Identification of Multiple Modes of Axisymmetric or Circularly Repetitive Structures

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The axisymmetric structures, or those composed with circularly repetitive elements, often display multiple modes, which are not easy to separate by modal identification of experimental responses.

To be able to solve in situ some problems related to the vibrational behaviour of reactor vessels or other such huge structures, ELECTRICITE DE FRANCE developed a few years ago, experimental capabilities providing heavy harmonic driving forces, and elaborate data acquisition, signal processing and modal identification software, self-contained in an integrated mobile test facility.

The modal analysis techniques we have developed with the LABORATOIRE DE MECANIQUE APPLIQUEE of University of BESANCON (FRANCE) were especially suited for identification of multiple or separation of quasi-multiple modes, i.e. very close and strongly coupled resonances.

Besides, the curve fitting methods involved, compute the same complex eigen-frequencies for all the vibration pick-ups, for better accuracy of the related eigen-vector components.

Moreover, the latest extensions of these algorithms give us the means to deal with non-linear behaviour.

The performances of these programs are drawn from some experimental results on axisymmetric or circularly repetitive structures, we tested in our laboratory to validate the computational hypothesis used in models for seismic responses of breeder reactor vessels.
INTRODUCTION

To be able to solve in situ the problems related to the vibrational behaviour of real size reactor vessels, or other such huge structures, "ELECTRICITE DE FRANCE" developed a few years ago, experimental capabilities providing heavy driving forces, and also elaborate data acquisition, signal processing and modal identification softwares, selfcontained in an integrated mobile test facility.

In this paper, we intend to show how the acquisition procedures and the modal identification methods that we developed with the aid of the "LABORATOIRE DE MECANIQUE APPLIQUEE" of University of BESANCON, were fitted to the special analysis of quasi-multiple eigen-modes of axisymmetric or circularly repetitive structures such as reactor vessels.

Following the review of the specific abilities of our hardware and software, related to experimental analysis of very close and strongly coupled resonances, the performances and limitation of these tools will be pointed out, from some simple tests which we carried out in our laboratory on preliminary models of breeder reactor vessels (Super Phenix II).

A. IDENTIFICATION OF MULTIPLE MODES OF AXISYMMETRIC OR CIRCULARLY REPETITIVE STRUCTURES SUCH AS REACTOR VESSELS.

1. Modal behaviour of axisymmetric or circularly repetitive structures

Most modes of axisymmetric or circularly repetitive structures are of multiplicity two. In the very special case of axisymmetric structures, one easily takes for obvious that the same modal deformations are likely to build up with any angular orientation. So it can be shown that all these possible occurrences may be considered as linear combinations of two distinct such mode shapes. In the case of circularly repetitive structures, most modes are also of multiplicity two. But as all possible modal deformations do not present obviously resemblant shapes, one ordinarily is not so easily convinced that like in the previous case, they may be considered as linear combinations of two particular eigen-functions.

However they are, and when actuating these multiple modes, one theoretically observes as simple modes the particular deformation patterns which tend to maximize the effectiveness of the driving forces. So in the case of a point force, the single degree of freedom on which it is acting gives a local maximum of the mode shape ; and moving this point force, one changes also the mode shape (one may even get local maximum deflections at previous nodal lines).

FIGURE 1 : Theoretical mode shapes of the first (double) bending mode of an equilateral triangle composed with three beams of same characteristics. All obtainable patterns are combinations of these two orthonormal mode shapes (computation results).
2. Identification of multiple modes of real structures

In fact, two practical circumstances yield further difficulties to interpretation of multiple modes of real structures.

First, the structures under test may be slightly non-symmetric or not perfectly circularly repetitive. Then, the multiple modes generally split into several simple modes with very close, however distinct, resonance frequencies; and the corresponding observed mode shapes should thus be well determined and not quite related to the driving forces but rather to the localisation of the dissymmetries.

