A Substructure Method to Compute the 3D Fluid-Structure Interaction During Blowdown

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Abstract
The waves generated by a sudden rupture of a PWR primary pipe have an important mechanical effect on the internal structures of the vessel.

This fluid-structure interaction has a strong 3D aspect.

3D finite element explicit methods can be applied. These methods take into account the non linearities of the problem but the calculation is heavy and expensive.

We describe in this paper another type of method based on a substructure procedure:

The vessel, internals and contained fluid are axisymmetrically described (AQUAMODE Computer Code).

The pipes and contained fluid are monodimensionally described (TEDEL-FLUIDE Computer Code).

These substructures are characterized by their natural modes.

Then, they are connected to another (connection of both structural and fluid nodes) in the TRISTANA Computer Code.

This method allows to compute correctly and cheaply the 3D fluid-structure effects. But the treatment of certain non linearities is difficult because of the modal characterization of the substructures. However variations of contact conditions versus time can be introduced.

We present here some validation tests and comparison with experimental results of the litterature.
I. Introduction

During an hypothetical "loss of coolant accident" (LOCA) the pressure transient generated by the sudden break of a pressurized water reactor inlet pipe may induce large mechanical effects on the internal structures of the vessel.

This fluid-structure coupled problem is generally treated by using 3D finite element explicit methods able to scope with the strong non linearities which take place both in the structures and in the fluid.

However simplified methods may also be quite relevant at least for dealing with the early phase of the LOCA which is dominated by the acoustical propagation of pressure waves and their interaction with the structures. Indeed for a time of about 50 ms after the break the fluid is still monophasic almost everywhere in the circuit and the flow has just started at the broken pipe. At this step non linearities are expected to occur only at a few locations, for instance at the broken pipe outlet as the fluid is concerned.

Here we present a numerical method devoted to this early phase of the LOCA, which main features are as follows:

- a subsystem procedure allows to deal with the strong 3D aspect of the problem,
- each subsystem is described by a modal basis consisting in a set of acoustical-mechanical coupled eigen modes. This allows to scope with the wave propagation and the fluid-structure interaction,
- strong non linearities, if localized, may be taken into account either as external loads or as non linear connections between subsystems.

This method has been implemented in the computer code system CASTEM developed at CEA Saclay. Its main advantage is to reduce significantly the cost of computation in comparison with the 3D explicit codes.

II. Fluid-structure interaction

The formalism used in CASTEM for fluid-structure coupled systems described in the linear acoustical approximation has been presented before (see ref. 1,2). This leads to the following symmetrical linear system:

\[
\begin{bmatrix}
  K & 0 & 0 \\
  0 & 0 & 0 \\
  0 & 0 & A
\end{bmatrix}
\begin{bmatrix}
  x \\
  \Pi \\
  P
\end{bmatrix}
+ \begin{bmatrix}
  M - C & 0 \\
  -C^T & L \\
  0 & A^T & 0
\end{bmatrix}
\begin{bmatrix}
  \dot{x} \\
  \dot{\Pi} \\
  \dot{P}
\end{bmatrix}
= \begin{bmatrix}
  \dot{F} \\
  0 \\
  0
\end{bmatrix}
\]

\[\text{eq. (1)}\]

K and M are the stiffness and the mass matrix of the structure; C is the fluid-structure coupling matrix \((C^T)\) transposed of it), L and A are the discretized space and time operators of the wave equation. \(\dot{x}\) stands for the nodal displacements of the structures \(\Pi\) and \(P\) for the nodal acoustical variables, \(\dot{P}\) being the pressure and \(\dot{\Pi}\) an auxiliary variable defined by the third line of the system.

The first line of eq. (1) gives the dynamical equilibrium of the structure under the external mechanical load \(\dot{F}\). The second line gives the wave propagation under the external acoustical source \(\dot{\Pi}\).

Acoustical-mechanical modes are produced by solving the standard eigenvalue problem

\[
\begin{bmatrix}
  [K_{mm}] \\
  [M_{mm}]
\end{bmatrix}
- \omega^2
\begin{bmatrix}
  [M_{mm}] \\
  [K_{mm}]
\end{bmatrix}
\begin{bmatrix}
  x \\
  \Pi
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0
\end{bmatrix}
\]

\[\text{eq. (2)}\]
$K_{am}$ and $\mathcal{M}_{am}$ are the acoustical mechanical "stiffness" and "mass" matrices of the fluid structure system defined by eq. (1).

