Eigenvibrations of Elastic Tube Bank in a Fluid

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SUMMARY

The determination of resonance frequencies of elastic tube bundle immersed in a liquid is generally difficult, because the number of cylinders is large, as it is the case for heat exchangers. In this paper, we are concerned with a method of computation, using homogenization technique which consists of likening the coupled liquid-structure as an equivalent fluid. Several numerical examples illustrate this computation technique, in a good agreement with experimental measurements.
1. Introduction

With the growth of the units of industrial equipments, the eigenfrequencies tend to enter into the range of usual excitations from few Hz to several thousands Hz ; the determination of these resonance frequencies is then of great importance with regard to the mechanical behaviour of the structures. Particularly, this fact is crucial, concerning the heat exchangers, condensers, etc... It is the reason for which the problem of eigenoscillations of tube bundle was intensively studied by numerous authors during these last years : Connors [1], Blevins [2], Chen [3], Paidoussis [4], Pettigrew [5], Gorman [6], etc... In France, experimental works are carried out at EDF [7], [8], [9] and C.E.A. [10]. The computation of natural vibrations of elastic tube arrays is generally difficult because the number of cylinders is large in industrial applications. To overcome this difficulty, numerous efforts were made in different ways : Schumann [11], Shinohara [12] and at EDF [13], [14]. The idea essentially is to liken the fluid-structure system as a kind of equivalent fluid whose one has to find the mechanical properties. In this paper, we present the methods developed at EDF ; numerical examples are described in order to illustrate them. Experimental verification are also presented.

2. Mathematical Model and Notations

The fluid, of density \( \rho_0 \), is supposed to be in a state of rest ; it occupies a domain \( \Omega_0 \) limited by a rigid wall \( \Gamma \) ; \( \gamma_\ell \) represents the surface of each tube \( \ell \). Then, the small pressure fluctuations of frequency \( \omega \) satisfy :

\[
\Delta p + \frac{\omega^2}{C^2} p = 0 \text{ in } \Omega_0
\]

with the boundary condition on \( \Gamma \)

\[
\frac{\partial p}{\partial n} = 0 \text{ on } \Gamma \left( \frac{\partial p}{\partial n} : \text{ normal derivative} \right);
\]

\( C \) is the sound speed (\( C \) is infinite for uncompressible fluid). The pressure \( p \) is obviously related to the displacement \( \mathbf{s}_\ell \) of each tube \( \ell \) by

\[
\frac{\partial p}{\partial n_x} = \rho_0 \omega^2 \mathbf{s}_\ell \cdot \mathbf{n} \text{ on } \gamma_\ell,
\]

where \( \mathbf{n} \) is the unit-normal vector oriented towards the interior of \( \gamma_\ell \). Two cases are considered hereafter.

2.1 2-Dimensional Problems

The tube movement equation is :

\[
m_\ell \frac{d^2 \mathbf{s}_\ell}{dt^2} = \int_{\gamma_\ell} p(y) \mathbf{\hat{r}}_x \mathbf{d} \gamma - k_\ell \mathbf{\hat{s}}_\ell, \quad \ell = 1 \text{ to } K,
\]

