables is justified by a micromechanical analysis. This law has been implemented in the" Code Aster" [5], a thermomechanical structural software using the f.e.m, developed at Electricité de France. In this paper we briefly describe the proposed model, and for a 316 stainless steel we present some comparisons between experimental data and simulations obtained by this model and some other models. There is two types of comparison, one using only uniaxial identification and the other one multiaxial identification.

2 MACROSCOPIC VARIABLES THROUGH MICROSTRUCTURE

The macroscopic variables are defined through a microstructural analysis using the theory of dislocations. This permits to have a minimal number of variable, which is very
important for parameter identification. We define now the macroscopic variables in relation with the microscopic analysis:

- \( \varepsilon^{p} \) usual plastic part of the strain, related to the gliding of dislocations,
- \( \sigma_{p} \) maximal past absolute value of stress supported by the material in his history, related to the actual mean size of cells, this variable is used partly as \( S = \sigma_{p} \) where \( S \) is the ratchetting stress.
- \( \varepsilon^{p}_{n} \) plastic deformation at the last unloading point. Here the significant variable is the difference \( \varepsilon^{p} - \varepsilon^{p}_{n} \), on stabilized cycles. It measures the amplitude of plastic deformation, which may be related to sweeping of cell volume by the active dislocations
- \( \lambda \), cumulated plastic strain, related to the density of dislocations. But to take into account the interaction between cell size and dislocation density we use instead the variable \( \lambda(1-\sigma_{p}/S) \)

3 UNIAXIAL CONSTITUTIVE LAW

3.1 Natural introduction of \( \varepsilon^{p}_{n} \)

In the case of ratchetting (constant increment of strain), after some cycles the tension and compression curves are translated at each cycle of a constant quantity. So we have (figure1):

\[
\sigma_{l} = f(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n}) \text{ and } \sigma_{c} = g(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n+1}) \text{ with } f', g'', g' > 0 \text{ and } f'' < 0
\]

where \( \varepsilon^{p}_{2n} \) (resp \( \varepsilon^{p}_{2n+1} \)) is the plastic strain at the last unloading on the compression (resp. tension) curve. We suppose that there is no ratchetting phenomena for a symmetrical loading, (for \( \sigma_{\min} = -\sigma_{\max} \) we obtain plastic or elastic shakedown). It is so possible to show that the general form of tension and compression curves in the ratchetting state are:

\[
\sigma_{l} = (\varepsilon^{p}_{n} - \varepsilon^{p}_{2n}) * Q(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n}) + R(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n})
\]

\[
\sigma_{c} = (\varepsilon^{p}_{n} - \varepsilon^{p}_{2n+1}) * Q(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n+1}) - R(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n+1})
\]

We studied the case \( Q=0 \) in [6]. A better choice is given by the case \( Q=K \) (\( K \) a non zero constant) [2]. This is the model which is used for the simulations. We have so:

\[
\sigma_{l} = K(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n}) + R(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n})
\]

\[
\sigma_{c} = K(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n+1}) - R(\varepsilon^{p}_{n} - \varepsilon^{p}_{2n+1})
\]

This obviously remembers the yield function with combined kinematic isotropic hardening in 3D situation:

\[
\sum_{ij}^{2} \left( \frac{s_{ij} - x_{ij}}{s_{ij} - x_{ij}} \right) - R = 0
\]

which gives in uniaxial case:
\[ \sigma_t = 1.5 \epsilon_{11} + R \text{ in tension and } \sigma_c = 1.5 \epsilon_{11} - R \text{ in compression.} \]

