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Structural reliability computations and response surfaces

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ABSTRACT : To compute reliability index in case of complex mechanical model, it is often interesting to introduce a response surface. The object of this paper is to test classical methods used in the computation of reliability index on usual polynomial response surface. These used examples show that a blind use of these methods can lead to crude errors.

1 INTRODUCTION

In structural reliability analysis, the reliability of a structure with respect to a particular limit state is carried out via a mechanical model describing the limit state in terms of a function, called the limit state function or failure function g , whose value depends on all basic random variables denoted by $\overline{X}$. The limit state surface is given by

(1) $\quad g(\overline{X})=0$

and separates in the random variable space the safe set D_S and the failure set D_F.

The failure probability is calculated as :

(2) $\quad P_F=\int_{\overline{x}\in D_F} f_{\overline{X}}(\overline{x})d\overline{x}$

where $f_{\overline{X}}$ is the joint probability density function of the basic variables.

In general this integral cannot be computed analytically, and numerical integration or Monte-Carlo simulations are generally very time consuming. First and second order reliability method (FORM/SORM) based on the reliability index of Hasofer and Lind β_{HL} often gives a good approximation of the failure probability. (Rubinstein 81, Madsen 86, Bjerager 89)

When the mechanical models are too complex, the computations of solicitations or resistances must use finite element codes. Calling the code when the reliability computations needs it leads to expensive computations costs. Since the computation of index is easier and more reliable with an explicit and simple failure function, it becomes interesting to define a polynomial response surface as an interface between the finite element code and the reliability code. The response surface methodology seeks to relate a mechanical response to a subset of the random variables according a presumed functional relationship. Such response surface simulates the real behaviour of the mechanical response. Second order polynomial surfaces are often used by reason of simplicity. The coefficients are estimated from selected computations by least squares method. (Box 87, Shinozuka 87, El Tawil 91)

The purpose of this paper is to examine the behaviour of classical methods with regard to the following type of polynomials limit state function :

(3) $$g(x, y) = b + x - a(y-c)^2$$

2 RELIABILITY INDEX CALCULATION

The approximated FORM/SORM methods are based on the idea to isolate the region of the design space where the failure probability is the most important and to calculate approximations of the failure probability on this approached region. The first stage consists in transforming the basic random variables space into a standard normal and uncorrelated variables space. Since the probability density in the standard normal space decays exponentially with the distance from the origin, the point at which failure is most likely to occur (*design point*) is the point on the limit state surface of minimum distance to the origin. Differents methods to find the design point are studied in Liu 88 and Abdo 90. In this standard normal space, the limit state surface is approached by a hyperplan or a parabolic surface at the design point. The FORM approximation to P_F is:

(4) $$P_F \approx \Phi(-\beta_{HL})$$

where Φ is the standard normal distribution function and β_{HL} is the distance of the design point to the origin. A second-order approximation to the failure probability is given in terms of β_{HL} and the curvatures κ_i at the design point and called Breitung's approximation :

(5) $$P_{FB} \approx \Phi(-\beta_{HL})\prod_{i=1}^{n-1}(1-\beta_{HL}\kappa_i)^{-1/2}$$

Reliabity index β_B associated to Breitung's approximation is obtained by :

(6) $$\beta_B = -\Phi^{-1}(P_{FB})$$

Use of Monte-Carlo simulations gives also an estimate β_{MC} of the reliability index :

(7) $$\beta_{MC} = -\Phi^{-1}(P_{F,MC})$$

In the following of the paper, all the random variables will be considered as statistically independent and standardized normally distributed. err($\beta_{Methode}$) is defined by :

(8) $$err(\beta_{Methode}) = \left|(\beta_{Methode} - \beta_{MC})/\beta_{MC}\right|$$

3 EXEMPLE 1

Taking $b = 3$ and $c = 0$. Then the surfaces are defined by :

(9) $$g(x, y) = 3 + x - a y^2$$

The gradient vector verify

(10) $$\nabla g(x, y) = \begin{pmatrix} 1 \\ -2ay \end{pmatrix}$$

So depending on the constant values a , the axis (0x) will be favoured or not. Then for an initial point belonging to x-axis, as $\nabla g(x, y) = (1 \quad 0)^t$, the different methods will converge to the point $(-3,0)^t$. But the point $(-3,0)^t$ is not always the design point.

