Characterization of Diffusive Fluid-Structure Wave Motions Using an Alternative Viscosity Term

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SUMMARY

In many diffusive wave motions, such as fluid-structure systems, an exponential decay is observed in field functions. This kind of decay cannot be properly described by the linearized Navier-Stokes wave equation. The Navier-Stokes wave solution does not provide a simple exponential decay which is expected for constant loss rate energy propagation. Instead the decay is frequency dependent with a sinusoidal function in the exponent. Hence there is a need to characterize a wave equation suitable for the case of constant loss rate wave propagation which is commonly observed in diffusive waves.

To overcome this problem an alternative loss term is proposed which provides a solution to the governing equation whose natural character is suggestive of an exponential decay.

In the new loss term the divergence of the viscosity (loss) force is assumed to be related to the combination of divergence of the velocity time potential and its time derivative with factors as functions of the field points. This assumption describes a large class of diffusive propagations whose complete coverage is beyond the scope of this study. The discussion is limited to a homogenous media where disturbances propagate spherically. The case of plane wave may easily be derived from the spherical results. The pressure, velocity and loss field functions, as well as the corresponding energy loss, are also described in this study.

It is also shown here that the wave equation corresponding to the thermal damping of a two phase flow is a special case of the above generalized wave equation.

Furthermore, for the validation of the proposed loss term, the test data of a fluid-structure system is compared to the predicted pressure field for a similar model in which the structural energy absorption and loss are simulated using the proposed viscosity term. The similarity of the predicted and measured responses indicates that the proposed viscosity term accurately represents the damped propagation of acoustic waves in dissipative media.
1. INTRODUCTION

It has been observed that an acoustic disturbance in fluid-structure systems decays exponentially. This decay cannot be properly described by the linearized Navier-Stokes wave equation. Various physical models have appeared in the literature attempting to explain the observed decay. However, it appears that no particular model, except that in Ref. [1], has yet completely predicted observations with a high degree of accuracy.

In Ref. [1] an alternative loss term is proposed which provides a solution to the governing equation whose natural character is suggestive of an exponential decay. This solution may describe many dissipative phenomena where nonthermodynamic variables are the origin of the dissipation.

The alternative viscosity concept requires experimental evaluation of constants for any specific diffusive wave propagation and appears to be equally valid with physical models which likewise require data for tuning. The alternative viscosity has the potential for application to problems involving not only fluid structure interaction (FSI), but also other dissipative processes which may include chemical reaction, radiation, content disso- lution, nonequilibrium molecular dynamics, etc.

2. THE PROMINENCE OF EXPONENTIAL DECAY IN DIFFUSIVE WAVES

For dissipative or combination of dissipative processes, in the limit of small amplitude motions, a single equation governing the motion can always be found. This equation for one dimension has the general form

\[ \sum_{n} \lambda_{m} n \left( \frac{\partial}{\partial t} + c_{mn} \frac{\partial}{\partial x} \right) v = 0. \quad (1) \]

The \( \lambda_{m} \)'s are known parameters and the \( c_{mn} \)'s are different wave speeds, W. Lick [2]. The general solution to this equation can be obtained by Laplace transform, G. E. Wither [3], M. G. Lighthill [4].

For example, for \( \lambda_{1} = 1 \), \( C_{11} = C_{1} \), \( C_{12} = C_{2} \), \( \lambda_{2} = \lambda \) and \( c_{21} = a \), the solution is

\[ v = \frac{v_{0}}{2\pi i} \int_{\gamma} e^{\omega t + i \chi x} \frac{d\omega}{\omega} \quad (2) \]

where, \( \Gamma \) is an integration path located to the right of all singularities along which \( Re \omega \) is constant and

\[ \gamma = -1/2 \left\{ \delta_{1} + (\delta_{1}^{2} - 4 \delta_{2})^{1/2} \right\} \]

\[ \delta_{1} = \omega (c_{1} + c_{2} + \lambda \alpha)/c_{1} c_{2} \]

\[ \delta_{2} = \omega (\omega + \lambda)/c_{1} c_{2} \quad (3) \]

The boundary conditions are \( t=0, v=0; x=0, v=v_{0}; x=\infty, v=0. \)

The solution (2) suggests that the exponential decay is a distinctive feature of the diffusive waves. Hence, it is rational to believe that this type of solution is a natural one for waves in homogenous media with continuous or piecewise-continuous loss of energy.

