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A model for the reliability analysis of a protective channel including its test period

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ABSTRACT: This paper presents a stochastic model that allows for performing the availability analysis of a single protective channel considering a time period that includes its test period. Typically, this latter is considered by adding a corrective term to the unavailability expression for it is much less than the interval of the analysis. However, this is just a linear approximation that does not take into account a number of features that may influence the results.

1. INTRODUCTION

Usually, the unavailability analysis of standby safety systems is not performed considering the test duration time of its components for these latter are much lower than their proof test intervals in general. In this sense, it is common practice to add a corrective term to the mean unavailability expression to take into account the test duration time when considering one component only.

Whenever the reliability figure of merit is the system mean unavailability and the proof test interval is an integer submultiple of the plant downtime where the single protective channel is installed, it suffices to consider only one interval to solve the problem satisfactorily.

However, for those cases where the time interval of the analysis does not comprise an integer number of proof test intervals, it would be necessary to consider a whole cycle that contains an integer number of proof test intervals.

The purpose of this paper is to present a model that allows for developing the analysis discussed above and that takes into account explicitly the test efficiency, the possibility of channel failure induction due to human errors during maintenance, besides a channel test override.

The test duration time is supposed to be constant and so the model cannot be a Markovian one anymore. The complete model is made up by two stochastic models in fact, namely, a Markovian model that takes into account the proof test interval and a Semi-markovian model for the test duration time.

2. DISCUSSION OF THE STOCHASTIC MODEL

Our purpose is to obtain the plant accident rate considering an interval that contains a channel test. The analysis in this interval is developed by considering a Semimarkovian model for the residence times in the system states do not necessarily follow an exponential distribution.

A point must be stressed here: the channel test is an operational procedure not a system state. The system states are defined in terms of failures and sucess of the protective channel.

The channel test is performed obviously to improve its performance but in terms of the developed model it affects some of the transition probabilities and residence times. The transition diagram in itself is not affected by the channel test.

The channel is working at t=0. At $t=t_1^i$, the channel test begins and it lasts until $t=t_1^f$. The analysis is to be developed until a time t* such that $t_1^f<t^*\leq t_2^i$.

As in Oliveira & Amaral Netto (1987), the channel may be in one of three states: state 1 means that it is working; in state 2 the channel is failed but the failure is unrevealed. Finally, a transition to state 3 indicates that the failure is revealed. λ is the channel failure rate and it is the transition rate from state 1 to state 2. ν is the channel demand rate and is the transition rate from state 2 to state 3. In this latter, repair begins. μ is the channel repair rate and γ is the probability that the channel repair is perfect. Two transitions are possible here: from state 3 to state 1, with rate equal to $\mu\gamma$, and from state 3 to state 2, with rate equal to $\mu\bar{\gamma}$, where $\bar{\gamma} = 1-\gamma$.

The equation to be solved for the first interval is:

$$\dot{\underline{p}}(t) = \underline{\underline{A}}\,\underline{p}(t) \tag{1}$$

subject to the initial condition $\underline{p}(0) = (1\ 0\ 0)$, that is, the channel is working at t=0. The dot over $\underline{p}(t)$ indicates a time derivative. The transition rate matrix $\underline{\underline{A}}$ is given by:

$$\underline{\underline{A}} = \begin{vmatrix} -\lambda & \lambda & 0 \\ 0 & -\nu & \nu \\ \gamma\mu & \bar{\gamma}\mu & -\mu \end{vmatrix} \tag{2}$$

