A Two-Dimensional Model for the Coupled Dynamics of Fluid-Filled Curved Pipes

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SUMMARY

This study is concerned with modelling the coupled dynamics of fluid-filled curved pipes subjected to extreme impulsive loads. In contrast with current quasi one-dimensional models, the present model is capable of simulating the effects of streamline curvature in the fluid, taking full account of fluid-pipe interaction. Comparison with experiments and with other calculations is included.

The model is essentially a representation of the three-dimensional system of the fluid and pipe by an equivalent two dimensional system of a fluid and a channel-like conduit.

Clearly, a two dimensional conduit can not represent the behavior of a three dimensional pipe. Therefore, a special infra-structure is added to the conduit in order to achieve the three dimensional character of the pipe in both the breathing and bending modes.

The mechanical properties, dimensions and masses of the various elements of the model are determined from the requirement that the partial differential equations governing the model and the real pipe yield similar solution for a set of preferred variables. This procedure facilitates the fitting of the model parameters so as to simulate the more important phenomena in the pipe response.

The model is implemented in the two-dimensional lagrangian code DISCO for fluid-shell interaction, which has a built-in capability for treating a multiple-branched shell. This feature has proven very efficient in representing the infra-structure and the conduit as a branching shell.

Two tests for the validity of the model were carried out. The first was a numerical test which compared predictions of the model with axisymmetric three dimensional calculations for a straight pipe. A very satisfactory agreement was found between the pressure and strain time-histories.

In the second test case, experiments with a straight and a 90° elbow pipes were conducted. The pipes were loaded by impinging a flying disc axisymmetrically on one of their ends. Measurements of pressure and strain time-histories in the pipes enable a critical assessment of the validity of the model. This second test is still in progress, but some results are given here. These show a very good agreement with the model calculations.
INTRODUCTION

The response of fluid carrying piping systems to high pressure pulses is one of the central problems in the safety analysis and assessment of nuclear reactors. The system which includes curved portions, valves, branches and other components is subjected to extreme values of time varying pressures which cause large dynamic elastic and inelastic responses. The pressure waves in the fluid are affected, in turn, by the piping response. The problem is complex and calls for a three-dimensional, non-linear, fluid-structure interaction analysis.

The mathematical problem thus defined is formidable and the need for simplifying assumptions is evident. A common approach has been to separate the stage of the waves propagating in the fluid from the flexural response of the piping system. The first step includes the "breathing mode" of the pipe, and the pressures resulting from this analysis are applied as loading to the second step. The determination of wave motion and dynamic pressures may be done by a one-dimensional fluid model which takes into account the changing area of the cross section due to the internal fluid pressure ("water hammer" analysis) [1], [2]. A more sophisticated model, which takes into account two-dimensional axi-symmetric fluid motion combined with a one-dimensional axi-symmetric shell analysis has also found wide usage. It facilitates more exact pressure and stress determinations, but still cannot take into account non-axisymmetric response [3].

The second stage accepts the pressures from the first stage as loadings [4]. A beam-type analysis is then performed with special attention being given to the stress and deformation analysis of the elbows.

More recently, programs which perform at least partial transient dynamic analysis of the two steps simultaneously have been emerging [5], but the main emphasis is still on the beam-axisymmetric shell mode of the analysis [6]. A problem area which needs further study in this respect is the details of the interaction at elbows.

The present work falls under the heading of combined interactive analysis. The main approximation involves a two-dimensional description of the flow in a plane channel (rather than the three-dimensional flow in a curved pipe). This facilitates the estimation of the effects of pipe's curvature on flow through the elbows. It is assumed that the flow parameters are independent of the direction normal to the plane of the channel, and they are taken as averages of the pipe flow parameters in that direction.

Details of the model and its implementation are discussed in subsequent sections. It is helpful, however, to state at the outset some of the requirements and considerations which led to the choice of the model and its parameters:

1. It should be implementable into existing computer programs dealing with axisymmetric or cylindrical forms of fluid-structure interaction.
2. It should handle (simultaneously) the dynamics of both shell-type "breathing" modes and beam-type bending modes without a substantial loss of accuracy.
3. It should be able to cope with geometrical and material nonlinearities.
4. For the same (averaged) nonuniform time-varying pressures, it should yield essentially the same transverse motion and the same relative changes in fluid cross-sectional area. The last requirement actually dictates the method of identifying model para-
meters with their corresponding pipe parameters.

(5) It should have sufficient growth potential for the future inclusion of more refined effects (if required).

The channel model was programmed into the DISCO computer code. This code has been developed for the explicit solution of axisymmetric and cylindrical dynamic fluid-structure interaction problems, see Kivity et al. [7],[8].