3. ALTERNATIVE WAVE EQUATION

In general, the linearized wave equation may be put in the following form,

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\[ \frac{\partial^{2} p}{\partial t^{2}} = \frac{1}{c_{2}^{2}} \frac{\partial^{2} p}{\partial x^{2}} + \nabla \cdot \bar{\psi}, \quad (4) \]
where \( \vec{\psi} \) is the loss or viscosity vector. As an alternative to Stokes assumption, we assume that \( \vec{\psi} \) is in the form:

\[
\vec{V} \cdot \vec{\psi} = \alpha \rho \, C^2 \{ M - \frac{N \, B}{C \, \beta t} \} \vec{V} \cdot \vec{\psi},
\]

(5)

where, \( \vec{\psi} \) is the time potential of the velocity with

\[
v_1 = \frac{\partial \psi}{\partial t},
\]

(6)

and \( M \) and \( N \) are functions of the coordinates. Substitution of (5) and (6) in (4) and the subsequent application of the mass conservation equation yield the following hyperbolic partial differential equation:

\[
\nabla^2 p = \frac{1}{C^2} \frac{\partial^2 p}{\partial t^2} + \alpha (M + \frac{N \, B}{C \, \beta t}) \, \frac{\partial p}{\partial t}.
\]

(7)

Use of (7) in a homogenous medium, where disturbances propagate spherically, provides the following solution for an outgoing wave:

\[
p = e^{\frac{-\alpha \int_{t}^{\infty} \hat{g} (t') dt'}{r}} \frac{f(t-r/c)}{r}.
\]

(8)

In the process of obtaining (8) we have assumed the following simplified forms for \( M \) and \( N \):

\[
M(r) = \frac{d g(r)}{d r} - \alpha [g(r)]^2
\]

\[
N(r) = 2 g(r),
\]

(9)

In this study only the case of \( \alpha = 1 \) (i.e. simple exponential decay) is considered. For this case, the pressure field is:

\[
p = \frac{f(t-r/c)}{r} \, e^{-\alpha r} = \frac{1}{r} \, \int_{-\infty}^{\infty} \hat{f}(\omega) \, e^{-i(\omega t-Kr)} \, d\omega
\]

(10)

in which \( \hat{f}(\omega) \) is the Fourier transform of the forcing function \( f(t) \) and \( K = \omega/C + i\alpha = k + i\alpha \).

It is possible to characterize the velocity field by the use of Euler's equation:

\[
v = \frac{1}{\rho \, C^2} \frac{1}{r} \, \int_{-\infty}^{\infty} \omega \hat{f}(\omega) \, e^{-i(\omega t-Kr)} \, d\omega
\]

(11)

where, \( v \) is the radial component of the velocity.

The loss force may be obtained by using Eqs. (5), (6) and (11). The radial component (the only one) of the \( \vec{\psi} \) is of the form

\[
\psi = \frac{1}{r} \, \int_{-\infty}^{\infty} \left[ \frac{1}{K} \left( k^2 - \frac{\omega^2}{C^2} \right) - \frac{1}{r} \right] \hat{f}(\omega) \, e^{-i(\omega t-Kr)} \, d\omega
\]

(12)

4. HARMONIC ANALYSIS

The harmonic analysis will provide more insight into the proposed wave equation. The corresponding modal wave functions are:

\[
p = \frac{1}{r} \, e^{-i(\omega t-Kr)}
\]

\[
v = \frac{1}{\rho \, C^2} \frac{1}{r} \, e^{-i(\omega t-Kr)} = \frac{1}{\rho \, C^2} \frac{\omega}{K} \, p
\]

\[
\psi = \left[ \frac{1}{K} \left( k^2 - \frac{\omega^2}{C^2} \right) - \frac{1}{r} \right] \frac{1}{r} \, e^{-i(\omega t-Kr)}
\]

(13)
Equations (13) show that the loss force has two constituents. One constituent is proportional to modal field function with a complex proportionality factor depending on wave number and damping factor. The other constituent is equal to modal field pressure with opposite sign times the inverse of radial distance which becomes insignificant for large distances.