Unfortunately, the fact that real structures are always more or less damped, yield the opposite effect of strongly coupling the close resonances. So, identification of quasi-multiple modes often result in complex eigen-functions for which the effects of the driving forces (coupling of close resonances) and those of the small lack of axisymmetry or repetitivity (splitting in multiple modes) are not very simple to discriminate.

Anyway, it is easily understood that experimental analysis of the vibrational behaviour of such structures absolutely requires performant acquisition procedures and elaborate data processing capabilities.

3. Acquisition of mechanical transfer functions

Our test procedures monitor the acquisition of forced responses of a given structure to sinusoidal mechanical loading. First because our hardware is rather ancient, but also because it seems to be the best way to excite a structure which displays close and strongly coupled modes. For huge structures, we use four hydraulic shakers comprising reaction masses, each able to provide sinusoidal point forces of 10000 newtons over the range 5 hertz to 200 hertz. The entry to these shakers are independently amplitude and phase regulated by analogue feed-back acting in most cases by reference to the measured input forces.

But it could also be switched onto other parameters, and we often manage complex multiple excitations in order to appropriate some modal responses, to create special torques or rotating forces, and sometimes to simulate ideal limit conditions such as perfect clamping.

The onboard computer gathers the response vectors of up to forty vibration transducers. The signals issued are preprocessed by analogue co-quad analysers, and digitalized as D.C. voltages representing their active and reactive components.

A numerical feed-back loop including a programmable synwave synthesiser, controls the step of the discrete frequency sweep, and automatically adapts the resolution of the analysis to the characteristics of the encountered resonances. The criterion used to compute the instant pitch depends on parameters chosen by the operator and is sensitive to amplitude as to phase variations of some selected structural responses.

Hence, mechanical admittance curves are obtained, for which the frequency points are more or less uniformly distributed over the occurred resonance loops. This way our data is better suited for modal identification of close resonances than if it were resulting from a constant pitch frequency sweep or F.F.T. techniques.

4. Multimodal identification techniques

The experimental data stored on magnetic disk cartridges may be processed by integrated multimodal identification tools, we implemented in our onboard minicomputer.

The basic assumption supporting these computations is that the vibrational behaviour of the structures we excite, can be approached by a linear model following equation [1]

\[
[M][\ddot{x}] + [C][\dot{x}] + [K][x] = [F]
\]

(1)

Where M, C and K stand for the real symmetric mass, damping and stiffness matrices.

Although damping is assumed to be low, we have to take in account the possibility of closely coupled resonances, and the fact that the damping matrix may not reduce to diagonal when projected on the set of eigen-solutions of the conservative system.
Under harmonic excitation of complex pulsation \( S = jw \) \( (j^2 = -1) \) we can derive from \( [1] \)
equation [2].

\[
\begin{bmatrix}
K & O \\
O & -M
\end{bmatrix}
+ S
\begin{bmatrix}
C & M \\
M & O
\end{bmatrix}
\begin{bmatrix}
x \\
x
\end{bmatrix}
= \begin{bmatrix}
F \\
Sx
\end{bmatrix}
\]  

(2)

The homogeneous related system has \( 2N \) complex eigen-solutions which are paired following conjugate eigen-values (\( N \) is the dimension of the original system).

Normalizing the eigen-vectors \( \mathbf{X}_v \) related to the eigen-values \( S_v \) in following way:

\[
\begin{bmatrix}
\mathbf{X}_v \\
\mathbf{X}_v
\end{bmatrix}
\begin{bmatrix}
C & M \\
M & O
\end{bmatrix}
\begin{bmatrix}
2S \\
1
\end{bmatrix}
\begin{bmatrix}
\mathbf{X}_v \\
\mathbf{X}_v
\end{bmatrix}
= 1
\]  

(3)

forced responses of the structure could be expressed like:

\[
\mathbf{X} = \sum_{v=1}^{n} \left( \begin{array}{c}
\mathbf{X}_v \\
\mathbf{X}_v
\end{array} \right) \begin{bmatrix}
F \\
F
\end{bmatrix}
+ \left( \begin{array}{c}
\mathbf{X}_v \\
\mathbf{X}_v
\end{array} \right) \begin{bmatrix}
F \\
F
\end{bmatrix}
\]  

(4)

(The notation "---" signals complex conjugate)

And this expression we use to identify the complex modal characteristics from the measured structural vibrations.