2. **THE CHANNEL MODEL-GEOMETRY AND ANALYSIS**

The model (fig. 1) consists, essentially, of two parallel, infinitely wide cylindrical shells which are spaced a distance \( H \) apart. The shells can undergo "cylindrical motion" only, in the plane normal to the shell generators. The locus of the points which are initially equally distant from both shells forms a cylindrical surface ("midsurface") such that its shape in the plane of motion is that of the axis of the simulated pipe. The fluid is forced to flow inside the region confined by the two shells. A two-dimensional channel or conduit is thus formed, wherein the fluid is in a two-dimensional (planar) state of motion, while the two shells can undergo cylindrical deformations.

An infra-structure of flexible transverse plates ("branches"), spaced at distances \( \Delta S \) apart and rigidly attached to both shells is also provided. The properties of the branches are determined in such a way that the proper "breathing mode" of the pipe is simulated. Each branch consists of two parts which are hinged along the middle surface.

The bending response of the system can be realized by two possible methods:

(a) The branches can be given dimensions and mechanical properties in such a way that the proper resistance to beam shearing action be simulated in a manner which is somewhat similar to a "Vierendel truss".

(b) A separate third thicker shell may be added to the infra-structure along the midsurface, which can take on the bending rigidity of the system.

A combination of both can also be envisaged. For simplicity and economy of detail, the first option was chosen at this stage.

The infra-structure does not interact with the fluid. This is taken care of by the program-logic of the DISCO code. Thus, in so far as the fluid is concerned, the infra-structure is imaginary and the fluid can flow through it. It is, however, real in so far as structural response is concerned.

A significant number of model parameters are in existence. These are: shell "envelope" properties (thickness \( t_1 \), elastic modulus \( E_1 \), mass density \( \rho_1 \)), branch properties (\( t_2, E_2, \rho_2 \)), branch spacing \( \Delta S \) and "envelope" height \( H \). Also to be considered are the pressures \( P_U, P_L \) which exist between the fluid and the "upper" and "lower" shell "envelopes". The parameters are determined so as to simulate the pipe response as closely as possible.

Obvious requirements of conservation of mass and momentum dictate that the fluid volume in the pipe and model (per unit length of generator) be the same, that mass per unit length of pipe and model be the same and, most importantly, that similar fluid pressures will produce similar structural motion. The latter is more difficult to achieve in the transverse response because of the three-dimensionality of fluid flow in the curved pipe. This is
achieved, however, in an average sense by considering an imaginary intermediate "square" pipe with the same internal fluid area. A system of such pipes, placed side by side, will produce a two-dimensional flow which simulates the pipe flow in an average sense. The model can then be compared, dimensionwise, with this pipe. This dictates the value of \( H \) at

\[
H = Rv' \tag{1}
\]

In order to obtain similar structural motion for similar average pressures, the equations of motion for the model have to be constructed. A necessary condition for proper response is that the model will function properly in its fundamental configuration of straight, linearly elastic form.

A derivation of the basic equations of motion for the model which started from first principles by considering each part separately and then combining the resulting equations, was performed. The details of the derivation will not be given here, but the essential principle was that of "smearing" the properties of the branches in the longitudinal direction. The final equations of motion for the linearized straight model subjected to general transverse non-axisymmetric loads are:

\[
\begin{align*}
\frac{\partial^2}{\partial x^2} & x^6 + \frac{3}{2} \frac{\partial^2}{\partial t^2} \left( \frac{U + V}{2} \right) - \frac{6}{t^2} \left( \frac{U + V}{2} \right) = 0 \\
\frac{\partial^4}{\partial x^4} & + \frac{2}{3} \frac{\partial^2}{\partial t^2} \left( \frac{U - V}{2} \right) = 0 \tag{2}
\end{align*}
\]

Where \( U, V \) are the transverse displacements of the cylindrical "envelope", \( \bar{T} = t_1 H^2 + \frac{1}{8} t_1^3 \) is the moment of inertia of the composite model, and the other parameters are given by

\[
\begin{align*}
\lambda^2 &= \frac{E_1 H^3 \Delta t}{12 E_2 t_2^3} \\
\mu &= \frac{E_1 \Delta S t_1}{24 E_2 t_2^3} \\
\rho &= \frac{P_L - P_U}{E_1} \\
\rho &= \frac{(P_L - P_U) H \Delta S}{2 E_2 t_2} \tag{3}
\end{align*}
\]

Where \( \rho = \frac{1}{E_1} (\Delta t_1 + \frac{\rho_2 t_2 H}{\Delta S}) \); \( \rho = \frac{H \Delta S}{2 E_2 t_2} (\rho_1 + \frac{\rho_2 t_2 H}{\Delta S}) \)