If a is equal to 1, then there are two design points $(-0.49;1.58)^t$ and $(-0.49;-1.58)^t$. For any initial point not belonging to x-axis, the algorithms converge always to one of the design point. The results obtained by Monte-Carlo simulations, FORM and SORM methods are given in table 1. The relative error is important: err(β_{FORM})=34.8% and err(β_{SORM}) = 64.9%. Indeed the FORM/SORM methods ignores the symmetry, and the curvatures at the design point cannot permit to obtain a correct parabolic approximation. Increasing the value of a will increase this relative error. Taking a equal to 5, the error is more important. The relative errors are err(β_{FORM})=519% and err(β_{SORM}) = 908%. The influence of the symmetry and the curvature in $(-3;0)^t$ grows, but the design points are not representative of the geometry. A system approach can give good results in terms of failure probability.

An uncertainty on the value of a implies a negligible variation on the results (table 1). The relative variation is near 4% if a varies between 0.9 and 1.1.

When the value of a decreases, the surface look almost like an hyperplane and the approximation made by FORM is quite correct. With a equal to 1/6, we obtain 3 and 2.70 respectively for β_{FORM} and β_{MC}. With a equal to 1/20, the results are 3 and 2.95 respectively for β_{FORM} and β_{MC}. The figure 1 shows the geometry for the different values of a. There exists a value a_0 such that for a less than a_0, an unique design point exists and has coordinates $(-3;0)^t$. For these values of a, FORM gives always a reliability index equal to 3. The value of a_0 is 1/6. An uncertainty on the value of a below has not influence on the results of FORM method and a weak influence on the results of SORM methods..

4 EXEMPLE 2

Taking now c equal to 1. The surfaces are defined by : (figure 2)

$$g(x,y)=3+x-a(y-1)^2 \tag{11}$$

When c differs of 0, an unique design point exists for any value of a. The results for different values of a are presented in tables 2 and 3.

If a is equal to 1, the FORM method gives better results that SORM method. The relative errors are err(β_{FORM}) = 6.5% and err(β_{SORM}) = 85.1%. The better behaviour of FORM is confirmed by other results. For a equal to 10, the values of the relative errors are respectively 29.6% and 60%. The reason of the worst behaviour of SORM method is the same that in exemple 1. The design point is not representative of the geometry of the surface. The knowledge of its coordinates and its curvatures don't permit to rebuild the limit state surface.

An uncertainty on the value of a implies a variation on the results (table 2). The relative variation is upper to 10% if the variation on the value of a is greater than 0.1.

When the value of a increases, the sensibility of the results to the variation on a vanishes. For a equal to 10 (a_2), the relative variation is 0.6% for FORM method and 0.1% for SORM method. The reason is the geometrical quasi-invariance of the parabola according a large interval near a_2. .

When the value of a decreases, FORM method gives better approximation of the reliability index. For a near to 0.5, the relative error is roughly 3%. The improvement comes from the fact that the branches of the parabola diverge and the values of the curvature tends to zero. An uncertainty on the value of a has a non negligible influence. The difference between the results for a equal to 0.45 and 0.5 is of 8.2%, and for a equal 0.5 and 0.6 is 13.3%.

5 CONCLUSION

From some simple and theoretical examples of polynomial surfaces, the computed results show that problems could arise in case of blind use of the FORM/SORM methods. A critical judgement of the results and the methods is always indispensable. The influence of the uncertainty on a coefficient of the polynomial representation can be not negligible. A such uncertainty appears often in the building of the response surface or when the choice of experiments is badly done. To verify and to improve the results, the use of another mathematical representations of the response surfaces could be necessary.

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Value of a	0.9	1	1.1	4.9	5	5.1
P_{MC}	0.104	0.119	0.0895	0.444	0.450	0.455
σ_{MC}	$0.40\ 10^{-3}$	$0.43\ 10^{-3}$	$0.45\ 10^{-3}$	$0.70\ 10^{-3}$	$0.70\ 10^{-3}$	$0.70\ 10^{-3}$
β_{MC}	1.254	1.177	1.348	0.139	0.124	0.110
P_F	0.048	0.056	0.041	0.218	0.221	0.223
β_F	1.658	1.587	1.739	0.775	0.768	0.760
P_B	0.022	0.026	0.018	0.104	0.105	0.106
β_B	2.004	1.941	2.077	1.257	1.251	1.245

Table 1 : Results for $g(x,y) = 3 + x - a\,y^2$

Value of a	0.9	1	1.1	9.9	10	10.1
P_{MC}	0.227	0.254	0.280	0.261	0.261	0.258
σ_{MC}	$0.59\ 10^{-3}$	$0.43\ 10^{-3}$	$0.63\ 10^{-3}$	$0.62\ 10^{-3}$	$0.62\ 10^{-3}$	$0.61\ 10^{-3}$
β_{MC}	0.747	0.659	0.520	0.638	0.640	0.646
P_F	0.215	0.241	0.265	0.327	0.326	0.325
β_F	0.788	0.702	0.627	0.447	0.450	0.453
P_B	0.098	0.111	0.122	0.152	0.151	0.151
β_B	1.280	1.220	1.162	1.027	1.029	1.031

