Flow-induced vibration in heat exchanger tubes can result in impact with the baffle plates and subsequent tube failure through fatigue, fracture and fretting wear. As a step towards the correlation between the random flow excitations and the rate of wear, this paper presents a general theory for predicting the tube motion and the tube baffle impact forces through a case of cantilever pipe with motion limiting stopper at the free end and simultaneously subjected to transverse fluid flow. The mathematical model has been developed using the theory of fluid-structure interactions with modal superposition technique. The pipe displacement induced by lift forces is evaluated by numerical integration. When displacement increases to greater than the pipe-stopper clearance, the pipe impacts on stopper. Assuming semi-elastic impact, the equation of pipe motion during impact is developed using extended Hertz’s theory to include the vibration of one of the colliding bodies. The stopper is assumed to be at rest before and after the impact. The constraint imposed on pipe motion, at the free end due to impact of the pipe on stopper, is considered as one of the boundary conditions and is used to evaluate the pipe natural frequencies. The non-linear equations are solved numerically. The response of the pipe due to wake induced lift forces superposed by the impact response is evaluated. The pipe natural frequency and the response are presented as a function of flow velocity and the pipe-stopper clearances. When the displacement is less than or equal to the pipe-stopper clearance, the response increases with increasing flow-velocity. However in case, when the displacement is such that the pipe impacts on stopper, it is shown that the natural frequency decreases and response increases with increasing clearance at constant excitations. The time history of the response and the impact force is shown to be random and impulsive respectively.
1. Introduction

A tubular heat-exchanger consists of closely spaced tubes running between two tube-sheets and supported at equal or unequal intervals by baffle plates. To facilitate assembly, baffle holes are made larger than the tube diameter. Due to fluid flow excitations, the tube oscillates within the clearance space of the baffle holes. At large amplitude of oscillations, the tube impacts on baffles causing the tube wall thinning due to localized chaffing which finally leads to the tube leakage due to fracture and forced shut-down of the plant. Leaks also occur when fatigue cracks develop at the tube to tube sheet joints or when tubes flatten at midspan because of collisions with neighboring tubes at still higher amplitude of oscillations.

The common repair procedure of tube plugging is often expensive and in nuclear power plants it is even impractical due to heavy radiation field. Therefore it is highly desirable to predict the amplitude of tube oscillations and the associated possibility of these modes of failure at design stage. This paper describes the complex behavior of tube/baffle interaction through a case of cantilever pipe subjected to cross-flow and having a motion limiting stopper at the free end. It is shown that the simulated model have explained the entire complex behavior of tubular beams with motion limiting stoppers observed in various experiments [1, 5] and presents a way for impact fretting model to predict the wear life of heat exchangers.

2. Equations of Motion

Consider a uniform cylindrical cantilever pipe immersed in a fluid, flowing at velocity \( V \) perpendicular to \( x-y \) plane as shown in Fig.1. The pipe has linear mass density per unit length \( m \), total length \( L \) and the \( x \)-coordinate coincides with the pipe axis at rest. Consider a small element \( dx \) of the deflected pipe, Fig.2, where \( \zeta \) is shear force, \( M \) is bending moment, \( C_d \) is equivalent viscous damping coefficient and the added mass per unit length \( m_x \) is given by

\[
m_x = \frac{D^2 \rho}{4} (\frac{D}{D} + \frac{D^2}{2} \frac{\partial}{\partial x}) \zeta
\]

where \( D \) is pipe outer diameter, \( \rho \) is the fluid density and \( \zeta_m \) is the added mass coefficient. Since the pipe vibrates in its own wake the vibratory excitation in the lift direction is five to ten times higher than in the drag direction. Therefore in this paper the pipe motion is considered in the lift direction only.

The pipe transverse (lift direction) motion \( y \) due to the lift force \( F_L(t) \), is described by force and moment balance equations expressed as
\[
\frac{\partial ^2 y}{\partial t^2} - (m + m_f) \frac{\partial ^2 y}{\partial t^2} - C_D \frac{\partial y}{\partial t} + F_L(t) = 0
\]  
\[Q - \frac{\partial ^2 N}{\partial y} = 0\]

Assuming the viscoelastic pipe obeying a stress-strain relation of Kelvin and Voigt type, the bending moment becomes

\[M = -I \left( \mathcal{E} \frac{\partial ^2 y}{\partial x^2} + \mathcal{U} \frac{\partial ^3 y}{\partial t \partial x^2} \right)\]

where \(\mathcal{E}\) is flexural rigidity and \(\mathcal{U}\) is the viscoelastic damping factor.