3.2 Introduction of a ratchetting stress \( S \)

We suppose that in the uniaxial case ratchetting (constant increment) is obtained when the maximal stress (or minimal stress in absolute value) reaches the value \( S \) [2]. But for the stresses smaller than this value we have elastic or plastic shakedown. The simplest way to obtain this is to transform the expression \( K (\epsilon^p - \epsilon^p_{2n}) \), into \( K (S \epsilon^p - \sigma_p \epsilon^p_{2n}) \)

where \( \sigma_p \) is the maximal stress obtained during the history of loading. The expression of tension and compression curves in the case of plastic shakedown are so:

\[
\sigma_t = K (S \epsilon^p - \sigma_p \epsilon^p_{2n}) + R (\epsilon^p - \epsilon^p_{2n}) \\
\sigma_c = K (S \epsilon^p - \sigma_p \epsilon^p_{2n+1}) - R (\epsilon^p - \epsilon^p_{2n+1})
\]

It is possible to describe schematically the obtention of ratchetting and plastic shake down. In fact the above two relations constitute a recurrent one. For \( X = \epsilon^p_{2n} \), \( Y = \epsilon^p_{2n+1} \), we get two curves:

\[
C1\leftarrow K (S \epsilon^p - \sigma_p X) + R (\epsilon^p - \epsilon^p_{2n}) - \sigma_{\text{max}} = 0, \\
C2\leftarrow K (S \epsilon^p - \sigma_p X) - R (\epsilon^p - \epsilon^p_{2n}) - \sigma_{\text{min}} = 0
\]

For \( S > \sigma_p \), \( C1 \) and \( C2 \) are two intersecting curves and with an initial state of strain \( \epsilon^p_0 \), plastic or elastic shakedown is obtained [2] (figure 2). For \( S = \sigma_p \), \( C1 \) and \( C2 \) are two parallel lines and ratchetting is obtained. However if \( \sigma_{\text{max}} = -\sigma_{\text{min}} \) the two lines are superposed and plastic or elastic shakedown is obtained.

3.3 Introduction of cumulated plastic strain \( \lambda \)

Introduction of this variable permits to get the hardening and the softening for strain controlled tests. However in a strain controlled test once stabilization is obtained for a greater amplitude of strain the stabilization will be obtained in one cycle. This difficulty will be surrounded if we use instead of \( \lambda \) the parameter \( \lambda (1 - \sigma_p / S) \). Finally as the simplest choice we may take [2]:

\[
\sigma_t = K (\lambda (1 - \sigma_p / S)) (S \epsilon^p - \sigma_p \epsilon^p_{2n}) + f (\lambda (1 - \sigma_p / S)) R (\epsilon^p - \epsilon^p_{2n}) \\
\sigma_c = K (\lambda (1 - \sigma_p / S)) (S \epsilon^p - \sigma_p \epsilon^p_{2n+1}) - f (\lambda (1 - \sigma_p / S)) R (\epsilon^p - \epsilon^p_{2n+1})
\]

3.4 Cyclic stress strain curve

The usual cyclic stress strain curve for a symmetrical loading is given by:

\[ y = (1.5K_{\infty} S x + R (2x)) / (1-1.5K_{\infty} x) \]

where \( y = \Delta \sigma / 2 \), \( x = \Delta \epsilon^p / 2 \), and \( x < (1/1.5 \ K_{\infty} x) \), and \( K_{\infty} = K (\lambda = \infty) \). If there is a
prehardening \( \sigma_p \), the cyclic curve is given by:

\[ y = 1.5 K_\infty (S+\sigma_p)x + R(2x) \]

We have also \( \sigma_{mean} = 1.5K_\infty (S-\sigma_p) \varepsilon_{mean} \), this gives a possibility of a good relaxation for important prehardening, which seems being the case for 316 stainless steel.