Table 2 : Results for $g(x,y) = 3 + x - a(y-1)^2$

Value of a	0.45	0.5	0.6
P_{MC}	0.082	0.099	0.134
σ_{MC}	$0.38\ 10^{-3}$	$0.40\ 10^{-3}$	$0.48\ 10^{-3}$
β_{MC}	1.386	1.283	1.105
P_F	0.074	0.091	0.124
β_F	1.44	1.33	1.15
P_B	0.033	0.041	0.056
β_B	1.83	1.73	1.58

Table 3 : Results for $g(x,y) = 3 + x - a(y-1)^2$

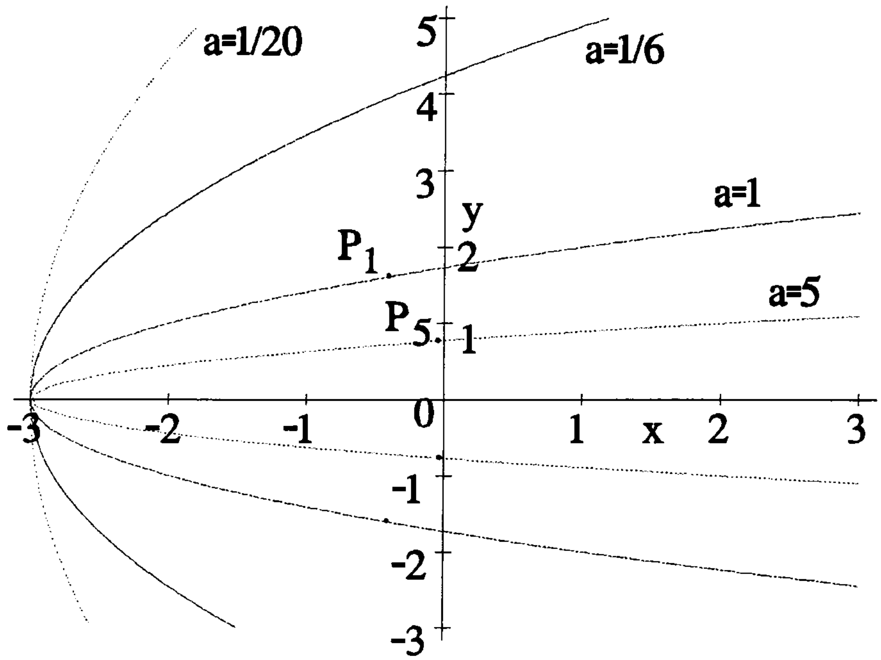

Figure 1 : Influence of a in $g(x, y) = 3 + x - a y^2$

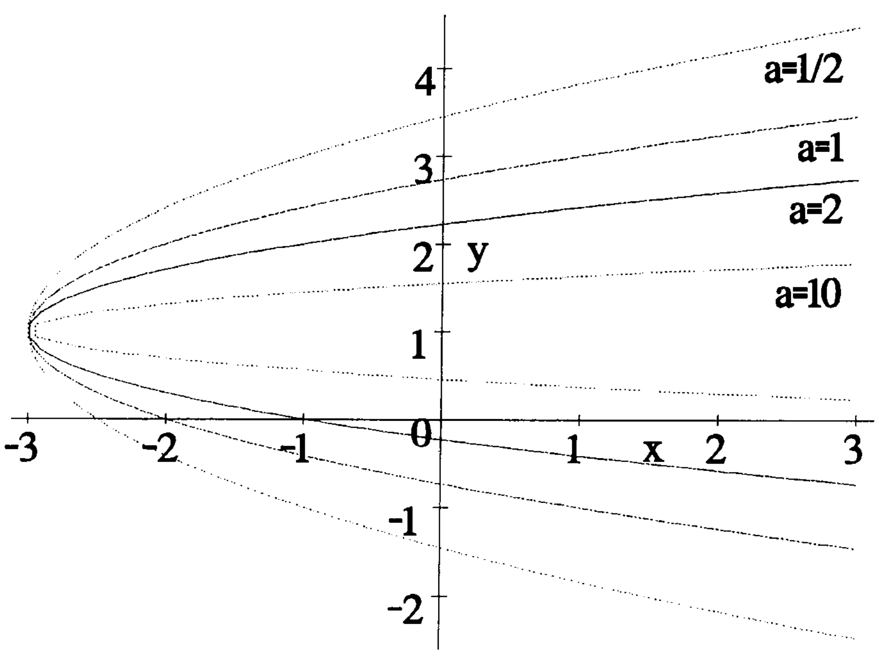

Figure 2 : Influence of a in $g(x, y) = 3 + x - a(y-1)^2$