Introducing (3) in (2) and using (4), one gets

\[(m + m_f) \frac{\partial ^2 y}{\partial t^2} + C_D \frac{\partial y}{\partial t} + I \frac{\partial ^5 y}{\partial t \partial x^4} + \mathcal{K} \frac{\partial ^4 y}{\partial x^4} = F_L(t)\]

Now, let a motion limiting stopper is placed at the free end of the pipe. If the deflection \(y\) at the free end is greater than the pipe-to-stopper hole clearance \(c\), the stopper imposes a non-linear constraint at the free end and the equation (5) becomes

\[(m + m_f) \frac{\partial ^2 y}{\partial t^2} + C_D \frac{\partial y}{\partial t} + I \frac{\partial ^5 y}{\partial t \partial x^4} + \mathcal{K} \frac{\partial ^4 y}{\partial x^4} + F_S \delta(x - L) = F_L(t)\]

where \(F_S\) is the reaction force exerted by the stopper on the pipe due to impact and \(\delta(x - L)\) denotes that the \(F_S\) acts at \(x = L\) only.

Assuming the stopper is rigid compared to pipe and is stationary before and after the impact of the pipe on it, the coefficient of restitution \(e\), for this semielastic vibroimpact, can be interpreted as an equivalent damping. For low velocities of impact, the balance between the hysteresis energy loss and the kinetic energy loss due to collision, leads to

\[F_S = k \left( 1 + 1.5 \beta \frac{\partial y}{\partial t} \right) (y - c)^{1.5}\]

where \(k\) is stiffness parameter based on Hertz law and \(\beta\) is a constant based on the slope of the variational curve of \(\beta\) against velocity of impact. For steel pipe impacting on hemispherical steel stopper of diameter \(D_S\), the \(\beta = 0.005\) and

\[k = \left[ \frac{2 D \frac{\beta}{\partial} \frac{E^2}{g(D+D_S)(1 - \nu^2)}}{9(1+D_S)^5} \right]^{0.5}\]

Substituting (7) in (6) one gets

\[(m + m_f) \frac{\partial ^2 y}{\partial t^2} + C_D \frac{\partial y}{\partial t} + \mathcal{U} \frac{\partial ^5 y}{\partial t \partial x^4} + \mathcal{K} \frac{\partial ^4 y}{\partial x^4} + k(1 + 1.5 \beta \frac{\partial y}{\partial t})(y - c)^{1.5}\]

\[\delta(x - L) = F_L(t)\]
where the lift force $F_L(t)$ is given by

$$F_L(t) = \frac{1}{2} C_L \rho V^2 D \cos(\omega t) \quad (10)$$

where $C_L$ is the lift coefficient and $\omega$ is the vortex-shedding frequency given in terms of Strouhal number $S$, by

$$\omega = \frac{2 \pi V S}{D} \quad (11)$$

3. Analysis

Modal superposition technique with Galerkin's method is used to express $y(x,t)$ as

$$y(x,t) = \sum_{x} q_x(t) \varphi_x(x) \quad (12)$$

where $\varphi_x(x)$ is the $x$th beam-like eigenfunction of the cantilever pipe and $q_x(t)$ is the weighting function denoting the contribution of the $x$th natural mode in the total displacement of the pipe.

3.1 Characteristic Frequency Determination

The characteristic frequency of the pipe is evaluated from the equation

$$(m + m_r) \frac{\partial^2 y}{\partial t^2} + C_D \frac{\partial y}{\partial t} + \mu I \frac{\partial^5 y}{\partial x^5} + EI \frac{\partial^3 y}{\partial x^3} = 0 \quad (13)$$

and the boundary conditions, when $y \leq C$:

$$y = \frac{\partial y}{\partial x} = 0 \quad \text{at} \quad x = 0 \quad \text{(fixed end)} \quad (13 \text{ a})$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial x^3} = 0 \quad \text{at} \quad x = L \quad \text{(free end)} \quad (13 \text{ b})$$

when $y > C$, the pipe impacts on the stopper receiving an equal and opposite reaction and the boundary condition at free-end becomes

$$\frac{\partial^2 y}{\partial x^2} = 0 \quad \text{and} \quad EI \frac{\partial^3 y}{\partial x^3} = F_s \quad (13 \text{ c})$$

3.2 Response Determination

The forced vibration response is evaluated by numerical integration of equations (5) and (9) after substituting the equation (10), using either a digital or an analog computer. Initially the pipe is assumed to be resting at its undeflected position. Under the influence of lift force, it starts accelerating. When the deflection $y(> C)$ becomes greater than the clearance, the pipe collides with stopper and rebounds with a velocity proportional to the velocity of impact. During each collision only a
fraction of pipe energy is lost and hence multi-collisions are possible during a cycle of forced oscillation. Over and above, each impact gives a transient loading and the pipe undergoes a free vibration. If $y_x$ and $y_T$ are the free amplitude and the rebound amplitude of oscillations, then the total displacement $W(x,t)$ at any location $x$ and time $t$ will be given by

$$W(x,t) = y_x(x,t) + y_T(x,t)$$  \hspace{1cm} (14 a)

where $y_T(x,t) = \int_0^T e^{\frac{3y}{3t}} dt$  \hspace{1cm} (14 b)

and the average displacement at any point $x$ will be given by

$$\bar{W}(x) = \frac{1}{T} \int_0^T W(x,t) dt$$  \hspace{1cm} (15)

where $T$ is the time period of forcing function.