In order to permit comparisons with pipe response, the static equation (2) was also solved for the case of a model of length \( \pm L/2 \) clamped at both sides and subjected to a uniform load. The resulting analytical solution was

\[
\frac{U + V}{2} = \frac{1}{24} \left[ \frac{X_0^4}{L^4} - 12 \frac{X_0^2}{L^2} \left( \frac{1}{24} + \varepsilon^2 \right) + 3 \frac{1}{16} \left( \frac{1}{44} + \varepsilon^2 \right) + 12 \varepsilon^2 (\cosh \frac{X_0}{L} - \cosh \frac{1}{2\varepsilon}) (\sinh \frac{1}{2\varepsilon})^{-1} \right] \tag{5}
\]

where \( \varepsilon = \sqrt{3} \frac{H}{t_1} \)

For comparison, the equations of motion for the axisymmetric response of a cylindrical shell [9] and the solution of a clamped-clamped uniform loaded Timoshenko beam of length \( \pm L/2 \) [10] are noted. This are respectively:

\[
\begin{align*}
\left( \frac{\partial^4}{\partial x^4} \right) x^6 & + \frac{3}{2} \frac{\partial^2}{\partial t^2} \left( \frac{U + V}{2} \right) = 0 \\
\left( \frac{\partial^2}{\partial x^2} \right) x^6 & + \frac{2}{3} \frac{\partial^2}{\partial t^2} \left( \frac{U - V}{2} \right) = 0 \tag{6}
\end{align*}
\]

\[
\begin{align*}
\left( \frac{\partial^4}{\partial x^4} \right) x^4 & + \frac{2}{3} \frac{\partial^2}{\partial t^2} \left( \frac{U - V}{2} \right) = 0 \\
\left( \frac{\partial^2}{\partial x^2} \right) x^4 & + \frac{1}{3} \frac{\partial^2}{\partial t^2} \left( \frac{U + V}{2} \right) = 0 \tag{7}
\end{align*}
\]

\[\text{— 212 —}\]
Here \( W, V \) are the axisymmetric and transverse displacements of the pipe, respectively. Also, \( P_c \) is the internal normal pressure, \( P \) is the transverse load per unit length of beam, \( \rho, R, t, E, I \) are the mass density, pipe radius, wall thickness, elastic modulus and moment of inertia, respectively, and \( K_AU \) is the Timoshenko shear rigidity [10]. The other parameters are:

\[
\begin{align*}
\mu &= \frac{R^2 t^2}{12(1 - \nu^2)}; \\
\varepsilon^2 &= \frac{E}{K} A \phi; \\
P_c &= \frac{P}{E I}; \\
P_c &= \frac{P}{E t}; \\
\rho_p &= \frac{\rho P^2}{E} \\
\rho &= \frac{\rho P^2}{E}
\end{align*}
\]  

Equations (3), (6) and (5)-(7) are of the same form, and choice of proper model parameters easily follows from the two basic requirements of fluid-structure interaction:

(a) For the same average transverse pressure on pipe (in the "squared" pipe sense) and model, the same average transverse motion should result. That is:

\[
\text{if } \frac{P}{R^2} = P_L + P_U \text{ then } V = (V_U + V_L)/2.
\]

(b) The same internal pressure in pipe and model should produce the same relative changes in area. That is:

\[
\text{if } \frac{P}{R^2} = (P_L - P_U) \text{ then } W = (V_U - V_L)/2H.
\]

Application of the two requirements to (3) and (5), together with (1) and the total mass requirement, leads to the determination of all the relevant parameters of the model.

The DISCO program has the provision for pointwise determination of the stresses from the displacements in the actual pipe configuration. This takes care of the construction of the proper yield envelopes for the material, so that elasto-plastic analysis can be performed.

It is notable that the partial differential equations of the model are of higher order than those of the corresponding pipe. This has led to the appearance of an extra term in (5) which may slightly affect the results near the model boundaries. The explanation is that the model is more refined as a beam-model than the Timoshenko beam, permitting warping of its cross-sections. This warping is controlled by the increased number of boundary conditions, so that more complex conditions of stresses or displacements can be considered at the pipe boundaries.

3. THE EXPERIMENTAL SETUP

The experiment setup incorporated a circular 316L stainless steel pipe, filled with fresh water at ambient conditions. Pressure activation was obtained using a piston driven by a colliding disk of known mass and precalibrated velocity. A thick ring has been used to increase the pipe rigidity in the piston zone.

Two sets of experiments were carried out. In the first, described in Fig. 4, a straight pipe with a thickness of 3.9mm and 60.3mm outer diameter has been used. It is referred to as PASP configuration. The second, described in Fig. 6, used a curved pipe of the same characteristics and is referred to as PACP configuration. Dynamic measurements of pressures as well as strains were made as follows:
Pressure measurements were taken via a commercial kistler 6203 pressure transducer, connected with a low-noise cable to a charge amplifier. Preliminary experiments have shown the presence of high frequency vibration mode peaks, superimposed on the pressure signal. Therefore the output was filtered with an analog 0-30KHz filter, prior to being recorded. Strain measurements utilized a commercial grid type guage, connected to a DC amplifier. No filtering was necessary.