4. THREE-DIMENSIONAL LAW

The extension to three dimensional situations of the previous uniaxial law has been done in the simplest possible way. In stead of scalar variable, the deviatoric part of the tensors are chosen to keep the uniaxial law's general form. The constitutive law is now simply described by an elastoplastic model where the yield function combines isotropic and kinematic hardening:

\[ F(\sigma, \varepsilon^p, \lambda, \sigma_p, \varepsilon_n^p) = |\sigma_D - X(\varepsilon^p, \varepsilon_n^p, \lambda, \sigma_p)| - R(\lambda, \sigma_p, |\varepsilon^p - \varepsilon_n^p|) \]

The usual normality and consistency relations are used for the remaining variables \( \varepsilon^p \) and \( \lambda [3] \). The variable \( \sigma_p \) is defined as the maximal past deviatoric norm of the stress experienced by the material - the norm is denoted by \( |\sigma_p| \) - With initial value \( \sigma_p^0 \) of \( \sigma_p \), the precise definition is:

\[ \sigma_p(t) = \left( \max_{u \in [0,t]} \left( \sigma_p^0, |\sigma_D(u)| \right) \right) \]

and we can rewrite this definition as a new yield function \( G \) in the deviatoric stress space centered at the origine:

\[ G(\varepsilon^p, \lambda, \sigma_p, \varepsilon_n^p) = \sigma_p - X1(\varepsilon^p, \lambda, \sigma_p)| - R1(\lambda, \sigma_p, |\varepsilon^p - \varepsilon_n^p|) \]

leading to the evolution equation for \( \sigma_p \) (H is the Heviside function):

\[ \dot{\sigma}_p = H(|\sigma_D| - \sigma_p) \frac{\sigma_D \dot{\sigma}}{|\sigma_D|} \]

The model is so in the deviatoric space a three surface model: loading surface, maximal surface represented by maximal stress defined previously and centered at origine, and ultimate surface centered at origine and defined by the ratchetting stress \( S \) (figure 3a).

Two problems arise from the definition of the evolution law of \( \varepsilon_n^p \) [3]. The first one is that the material behavior admits some undershooting of the monotonic stress-strain curve after an elastic unloading followed by reloading. However this is not always a disadvantage [4]. The second problem, more important from a physical point of view, is the requirement of continuity of the stress-strain curve with respect to very small unloadings. With full discrete memory, this requirement is generally not fulfilled: any unloading, even as small as possible, leads to an (discontinuous) evolution of the memory variable which induces in turn a discontinuity on the value of the yield function \( F \). This last discontinuity can finally causes the violation of the yield condition \( F < 0 \). To overcome this last difficulty, we have transformed [3] the discrete evolution law for \( \varepsilon_n^p \) to a semi discrete one - the word semi-discrete is used because of the saturation of the memory ensuing from the definition of the evolution. We introduced a scalar differential evolution equation and a consistancy condition ensuring the fulfillement of the yield condition.
\[ \dot{\varepsilon}_n^P = \zeta \left( \varepsilon^P - \varepsilon_n^P \right) \quad \text{if} \quad F = 0 \quad \left( \frac{\partial F}{\partial \sigma} \right) \leq 0 \]
\[ \zeta \geq 0 \quad \alpha F = 0 \quad F \leq 0 \]

It can be seen that, with appropriate generalized hardening conditions on the yield function \( F \) [3] we have:
- the yield condition \( (F < 0) \) is never violated,
- the continuity with respect to the chronology parameter is restored,
- the memory shows a saturation effect: when, during the unloading, the value of \( \varepsilon_n^P \) reaches \( \varepsilon^P \), then \( \varepsilon_n^P \) stays at this value and the unloading becomes purely elastic with no internal variable evolution,
- for uniaxial cycling loadings, the discrete memory is recovered between two successive unloadings, provided the cycle is large enough.

So we can exhibit four types of increment for this model, by opposition with the two states, elastic and elastic-plastic, of a standard plasticity model see figure 4b, 4c, 4d, 4e.

- A Purely Elastic increment, where only the variable \( \sigma \) is incremented, b
- A Pseudo-Elastic increment, where \( \varepsilon_n^P \) and \( \sigma \) are actualized, c
- An Elasto-Plastic increment, where \( \sigma, \lambda \) and \( \varepsilon^P \) are actualized, d
- And a Pseudo-Elasto-Plastic increment, where \( \sigma, \lambda, \varepsilon^P \) and \( \sigma_p \) are actualized e.