Using the Kirchhoff corollary, [5], the energy loss may be characterized by:

$$L = \dot{w} + \nabla \cdot \vec{v}$$

(14)

where,

$$w = \frac{1}{2} \left( \rho_o \nu^2 + \frac{p^2}{\rho_o c^2} \right), \quad \vec{v} = \vec{p}$$

and $L$ is the unit energy loss. Substituting (13) in (14) yields:

$$L = -\frac{1}{\rho_o c^2} \omega^2 K \left[ \frac{1}{K} \left( \frac{\omega^2}{c^2} - \kappa^2 \right) + \frac{1}{r} \right] p^2$$

(15)

Notice that also $L = \frac{\partial}{\partial \omega} \vec{v}$. Equation (15) shows that the energy loss is proportional to the square of modal field pressure. This equation has a similar form as classical acoustics energy function.

5. THERMAL DAMPING EXAMPLE

An example of the application of the wave Equation (7) may be found in thermal damping of a liquid containing small quantities of undissolved gas. In this case, the damping mechanism is thermal in nature. Fluid pressure causes the bubble temperature to vary. This in turn causes irreversible, dissipative heat transfer between the gas bubbles and surrounding fluid. This dissipation causes damping of acoustic disturbances. The wave equation corresponding to the above-mentioned two phase fluid is derived by F. J. Moody [6] as follows:

$$\frac{1}{C^2} \frac{\partial^2}{\partial \tau^2} p + \frac{D}{C^2} \frac{\partial}{\partial \tau} p = \nu^2 p$$

(16)

where,

$$D = \frac{c^2}{g_o} F \frac{\rho g H}{N D g}$$

(17)

and $A$ is the gas-liquid contact area, $M$, mixture mass, $F$, the ratio of gas specific heats, $E$, the gas constant, $\rho_g$, the gas density, $C$, the acoustic speed, $g_o$, Newton's constant and $H$, the convective heat transfer coefficient.

For $g(r) = 1$, (7) becomes:

$$\frac{1}{C^2} \frac{\partial^2}{\partial \tau^2} p + \frac{2a}{C} \frac{\partial}{\partial \tau} p + a^2 p = \nu^2 p$$

(18)

For negligible values of the term $a^2 p$, (18) reduces to (16). The value of $D$ is in the order of 25 sec$^{-1}$ [6]. The corresponding $a^2$ will be $6.25 \times 10^{-6}$ ft$^{-2}$. For harmonic propagation the wave number of the solution of (16) is:

$$K^2 = \frac{1}{C^2} \left( \omega^2 + \frac{D}{a} \right)$$

(19)

while the solution wave number of (18) may be found to be for $a = \frac{D}{2C}$:

$$K^2 = \frac{1}{C^2} \left( \omega^2 - \frac{D^2}{4} \right) + \frac{iD}{a}$$

(20)

For $\omega >> \frac{D}{2C}$, (20) yields the same wave number as (19), this means the corresponding frequency should be much bigger than 1.989 Hz, which is within the practical range.
6. COMPARISON OF PREDICTIONS AND TEST RESULTS

One example is the propagation of disturbances created by vapor void collapse, submerged gas expansion or valve action in the containment of light water reactor systems. The dissipation of the energy takes place by energy transfer to mechanical systems in contact with water (such as containing vessel or submerged structures), thermal damping due to heat transfer to small gas bubbles present in the fluid and viscous dissipation. Figure 1 depicts a measured chug pressure trace on the flexible base of a cylindrical tank. The decaying pressure oscillations clearly undergo an exponential decay known as ringout.

Equation (10) has been applied to the analysis of a pool of water subjected to impulsive acoustic excitation. The analytical model is selected to be a rectangular pool approximating the pie-shaped experimental facility with average dimensions. Figure 2 depicts a typical test wall pressure from a case where the pool was subjected to an impulsive excitation. For the analysis, the image method was used to satisfy the boundary conditions at the pool walls and free surface. The damping coefficient, , was selected as 0.01. The predicted response to an impulse excitation of 2 ms duration, at one end of the pool, is given in Figure 3. The similarity of the predicted and measured acoustic responses for this type of excitation, in which the frequency content is considerably high (500 Hz to 600 Hz), indicates that the proposed mode of propagation accurately represents the damped propagation of acoustic waves in dissipative media. A complete comparative study is given in Ref. [7] for the results of image method and eigenmode expansion solution to solve the wave equation (16) in a rigid wall rectangular pool. The image method contained the alternative viscosity term and the eigen-expansion, the Navier-Stokes equation.

REFERENCES

Fig. 1 - Chug Pressure Trace.
Fig. 2- Normalized Experimental Pressure Response Time History.

Fig. 3- Normalized Predicted Acoustic Response Time History in a Rectangular Pool Due to a 2ms Source Spike.