This problem is solved in the code TEDEL as the pipes are concerned and in the code AQUAMODE as the axisymmetrical system are concerned.

As the mechanical aspect is concerned modes at zero frequency may occur in the case of non supported structures. Concerning the fluid there is also a mode at zero frequency in the case of grad $p = 0$ boundary conditions, for such a mode $\Pi$ and $p$ are uniform. In addition the use of the auxiliary variable $\Pi$ introduces a set of numerical modes, hereafter called the "pseudo modes" which are also at zero frequency. For such pseudomodes $X$ and $p$ variables are equal to zero. As it will be seen below such zero frequency modes have to be taken into account in the connection process of the subsystems.

III. Substructuration procedure

The complete system is divided into independent subsystems which are connected by some link forces and some acoustical mass flow rates located at the connection points.

a) Representation of the subsystems by their eigen modes.

For each subsystem, the modal basis chosen correspond to a free basis : neither forces nor acoustical sources exist at the connections.

The eigen modes of the complete system without a connection are given by the global equation:

$$
\begin{bmatrix}
X \\
\Pi \\
p
\end{bmatrix}
- \omega^2
\begin{bmatrix}
\mathcal{M} \\
\Pi \\
p
\end{bmatrix}
= 0
$$

As $K$ is the assembly of the elementary matrices $[K_{am}]$ of the subsystems, the modal decomposition can be made on each of them. The set of the eigen modes gives a complete basis for the displacement and the acoustical variables.

Due to the presence of fluid, the eigen modes are divided into two kinds:

- the physical modes $\phi_i = \begin{pmatrix} X_i \\ \Pi_i \\ p_i \end{pmatrix}$ characterized by:

$$
(K - \omega_i^2 \mathcal{M}) \phi_i = 0 \quad \text{with} \quad \omega_i \neq 0
$$

$$
[K_G] = [\phi_i^T] [K] [\phi_i]
$$

$$
[M_G] = [\phi_i^T] [\mathcal{M}] [\phi_i]
$$

$$
K_{Gi} = \omega_i^2 M_{Gi}
$$

- the pseudo modes $\overline{\phi_i} = \begin{pmatrix} 0 \\ \overline{\Pi_i} \\ 0 \end{pmatrix}$ characterized by:

$$
(K) \overline{\phi_i} = 0 \quad \text{and} \quad \omega_i = 0
$$

$$
[K_G] = 0 \quad \text{and} \quad [\mathcal{M}_G] = [\phi_i^T] [\mathcal{M}] [\phi_i]
$$

All these modes are orthogonal with respect to the matrix $\phi_i^T \phi_i$. They form the modal basis.

Now, it is necessary to define how the subsystems are connected.

b) Formulation of the connections.

Two kinds of links have to be taken into account:

- first : the mechanical links formulated in terms of linear springs (see ref. [3]). The relative displacements $X_{Li}$ between two substructures at each connection are related to the link forces by:

$$
- 41 -
$$
\[ X_L = [L_X] X = [L_L^{-1}] F_L \]

where \([K_L^{-1}]\) is the flexibility matrix and \(L_X\) the projection matrix which relates the degrees of freedom of the substructures at the connection points.

- Second: the fluid connections.

At the fluid connections, the continuity of the fluctuating pressure must be assumed. This condition is formulated on the \(\Pi\) variable in order to keep a symmetrical system. In the case of connection between 2D or 3D subsystems (a vessel for example) and a 1D system (a pipe) a spatial average of \(\Pi\) over the connecting fluid area is applied before equalizing the corresponding \(\Pi\) values. This continuity is expressed by:

\[
[L_{\Pi}] \Pi = 0
\]

where \([L_{\Pi}]\) is the projection matrix which relates the \(\Pi\) variables of subsystems at the connection.

In addition, the conservation of the fluctuating mass flow rate \(Q_L\) through the connecting fluid areas has to be written down. This is done by an internal source vector:

\[
[L_{\Pi}] Q_L.
\]

Then, the complete system has to be in equilibrium under the external mechanical and acoustical loads and the connecting loads, this is expressed as follows:

\[
\begin{bmatrix}
[X]
\end{bmatrix}
\begin{bmatrix}
\Pi
\end{bmatrix}_P + \begin{bmatrix}
[M]\end{bmatrix}
\begin{bmatrix}
\Pi
\end{bmatrix}_P - \begin{bmatrix}
[L_L^{-1}] F_L
\end{bmatrix} = \begin{bmatrix}
S_{\text{ext}}
\end{bmatrix}
\]

\[
[L_X] X - [L_L^{-1}] F_L = 0
\]

\[
[L_{\Pi}] \Pi = 0
\]

eq. (4)

c) Modal projection

Here the eigen analysis of the complete system will be studied. Any vector \(\begin{bmatrix}
X
\end{bmatrix}
\) can be decomposed on the natural basis.