Both pressure and strain signals were recorded with a digital transient recorder. In both experimental configurations strain gauges were cemented to the pipes. They are referred to as SG_i (i=1,2,...), and are located and oriented as shown in figs. 2 and 3. Pressure was measured at one location PR_1 in the PASP configuration, and at two locations PR_{11} and PR_{12} at the PACP's. Unfortunately, PACP/SG_2 and PACP/SG_3 strain-gauges failed. Consequently, there are no strain measurements in the elbow zone at this moment.

4. COMPARISON OF COMPUTATION AND EXPERIMENTS

Figs. 5 and 7 compare the computed and measured strains and pressures of the PASP's and the PACP's experiments. In each frame the line curve is the model computation while the circles and triangles represent experimental data.

The shell material is assumed to be elasto-plastic with linear strain hardening and rate sensitivity [11], as shown in fig. 2.

\[ \sigma = (E-E_y) \varepsilon^0 + E_y \varepsilon^T \]  

(9)

where \( \varepsilon^0 \) is the dynamic yield strain, and \( E_y \) is the tangential modulus \( (E_y=2.9) \).

\( \varepsilon_y^0 \) was calculated from the expression

\[ \varepsilon_y^0 = \varepsilon_0^m \left[ 1 + C_m \log \left( \frac{\varepsilon}{\varepsilon_0^m} \right) \right] \]

(10)

where \( C_m \) and \( D \) are material constants which were found from correlation with experimental data

\( C_m = 0.146 \)

\( D = 0.05 \times 10^{-6} \text{sec}^{-1} \)

other material constant are

\( E = 210 \text{GPa} \)

\( \varepsilon_0^m = 0.0017 \)

Calibration tests established the velocity of the colliding disc to be 330.0m/sec, while the disc mass was 0.165kg.

As seen from the figures, the experimental results agree, in most cases, quite satisfactorily with the model calculations. The timing and characteristics of the measured signals match the computed ones and the differences in amplitudes are relatively small,
considering the complexity of the problem. The use of more strain gauges per cross section might lead to even better correlation.

The shape of the PASP/PR pressure-time curve strongly implies a signal cut-off in the measuring system. This is further evidenced by the good agreement between experiment and calculation at the dead-end of the PACP test configuration, where a longer but identical pipe was used and the pressure measured was twice that of the PASP experimental curve.

In general, the calculated pressure-time curves are somewhat higher in amplitude (but very similar in shape) than the experimental curves. A more refined material model and the input of a pressure-time signal from the first portion of the pipe should further improve the correlation.

5. CONCLUSIONS AND FURTHER DEVELOPMENTS

The validation of the model and of its implementation in the DISCO computer code has required both analysis and experimentation.

(a) It was shown analytically that by a proper choice of parameters a simulation of the pipe-fluid interactive motion can be achieved (in the average sense) for the fundamental initially straight linearly elastic pipe.

(b) In order to complement the analysis, computer runs of identical cases were performed with the model and with the axisymmetric DISCO code. The case considered was that of the axisymmetric inelastic response of a straight pipe to pressure pulses, and was identical to the experimental configuration discussed earlier. Details of the comparisons are not supplied here due to lack of space, and only a typical result is shown (fig. 3). Throughout, the matching was highly satisfactory, which helps to verify the model for this case.

(c) A series of experiments on straight and curved pipes subjected to impulsive fluid excitation was used to check the model in the inelastic, nonlinear, nonstraight, highly time-varying range. The comparisons showed good agreement between measurements and calculation at various control points.

Considering the complexity of the three-dimensional problem and the inherent problems associated with the complex test systems and their measurement, the results of the comparisons are noteworthy. It is concluded that the proposed model shows very good promise of being able to predict the non-symmetric response of curved pipes to pressure pulses.

Further work now in progress or envisaged for the near future includes:
- additional testing on curved pipes. These will be used for obtaining more detailed data on the behavior at the elbow and for further validation studies.
- Introduction of the (b) model (section 2) as an alternative to model (a).
- a more detailed check of the nonlinear large deformation response of the model at elbows. This should be compared with the results of shell-type analysis for loss of rigidity and reduction in area. If necessary, adjustment of some of the model parameters will be made to accommodate the various possibilities.
REFERENCES


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Fig. 1: Channel Model Geometry.

Fig. 2: Stress-Strain Diagram

Fig. 3: Comparison Between Model and DISCO Calculation.
Fig. 4: Configuration of Straight-Pipe Experiment.

Fig. 5: Comparisons Between Model Calculation and Straight-Pipe Experiment.
Fig. 6: Configuration of Curved-Pipe Experiment.

Fig. 7: Comparisons Between Model Calculation and Curved-Pipe Experiment.