\( m_\ell \) and \( k_\ell \) are respectively the mass and the stiffness coefficient per tube length unit. The eigenfrequency equation becomes :
\[
\begin{aligned}
&\text{\(C^2\Delta p + \omega^2 p = 0\) in } \Omega, \quad \frac{\partial p}{\partial n} = 0 \text{ on } \Gamma, \\
&\frac{\partial p}{\partial n} = \frac{\rho_0 \omega^2}{K} \cdot \int_{\gamma_y} p(\gamma) \hat{n}_y \, d\gamma_y \text{ on each } \gamma_y.
\end{aligned}
\]
In case of an incompressible fluid, eq. (5) can be transformed into a standard eigenvalue problem by introducing the influence functions \(u_{kj}\) defined as follow; for \(k = 1, \ldots, K\) and \(j = 1, 2\) [15]:
\[
\begin{aligned}
&\Delta u_{kj} = 0 \text{ in } \Omega, \quad \frac{\partial u_{kj}}{\partial n} = 0 \text{ on } \Gamma, \\
&\frac{\partial u_{kj}}{\partial n} = \cos(\frac{\pi}{K}x) \delta_{k,k} \text{ on each } \gamma_y, \quad \delta_{k,k} \text{ is the Kronecker symbol},
\end{aligned}
\]
Then \(\omega\) satisfied
\[
\frac{1}{\omega^2} \text{diag}(\chi_1, \chi_2, \ldots, \chi_K) \hat{u} = \left( K + \text{diag}(\frac{m_0}{\rho_0}) \right) \hat{u},
\]
where \(\hat{u} = \text{col}(\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_K)\) and \(K\) is the added mass matrix whose entries are the numbers
\[
\int_{\gamma_y} u_{kj} n_1 \, d\gamma_y. \quad \text{For compressible fluid we can also define } K \text{ which then depends on } \omega \text{ but } u_{kj}
\]
satisfies a Helmholtz equation in \(\Omega\). Here, \(n_1\) is the \(i\)-component of \(\hat{n}\).

2.2 3-Dimensional Problem

If the tube bending is taken into account, the tube equation becomes:
\[
-m \omega^2 \hat{s} + EI \frac{d^2 \hat{s}}{dz^2} = \int_{\gamma(z)} p(x; z) \hat{n}_x \, d\gamma(z),
\]
with 2 boundary conditions at each end. Then, it is necessary to expand \(\hat{s}\) with the tube eigenmodes:
\[
EI \frac{d^2 \omega_i(x)}{dz^2} = m \int_1^2 \omega_i(x), \quad i = 1, 2, \text{etc}...
\]
It is possible to eliminate the pressure when the fluid is incompressible (see [15]) by using the technique of Chen, Paidoussis [3][4].

3. Case of Large Bundle: Homogenization Method

When the number of tubes is of the order of some thousands, numerical computation is unfeasible because the size of matrix \(K\) is very large. This difficulty can be overcome by homogenization; that supposes the bundle is spatially periodic (period \(\varepsilon\)), and the tubes are identical (same mass and stiffness).

3.1 2-Dimension Problems

With these assumptions above, the homogenized equation is [13],[14]:
\[
\left( C_{eq}^2 \frac{\rho_0 C^2 \omega^2}{k-\varepsilon} \frac{1}{\varepsilon^2 \pi r^2} \right) \Delta p + \omega^2 p = 0 \text{ in } \Omega, \quad \frac{\partial p}{\partial n} = 0 \text{ on } \Gamma.
\]
\(\Omega\) is the domain occupied by the bundle and fluid; \(C_{eq}\) is the equivalent speed of sound through the tubes (\(C\) is not affected along the tubes); \(r\) is the radius tube. The computation
of $C_{eq}$ is given in ref. [13]. The eigenvalues $\lambda$ of the Laplacian operator on $\Omega$ can be computed, so that the $\omega$'s are the roots of algebraic equations:

$$\omega^2 = \frac{kT + \lambda(\rho T C_{eq}^2 + b)\lambda}{2\rho T} \left[ \frac{1}{1 - \frac{4 \rho T kT C_{eq}^2 \lambda}{[kT + \lambda(\rho T C_{eq}^2 + b)]^2}} \right],$$  \hspace{1cm} (11)

with $b = \frac{\pi r^2 \rho_0 C_{eq}^2}{\varepsilon^2 - \pi r^2}$, $kT = k/\pi r^2$, $\rho T = m/\pi r^2$.

Because the sequence of $\lambda$'s tends to infinity, it is seen that $\omega^2 \rightarrow + \infty$ while $\omega \rightarrow kT/(\rho T + b/C_{eq}^2)$.