It is useful to distinguish the physical and the pseudo modes as follows:

\[
\begin{bmatrix}
X
\end{bmatrix} = \begin{bmatrix}
\alpha
\end{bmatrix} \alpha + \begin{bmatrix}
\tilde{\alpha}
\end{bmatrix} \tilde{\alpha}
\]

where \(\alpha\) and \(\tilde{\alpha}\) are the vector of modal contributions. The system (4) is transformed by Fourier and projected on the natural modes. Then, after elimination of \(\tilde{\alpha}\), it becomes:

\[
\begin{bmatrix}
[K_G] - \omega^2 [M_G] \end{bmatrix} \alpha + \begin{bmatrix}
[S]^T
\end{bmatrix} F = 0
\]

\[
\begin{bmatrix}
[A]
\end{bmatrix} \alpha + \frac{1}{\omega^2} \begin{bmatrix}
[A]
\end{bmatrix} F - \begin{bmatrix}
[K_L^{-1}]
\end{bmatrix} F = 0
\]

eq. (5)

with

\[
\begin{bmatrix}
[A]
\end{bmatrix} = \begin{bmatrix}
[L_X] [\Phi]\end{bmatrix} [K_L^{-1}] [\Phi]^T [L_X]^T
\]

\[
[S] = [L_X] [\Phi]
\]

\[
[F_L] = [L_{\Pi}] [0] [L_{\Pi}^{-1}]
\]

It is worth emphasizing that the \(A\) matrix characterizes the contribution of the pseudo-modes.

To preserve the symmetry of the system (5), the auxiliary variable \(G = \frac{1}{\omega^2} F\) is introduced, then it can be written under the standard form:

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d) Calculation of the matrix \([A]\)

\(A_{ij}\) may be computed from the quasi static responses for each subsystem at fluid connection \(i\) to unitary mass flow rate sources located at connection \(j\).

This can be seen by solving the equation for a subsystem:
\[
(\begin{bmatrix} K_G & 0 \\ 0 & A^T \end{bmatrix} - \omega^2 \begin{bmatrix} M_G & 0 \\ 0 & A^T \end{bmatrix}) \begin{bmatrix} X_{gs} \end{bmatrix} = \omega_s^2 \begin{bmatrix} I \end{bmatrix}
\]

with \(I\) : matrix of unit mass flow rates at fluid connections

By using a modal development the \(X_{gs}\) response matrix can be written:
\[
\begin{bmatrix} X_{gs} \end{bmatrix} = \omega_s^2 \begin{bmatrix} \phi \end{bmatrix} \begin{bmatrix} K_G - \omega_s^2 M_G \end{bmatrix}^{-1} \begin{bmatrix} \phi^T \end{bmatrix} = \begin{bmatrix} \phi \end{bmatrix} \begin{bmatrix} M_G^{-1} \end{bmatrix} \begin{bmatrix} \phi^T \end{bmatrix}
\]

If \(\omega_s\) is small enough the first term may be neglected

\[
\begin{bmatrix} X_{gs} \end{bmatrix} = \begin{bmatrix} \phi \end{bmatrix} \begin{bmatrix} M_G^{-1} \end{bmatrix} \begin{bmatrix} \phi^T \end{bmatrix}
\]

Then \(A\) can be obtained by adding the elementary contributions of each subsystem:
\[
A = (L) \begin{bmatrix} X_{gs} \end{bmatrix} (L^T) = (L) \begin{bmatrix} \phi \end{bmatrix} \begin{bmatrix} M_G^{-1} \end{bmatrix} \begin{bmatrix} \phi^T \end{bmatrix} (L^T)
\]

e) Modal base truncation

According to the formalism already presented in ref. [3], the effect of the neglected modes is essentially a quasi static one. This leads to add an extra flexibility matrix obtained through a quasi static computation. It is done independently from the calculation giving the \(A\) matrix.

IV. Tests

Now, two applications concerning purely acoustical problems are given.

a) Calculation of the eigen modes of two interconnected identical straight pipes (length \(L = 1\) m).