5. Practical identification: general features of our algorithms.

When limiting the frequency domain of the excitation pulsation \( S \) to the vicinity of one or maybe two or three well coupled resonances, the \( (S - S_v) \) factors in expression [4] are much smaller for the modes inside this domain than for those outside, and moreover, much smaller than every \( (S - \bar{S}_v) \) factors, because:

\[
| S - S_v | = | \bar{S}_v w_v + j (w - w_v) | \leq \bar{S}_v | w_v | \quad \text{if} \quad w \rightarrow w_v
\]

and

\[
| S - \bar{S}_v | = | \bar{S}_v w_v + j (w + w_v) | \leq 2 | w_v | \quad \text{if} \quad \bar{S}_v \ll 1
\]

Thus the summation of expression [4] may be restricted to the few more important terms related to the eigen-modes inside the frequency window, and the contribution of all the other terms be considered as a slowly varying function of the excitation pulsation.

The practical curve fitting computations for forced vibrations of a structure, finally rests upon following formula:

\[
\mathbf{X}_i = \mathbf{U}_i + S \mathbf{V}_i + \sum_{v=1}^{n} \frac{T_{vi}}{S - S_{vi}} \begin{bmatrix}
\mathbf{X}_v \\
\mathbf{X}_v
\end{bmatrix} \quad n = 1, 2 \text{ or } 3
\]  

(5)

Where the index \( i \) labels the degrees of freedom; a simple function with complex coefficients takes in account the external modes; and the modal participation coefficients \( T_{vi} \) are products of the components of the eigen-function by the generalized force acting on mode \( v \) (this latter being the scalar product of the eigen-function \( \mathbf{X}_v \) by the spatial distribution of loading).

Provided that there are more experimental excitation pulsations than parameters to identify, we get an overdetermined set of non-linear equations. To linearize our problem, the identification is handled by an iterative smoothing method based on gradient techniques associated to a least square criterion.
6. Special features of our algorithm

In order to get best accuracy even in case of noisy experimental data, we implemented following special features:

First, for curve fitting of each individual response, we use a special row conditioning, giving most importance to the responses to pulsations near the anticipated resonances (which are progressively updated). Beside the immunity toward experimental errors, we gain this way also very quick convergence, and good reliability of the modal characteristics in case of strongly coupled resonances.

Furthermore, noting that the primary identification process often converges toward slightly different eigenvalues $\lambda_{v1}$ for each response, and that it may have a notable influence on the accuracy of the modal participation coefficients, we recompute the smoothing, while forcing the eigen-values $\lambda_v$ to a mean of some of the $\lambda_{v1}$ obtained during first run, on the whole structure.

As the dispersion of the $\lambda_{v1}$ could be important for eigen-modes for which some of the transducers are close to nodes of vibration, the evaluation of the mean eigenvalues is first referenced to an empirical fitness criterion (comparing the mean distance between experimental data and simulations based on identified values, to the diameter of the concerned Nyquist resonance loops).

After the second run, we thus get modal characteristics which are not strictly the best for every individual response, but rather the best for the structure on the whole. The modal participation parameters $t_v$ let us evaluate the components of the eigen-vectors versus the instrumented degrees of freedom.

B. SOME EXPERIMENTAL RESULTS.

In order to qualify some simplifying assumptions used in numerical models for prediction of the vibrational responses of circularly repetitive structures (maybe seismic responses of reactor vessels), we at the moment are performing in our laboratory some tests on very simple such structures.

Some of the experimental results already obtained provide good examples for pointing out the particular modal behaviour of axisymmetric structures and the performances of our identification tools.

1. Modal deformations of a uniform disc (5mm thick steel plate) with free boundary conditions.

This plate is one of the simplest internal part of our academic model of a reactor vessel, and before clamping it inside the external tub (for information, see figure 5 ahead), we vibrated it under free boundary conditions, suspending it by its center by means of a very soft spring.