For the second interval (the test interval), the equation to be solved is:

$$\underline{p}\left(t_1^f\right) = \underline{p}\left(t_1^i\right).\underline{\underline{\Phi}}_s(n\Delta t) \tag{3}$$

where the time interval Δt is equal to $\left(t_1^f - t_1^i\right)/n$. The initial condition for solving Eq. (1) is given by $\underline{p}(t=t_1^i)$ which is the solution of Eq. (1) at the end of the intertest interval. The transition probability matrix $\underline{\underline{\Phi}}_s(n\Delta t)$ that appears in Eq. (3) is given by

$$\underline{\underline{\Phi}}_S(n) = {}^{>}\underline{\underline{W}}(n) + \sum_{m=0}^{n}\left[\underline{\underline{\Phi}}_M^* \otimes \underline{\underline{H}}(m\Delta t)\right].\underline{\underline{\Phi}}_s[(n-m)\Delta t] \tag{4}$$

where the symbol $\otimes$ indicates the congruent multiplication of matrices, Howard (1971), that is, for $\underline{\underline{A}} = \{a_{ij}\}$, $\underline{\underline{B}} = \{b_{ij}\}$, *and* $\underline{\underline{C}} = \{c_{ij}\}$, $\underline{\underline{C}} = \underline{\underline{A}} \otimes \underline{\underline{B}}$ implies that $c_{ij}=a_{ij}.b_{ij}$.

Eq. (4) can be understood if one writes it down in terms of the matrices elements:

$$\phi_{S_{ij}} = \delta_{ij}\, {}^{>}w_i(n\Delta t) + \sum_{k=1}^{N} \phi^{*}_{M_{ij}} \sum_{m=1}^{n} h_{ik}(m\Delta t)\phi_{S_{ij}}[(n-m)\Delta t] \tag{5}$$

We will now discuss Eqs. (4) and (5) in detail.

If the system were in state i at $t=t_1^{i}$, how could it reach state j at $t=t_1^{f}$? The first possibility is that i=j and so the system will never leave state i during this time interval, so that its first transition is necessarily performed for a $t > t_1^{f}$. Any other transition alternative from state i to state j in the interval requires that the system undergoes at least one transition during it. For example, the system could have undergone its first transition from state i to some state k in a time t' such that $t_1^{i}<t'\leq t_1^{f}$ and then by one or more transitions it could reach state j at $t=t_1^{f}$.

The Kronecker delta δ_{ij} assures that the term in which it appears will be nonzero only when i=j. The element ${}^{>}w_i(n\Delta t)$ is termed the complementary acummulated probability of the holding times, being the probability that the system will leave the initial state i at a $t>t_1^{f}$, Howard (1971).

The second term in Eq. (5) represents the occurrence probability of a sequence of events where the system undergoes its first transition from state i to some intermediate state k (where it is possible to have k=i) at a time t' and then by other transitions, during a time period equal to (n-m)Δt it reaches state j at $t=t_1^{f}$. This probability is summed over all the intermediate k states to which the initial transition could have been done and over all the first transition times m between 1 and n.

The transition probability matrix $\underline{\underline{\Phi}}^{*}_{M}$ comprises three Markovian models in fact, Azarm & Lofgren (1988):

1) system evolution in terms of failures and repairs of the protective channel;
2) channel test; and
3) test override (that will be denoted by TO).

Each of the above contributions will be modeled by a transition matrix, $\underline{\underline{\Phi}}_{M}$, $\underline{\underline{M}}_{T}$ and $\underline{\underline{M}}_{TO}$, respectively. These matrices are given by:

$$\underline{\underline{\Phi}}_{M} = \begin{vmatrix} 1-\lambda\Delta t & \lambda\Delta t & 0 \\ 0 & 1-\nu\Delta t & \nu\Delta t \\ \gamma\,\mu\Delta t & \bar{\gamma}\,\mu\Delta t & 1-\mu\Delta t \end{vmatrix} \tag{6}$$

and

$$\underline{\underline{M}}_{T} = \begin{vmatrix} p_1 & q_1 & 0 \\ 0 & q_2 & p_2 \\ 0 & 0 & 1 \end{vmatrix} \qquad \underline{\underline{M}}_{TO} = \begin{vmatrix} p_0 & q_0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{7}$$

The transition matrix $\underline{\underline{\Phi}}_M^*$ is written as the product $\underline{\underline{M}}_{TO}\underline{\underline{M}}_T\underline{\underline{\Phi}}_M$.