On this simple 1D case, the contribution of the pseudo modes appears in evidence.

The pipe contain a compressible fluid \((c = 100\) m/s). At the free extremities we assume \(p = 0\).

For each subsystem, only the first two modes are taken into account in the numerical computation. They present a maximum mass flow rate at the connecting point.

The table below allows for a comparison between analytical and numerical results concerning the eigen frequencies of the first two modes of the complete system.

<table>
<thead>
<tr>
<th>(f) (Hz)</th>
<th>Symmetrical mode (Q_L = 0)</th>
<th>Antisymmetrical mode (Q_L \neq 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculation</td>
<td>25,0</td>
<td>50,0</td>
</tr>
<tr>
<td>Substructuration with ([A])</td>
<td>25,0</td>
<td>50,0</td>
</tr>
<tr>
<td>Substructuration without ([A])</td>
<td>25,0</td>
<td>62,5</td>
</tr>
</tbody>
</table>
(Q_L being the mass flow rate at the connection).

This table shows the numerical importance of the contribution of the pseudo-modes for the calculation of the antisymmetrical mode.

The theoretical mode shapes of the complete pipe are given by:

\[ p(x) = \sin \frac{n \pi (x + L)}{2L} \]

As the symmetrical mode correspond to \( Q_L = 0 \), its spatial profile is numerically obtained with a high accuracy. For the second mode, there is a discontinuity in the pressure profile at the connection (see figures 2, 3). The magnitude of this discontinuity may be reduced by increasing the modal basis.

b) Calculation of the eigen modes of the assembling of a pipe and a cylindrical annular cavity.

The geometry of this system is very similar to that of a PWR. This test illustrates the connection between the plane waves of the pipe and the axisymmetrical 2D waves of the downcomer.

The geometry of this tridimensional system and its boundary conditions are given on figure 4. The assembling is filled with a compressible fluid \( c = 500 \text{ m/s} \).

9 pipe modes and 85 cavity modes have been needed for this calculation.

As it has been told previously, the pressure of the cavity has to be averaged over the tube cross section.

The first natural frequencies of the system are given in the table below:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRISTANA</td>
<td>18.35</td>
<td>41.11</td>
<td>46.58</td>
<td>50.96</td>
<td>55.04</td>
<td>68.75</td>
</tr>
<tr>
<td>Analytical results</td>
<td>18.33</td>
<td>40.78</td>
<td>46.58</td>
<td>50.55</td>
<td>54.92</td>
<td>68.75</td>
</tr>
</tbody>
</table>

The modes can be separated into two kinds:

- the symmetrical modes: their azimuthal profiles are symmetric, the origin of the azimuthal angle being defined by the pipe (see figure 4).
- the antisymmetrical modes: their azimuthal profiles are antisymmetric. Obviously, they present a null pressure in the pipe, so they are pure cavity modes. The modes number 3 and 6 belong to this kind.

The symmetrical modes are compared in figures (5-9)

dotted line: numerical results
full line: analytical results.

The pressure distributions in the cavity often show a strong axial variation in the neighbourhood of the connection area. This is due to the mass flow rate induced by the pipe. On this 3D acoustical problem, the numerical and the analytical results are in good agreement.

Conclusion

With this method, introduced in the TRISTANA CEASEMT code, it will be possible to calculate cheaply, the dynamic behaviour of complex fluid structure systems. TRISTANA code will allow us to study closely the movement of PWR vessel internals, during the first period of the blowdown. The vessel, especially the downcomer, and the pipe will constitute the complete system.
References

[1] JEANPIERRE, R.J. GIBERT, HOFFMAN, LIVOLANT - Fluid structure interaction. A general method used in the CEASENT computer programs SMIRT 5 BERLIN 1979


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Figure 1 - Geometry of the first assembling

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Figure 2 - 1st mode axial pressure distribution

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Figure 3 - 2nd mode axial pressure distribution

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Figure 4 - Geometry of the second assembling

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\( a = 0.1 \text{ m} \)
\( R = 1.9 \text{ m} \)
\( H = 7.0 \text{ m} \)
\( R_s = 5.7 \text{ m} \)
\( l = 5.0 \text{ m} \)
\( r = 0.2 \text{ m} \)

\( q : \) mass flow rate
\( p : \) pressure
\( \theta : \) azimuthal angle
\( z : \) vertical axis