The model presented here has been implemented in the Code Aster ©, developed at Electricité de France [5].

5 COMPARISON WITH EXPERIMENTS AND OTHER MODELS

We use the following definitions for isotropic and kinematic hardenings:

\[ R = D \left( A |\varepsilon_p - \varepsilon_n^P|^{\alpha} + R_o \right) \]
\[ x = C \left( S e_p - \sigma_p \varepsilon_n^P \right) \]

\[ C = C_m + C_1 \varepsilon \]
\[ D = 1 - m e \]
\[ -b \lambda \left( 1 - \frac{\sigma}{S} \right) \]
\[ -b \lambda \left( 1 - \frac{\sigma}{S} \right) \]

Two types of comparison with experimental data are presented for a 316 stainless steel.

A) Identification is done only on uniaxial curves from some references, but the comparisons with non proportional data are done on other references. This is to test the robustness of the method.
B) Identification is done on uniaxial or non proportional loadings for some loading amplitudes. The comparison is done for other values of loading amplitude from the same reference. This is to test the precision of the method.

5.1 Case A uniaxial identification

For the identification, uniaxial strain controlled tests and stress controlled tests have been used from two different auteurs [8]. The parameters are:
at 20°C, $E = 188000$ MPa, $\nu = 0.3$

\[
S = 800 \text{ MPa} \\
A = 341 \text{ MPa} \\
C_1 = 6, 8 \\
m = 0, 264 \\
\alpha = 0, 122 \\
R = 150 \text{ MPa} \\
b = 11
\]

at 350°C, $E = 164000$ MPa, $\nu = 0.3$

\[
S = 613 \text{ MPa} \\
A = 858 \text{ MPa} \\
C_1 = 3, 6 \\
m = 0, 464 \\
\alpha = 0, 303 \\
R = 112 \text{ MPa} \\
b = 9, 6
\]

5.1.1 Results for a material point (homogenous case)

Figures 4a, 4b, 4c, show the results for constant tension and symmetrical cyclic torsion (strain controlled) tests. We compare the experimental data [9], with simulations obtained by the present model, and two other constitutive laws. The first one is a Chaboche model with two kinematical variables [10] identified on uniaxial experiments. The second one is a two kinematical Chaboche law modified by introduction of a radial evanescent memory named Burlet-Cailletaud[11], [12]. The two models are the same in uniaxial case. It worths noting that for this second model the tests 30, 35, 36 have been used for identifications.

On figure 5 we show the results for a strain controlled one step loading ($\Delta e = 0.8\%$) [13,]. We compare the experimental stabilized state in stress space with simulations obtained by the present model and the one in [13], for which non proportional experiments have been used for identifications. Other results for the cases of square loading, two-step loading, and circular loading are presented in [4].

5.1.2. Results for a structure (anisothermal non homogenous case) BI-TUBE

**Experimental device**: the BITUBE (figure 6a) is a particular structure which may produce a non homogenous anisothermal situation. This has been designed at INSA LYON to test the ability of constitutive laws in relation with ratchetting. It is constituted of two excentrique tubes related to two plates at the top and the bottom to impose the same vertical displacement to the two tubes. The loading is the combination of two types of loading, thermal loading applied by a self-induction on the outer tube where the inner one is kept at room temperature, and a constant tension applied vertically on the two tubes through a weight.

**Loading**: the temperature vary between 20 and 350 degrees C. A constante weight have been applied to the lower plate, which in some way simulate the pressure in a tube.

**Material caracteristiques**: the tubes are of 316L stainless steel, quenched. The elastic limit is, $R_0 = 225$ MPa at 20 degrees.