Eq. (6) is the transition probability matrix that models failures and repairs of the protective channel. It is written for a small time interval Δt so that the system probability vectoı can be obtained for time points equally spaced in the first time interval (that is, before the test). In this sense, the problem is solved via a Markov chain.

Let us now discuss Eq. (7). The element $m_{T,11}=p_1$ is the probability that the channel remains in state 1 after its test. q_2 is the probability that the test induces a channel failure. On the other hand, there is a probability equal to q_2 that the channel is failed and the test does not reveal the failure. The complementary probability p_2 is the probability that the test reveals the channel failure. The element $m_{T,33}$ is equal to 1 because if the channel is found in state 3 at the beginning of the test it does not make sense to perform it. The test override is modeled by the second matrix in Eq. (7). A test override is considered only when the channel is in state 1 at the beginning of the test.

With the system in state 1, a test override can induce it to remain in state 1, or to make a transition to state 2 or, finally, it may not occur (so to say a test override failure occurs). The first and the third alternatives have the same effect, that is, the channel remains in state 1, so that they will be merged. This discussion explains the test override matrix. It should be noted that if the test override is perfect, then $q_0=0$ and the identity matrix is obtained.

Now the elements of the transition matrix $\underline{\underline{\Phi}}_M^*$ can be explained. For example,

$$\phi^*_{M11}= m_{TO_{11}}m_{T_{11}}\phi_{M_{11}}+m_{TO_{12}}m_{T_{23}}\phi_{M_{31}} \tag{8}$$

The first term of Eq. (8) is the probability that the channel was in state 1 at $t=t_1^i$, remained in it ($\varphi_{M,11}$), then a test was successfully performed and also a successfull test override occurred. The second term is the other possibility: a test override occurs, it induces a channel failure, the test reveals the failure and repair begins and the channel returns to state 1. The remaining elements can be explained in a similar way.

Anothe matrix appears in Eq. (4). It is termed the residence times matrix, $\underline{\underline{H}}(m\Delta t)$, Howard (1971). An element $h_{ij}(m\Delta t)$ represents the probability density of the residence time in state i before the transition to state j. This means that the channel chooses a transition and holds in the initial state for a time whose probability distribution has a density given by this matrix. The residence times matrix for this model is given by:

$$\underline{\underline{H}}(m\Delta t)=\begin{vmatrix}\delta[(m-n)\Delta t] & \delta[(m-n)\Delta t] & \delta[(m-n)\Delta t]\\ \delta[(m-n)\Delta t] & \delta[(m-n)\Delta t] & \delta[(m-n)\Delta t]\\ \delta[(m-1)\Delta t] & \delta[(m-1)\Delta t] & \delta[(m-1)\Delta t]\end{vmatrix} \tag{9}$$

The integer n represents the time interval under consideration (the test interval in this case). The function δ(mΔt) is a discretized Dirac delta function, Howard (1971):

$$\delta(m\Delta t)=\begin{cases}1, & if\ m=0\\ 0, & otherwise.\end{cases} \tag{10}$$

The analysis of the possible channel transitions reveals that all transitions from state 1 to state 2 during the test period are under a Semimarkovian model, as can be seen from Eq. (9).

It now remains to write down matrix ${}^{>}\underline{\underline{W}}(n\Delta t)$ of Eq. (4):

$$ {}^{>}\underline{\underline{W}}(n\Delta t) = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \delta(n\Delta t) \end{vmatrix} \tag{11} $$

RESULTS OBTAINED BY THE MODEL

The model above was applied to a single channel considering that its failure equal is λ=10 per year and its repair rate is μ=365 per year. On the other hand, the proof test interval has been set equal to 30 days and the interval of the analysis, is equal to 45 days. The demand rate was varied in a wide range to observe the plant behavior as to its accident rate, Oliveira & Amaral Netto (1987).