4. Results and Discussions

A 10 meter long cantilever steel pipe of 10 cm diameter subjected to liquid cross flow at room temperature with a 10 cm diameter hemispherical motion limiting steel stopper at the free end is studied. The other parameters considered are as follows –

$$m = 0.25 \times 10^{-3} \text{ kg - sec}^2/\text{cm}^2; \quad m_T = 0.06 \times 10^{-3} \text{ kg-sec}^2/\text{cm}^2$$

$$W = 2 \times 10^9 \text{ kg - cm}^2; \quad \mu I = 4 \times 10^6 \text{ kg-cm}^2 - \text{sec}; \quad C_L = 0.01 \text{ kg-sec/cm}^2$$

and $$C_L = 1.5, \quad S = 0.2$$

As shown in fig.3, when displacement $Y$ is less than or equal to pipe-to-stopper clearance $C$, the pipe behaves as a cantilever beam subjected to cross-flow whose natural frequency $\omega_n$ is constant and the average response $\bar{W}$ increases with increasing fluid velocity $V$. When $Y > C$, the pipe impacts on stopper and the $\omega_n$ becomes a function of $C$ and impact force. Fig. 4 shows the instantaneous value of $\omega_n$ as a function $C$. It can be seen that instantaneous $\omega_n$ decreases by increasing $C$ at constant excitation. This is obvious because the impact force decreases by increasing $C$ at constant excitation thereby decreasing the fixity of the pipe at the stopper location. This is in good agreement with the experimental results $[1,2]$. Fig. 5 and 6 shows respectively the dynamic response $W$ and the corresponding impact force $F_y$ as a function of time at constant excitation and constant pipe-stopper clearance $C$. It can be seen that the displacement is non-linear and random which agrees with the experiments $[3,4]$ whereas the force is impulsive. It can also be seen that pipe hits several times before passing over to strike at other side $[2,4]$. The physical explanation of these phenomena are as follows.
The pipe-to-stopper vibroimpacts are semi-elastic in nature. In each impact a part of pipe dissipated energy is transferred to stopper and the pipe experiences an impact reaction, causing secondary vibrations. This secondary vibration is free and decaying with time. The superposition of this free decay over the rebound displacement generates a highly non-linear and random displacement pattern. Also the impact imposes a constraint to pipe motion at stopper location which changes the pipe natural characteristics. Therefore, the pipe natural characteristics and its response to excitation are interdependent, making the problem highly complex and non-linear.

5. Conclusions

A non-linear theory is presented to simulate the pipe-stopper interaction. The model have explained the entire vibroimpact behaviour of tubular beams with motion limiting stoppers observed in various experiments [1,4] The following main conclusions are drawn -

1. The natural frequencies depends on the vibration amplitude and decreases with increasing pipe-to-stopper hole clearance.
2. The entire pipe rebounds with a velocity proportional to the striking velocity and multi impacts may occur during a period of forced oscillations.
3. The overall displacement is non-linear and random.
4. The impact force is impulsive and strongly depends on the relative approach of stopper into the pipe.

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7. References

FIG. 1: CANTILEVER PIPE IN FLUID CROSS-FLOW WITH MOTION LIMITING STOPPER

FIG. 2: FORCES AND MOMENTS ACTING ON PIPE ELEMENT dx IN ITS DEFLECTED POSITION.

FIG. 3: AVERAGE RESPONSE $\bar{w}$ AS A FUNCTION OF CROSS-FLOW VELOCITY

FIG. 4: AVERAGE RESPONSE $\bar{w}$ AND INSTANTANEOUS NATURAL FREQUENCY AS A FUNCTION OF CLEARANCE C

FIG. 5: DISPLACEMENT AS A FUNCTION OF TIME (b) A MAGNIFIED VIEW WITHIN THE TIME INTERVAL $t_1$

FIG. 6: IMPACT FORCE AS A FUNCTION OF TIME (b) A MAGNIFIED VIEW WITHIN THE TIME OF CONTACT $t_2$