The theoretical mode shapes of this simple axisymmetric element are shown in figure 2, where obviously the modes presenting modal diameters are of multiplicity two (annotated D).

The corresponding experimental resonances were in these cases splitted into pairs of simple modes with close but distinct resonance frequencies, and their mode shapes were angularly well determined, due to the fact that this disc was cut from a laminated plate.

On figure 3, the experimental modal deformations for such a pair of modes, display very important differences of shape (especially the nodal lines) and good reliability between their angular orientation, and the direction of laminating (observable under grazing light incidence)

What can also be underlined, is the difference of relative damping of these two modes; actually, the central suspension, although very soft, provides a rather important local damping. This has no effect on the second mode for which this central point belongs to the nodal lines, but has a notable effect on the first one, for which there is modal deformation at that point.
Figure 2: Experimental mounting and theoretical mode shapes of a uniform disc with free boundary conditions.

Figure 3: One pair of experimental mode shapes (mode 1 and 2)

On figure 4, the experimental modal deformation of the third mode is without nodal diameters (a simple mode) and however presents the same directivity: the nodal line should be circular if the plate were isotropic.

Let us point out that the effectiveness of the central damper is even more important for this mode than for the previous ones, because the central point is this time a local maximum of the mode shape.

Figure 4: Shape of mode 3
2. Experimental identification of strongly coupled modes

The limits of ability of our program to identify coupled modes, were approached, exciting an axisymmetric structure constituted of a circular plate and a cylindrical chimney, the disc being clamped along its edge inside the external tub which was filled with water (see figure 5).

Figure 5: Experimental mounting of a simple model of vessel with internal parts

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Table 1: A pair of strongly coupled modes of the internal component; Table of eigen-vector

Figure 6: Nyquist plot of resonance loops of two strongly coupled modes.
The clamping boundary conditions correlativey lessened the non-isotropic effects due to laminating, and besides, the closer resonance frequencies were also more strongly coupled because of the additional damping provided by the fluid.

The Nyquist plots of figure 6 display superposed experimental and identified resonance loops of six of the responses of this structure, for two closely coupled modes.

Table I gives the experimental resonance frequencies and relative damping coefficients. It is to be noticed that the gap between the resonance frequencies (0.25 Hz) has the same order of magnitude as the typical modal bandwidth due to damping (0.27 Hz). Not surprisingly, these two modes present strongly complex (coupled) eigen-vectors; table I gives their components at eight points equally distributed over a circle at approximately the middle of the radius of the disc.

3. Tricks to split multiple modes of a structure

When exciting quasi-multiple eigen-modes of a structure, the identification program may be able neither to decouple them correctly, nor to consider them as a particular occurrence of a true multiple mode, performing like a simple one. Fortunately there are experimental tricks to split any multiple modes.

In the case of axisymmetric structures, one can break the symmetry, by introducing a small point mass, stiffness or damping (generally, adding mass is most suitable).

But actually modifying the structures may be deceiving, because the added elements do not bear reliable or simple mechanical characteristics and because the resonance frequencies may even be more coupled than before, by non proportional damping.

Our analogue amplitude and phase controlling feed-back for four independent parameters of the excited structure, let us simulate the addition of local mechanical impedances, simply by feeding actuators with the computed reaction forces that such pure elements would provide.

For instance, if the controlled input parameter is a linear combination of the input force and the acceleration signal at that point, this produces the same behaviour as adding a point mass. Moreover, to ensure that this simulated structural modification does not increase the coupling of the modes by non proportional damping, one has the opportunity to add to the combination of force and acceleration responses, some amount of velocity response (integrating the acceleration signal), in order to get so much positive or negative local damping, that were suitable to minimize the non-diagonal terms of the damping matrix, and thus the coupling between modes.

Our acquisition program automatically takes in account these modifications of the excitation control parameters; so after identification, a structural modification program could very easily retrieve the perturbations from the modified vibrational behaviour and simulate the original multiple modes with much better accuracy.

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