Figure 1 displays the results considering that the repair policy allows for repair in any condition.

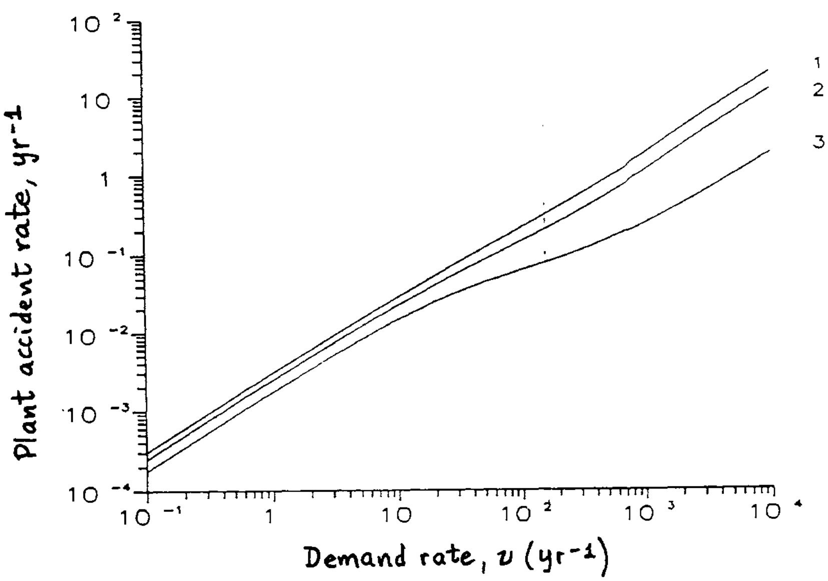

Figure 1. Plant accident rate for $\gamma=p_o=p_1=p_2$=0.6 (curve 1), 0.8 (curve 2) and 1.0 (curve 3) and online channel repair allowed.

Figure 2 presents results considering that the repair policy allows offline repair only. An ideal test is realized (and a possible test override also). The curves display the results considering that the test duration time ranges from intantaneous (curve 3) to 2 days (curve 1). An instantaneous test produces the better results as expected, although a significant variation is observed for higher demand rates only.

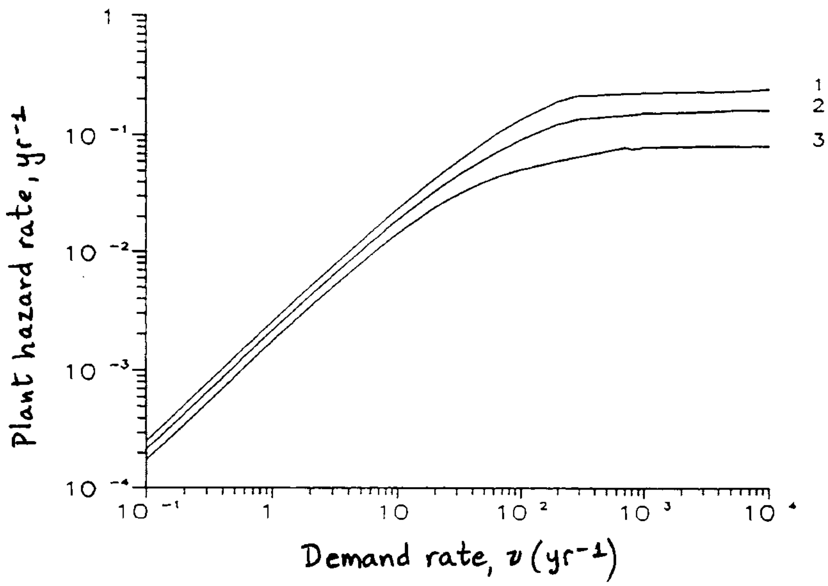

Figure 2. Influence of the channel test duration time on the plant accident rate, t_D=2 days (curve 1), 1 day (curve 2) and instantaneous (curve 3).

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