**Calculus**: the mesh is presented on the figure 6b. The upper plate is considered to be rigid elastic with a very important elastic modulus. Limite conditions are so that the axial displacements are zero at the top and are equal at the bottom. The horizontal displacements are free at the top and at the bottom.

**Resultats**: at point A we compare the axial strain increment (figure 6c) and strain stress curve (figure 6d) for experimental data [14] and for simulations obtained by the present
model, and the Chaboche model [15]. We may remind that the state of stress is practically uniaxial so the Burlet Cailletaud model gives practically the same result as Chaboche one. The calculus with the present model have been achieved for 7 cycles. The numerical convergence is more difficult for the present model than for the Chaboche model.

5.2 case B multiaxial identification

An identification has been realised using previous uniaxial tests and constant pressure cyclic tension tests [16]. The parameters are the followings:

at 20°C, $E=192000$ MPa, $\nu=0.3$

$$S = 1100 \text{ MPa} \quad A = 236 \text{ MPa} \quad C_0 = 6, 9 \quad R = 190 \text{ MPa}$$

$$C_1 = 10 \quad m = 0, 264 \quad \alpha = 0, 144 \quad b = 6$$

On figures 7a,7b,7c,7d we compare experimental constant pressure cyclic tension tests with simulations obtained by the present model using uniaxial identification (5.1) and also multiaxial identification (5.2). We also present on the same figures simulations obtained by Chaboche model identified on uniaxial data and Burlet-Cailletaud model identified on multiaxial data. We simulate also, using these new parameters the constant tension cyclic torsion tests, see figures 4a,4b,4c.

6 CONCLUSION

To get acceptable results for ratchetting during in service loading we propose to identify the model on uniaxial data where during identification we give more importance to tests for which the loadings are equivalent to the maximal of in service loading. In this way we obtain for smaller loadings un overestimating of strain, so a conservativ response.

REFERENCES

92.39A.
Figure 3a

Figure b

Figure c

Figure d
Constant tension cyclic torsion
\[ \sigma_{zz}= 157 \text{ MPa} \Delta \varepsilon_{\theta z} = 0.4\% \]
Strain % versus number of cycle

\textbf{FIGURE 4a} Test 30

Constant tension cyclic torsion
\[ \sigma_{zz}= 204 \text{ MPa} \Delta \varepsilon_{\theta z} = 0.4\% \]
Strain % versus number of cycle

\textbf{FIGURE 4b} Test 36

Constant tension cyclic torsion
\[ \sigma_{zz}= 204 \text{ MPa} \Delta \varepsilon_{\theta z} = 0.2\% \]
Strain % versus number of cycle

\textbf{FIGURE 4c} Test 35

One step loading, Strain controlled
\[ \Delta \varepsilon_{\text{eff}} = 0.8\% \]
Axial strain versus shear stress

\textbf{FIGURE 5} Test 30
Figure 6a Device

Figure 6b The mesh

Figure 6c Axial ratchetting gap versus number of cycle at point A

Figure 6d Axial stress strain curve at point A

Figure 6 BI-TUBE, uniaxial identification used
Constant pressure cyclic tension
\( P = 21.5 \, \text{MPa} \, \Delta \varepsilon \, zz = 0.9\% \)
Strain % versus number of cycle
**FIGURE 7a** Test 29

Constant pressure cyclic tension
\( P = 21.5 \, \text{MPa} \, \Delta \varepsilon \, zz = 0.5\% \)
Strain % versus number of cycle
**FIGURE 7b** Test 36

Constant pressure cyclic tension
\( P = 21.5 \, \text{MPa} \, \Delta \varepsilon \, zz = 0.72\% \)
Strain % versus number of cycle
**FIGURE 7c** Test 31

Constant pressure cyclic tension
\( P = 21.5 \, \text{MPa} \, \Delta \varepsilon \, zz = 0.82\% \)
Strain % versus number of cycle
**FIGURE 7d** Test 34