Homogenized dynamical equation: it is derived by using the Laplace transform to the dynamical equation, and homogenizing the equations so obtained. The inverse Laplace transform leads to:

$$C_{eq}^2 \Delta p - \frac{\rho_0 C_{eq}^2}{\rho T} \frac{\pi r^2}{\varepsilon^2 - \pi r^2} \frac{kT}{\rho T} \int_0^\tau \sin \left( \sqrt{\frac{kT}{\rho T}} \xi \right) \Delta p(x; t - \xi) d\xi = \frac{2^2 p}{\partial t^2}$$  \hspace{1cm} (12)

This equation may be solved by projection on the eigenfunctions of the Laplacian operator.

### 3.2 3-Dimension Problems

When the bending is considered, it is shown [13], by expanding $\mathbf{w}(z)$ with the tube eigenmodes $\omega_1(z)$, that

$$C^2 \frac{\partial^2 p(x, z)}{\partial z^2} + C_{eq}^2 \frac{\partial^2 p(x, z)}{\partial x^2} \sum_{i=1}^n \frac{\pi r^2 \rho_0 C_{eq}^2}{m(\varepsilon^2 - \pi r^2)} \omega_1^2(z) + \omega^2 \frac{\partial^2 p(x; z)}{\partial z^2} = 0,$$  \hspace{1cm} (13)

where $\Delta p = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $p_1(x) = \int_0^L p(x; z) \omega_1(z) dz$.

This eigenvalue problem is nonlinear and can be solved by frequency sweep technique.

**Remark:** This technique may also be applied for PWR fuel assemblies where the tubes are mechanically connected; in this situation, the stiffness matrix of the connections is transformed in a Laplacian operator (see [18]) because the displacement of each cylinder depends on the position of the nearest ones.

### 4. Numerical Results

Numerical computations have been done for a bundle of 10 x 10 tubes with $\varepsilon = 1.44$ cm, $r = 0.5$ cm, $m = 0.22$ kg, $k = 27,800$ N/m; the mechanical eigenfrequency of tube in vacuum is $f_0 = 56.57$ Hz; the tube array is placed in a box with a square cross-section of side length $D = 14.4$ cm.

#### 4.1 Case of Uncompressible Fluid (Water)

Exact calculation was carried out by means of the influence functions $u_k$ and $k_k$. The 200 eigenfrequencies are situated inside the interval 39.85 - 52.66 Hz. Some eigenmodes are pictured on fig. 4.5. The computer time is relatively large: about 20 minutes with IBM 30-81.

#### 4.2 Compressible Fluid

Homogenization is used for air ($\rho_0 = 1.33$ kg/m$^3$, $C = 340$ m/s) and water ($\rho_0 = 1,000$ kg/m$^3$, $C = 1,200$ m/s). The results appear in Tables I, II. The computer time is very short: about 3 seconds.
5. Experimental Verification

The acoustic resonance frequencies of the 10 x 10 tube bundle, described in the previous section, have been measured; the details of the experiment are given in [14]. A good agreement between measurements and calculations is observed (fig. 6). Measurements concerning the water-tubes interaction will be done in the future. However, Fig. 7 (from [7]) shows the broadening effect of the mechanical eigenfrequency (f_0 = 40 Hz) in the case of a 7 x 7 tubes bundle placed in a transverse flow (see [7]): it is observed that the peaks of resonance are lied in the interval 27 Hz-35 Hz, while the eigenfrequencies computed by the influence functions method (water at rest) are situated inside the band 30.4 Hz - 37.4 Hz; the effect of flow is a small left-shift of this frequency interval.

6. Conclusion

This paper shows that the homogenization techniques are full of promise in engineering design, and they can be extended to other situations such as the mechanical behaviour of fuel assemblies or the propagation, in a PBR heat exchanger, of the pressure due to the explosive sodium-water reaction. However, several difficult problems are to be solved, such as for instance the presence of flow through a cylinder array.

Acknowledgement: The authors are acknowledged to M. Ibnou Zahir for coding of the influence functions method.

References


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Fig. 1 - 2 - dimension case.

Fig. 2 - 3 - dimension case.

Fig. 3 - Equivalent sound speed.
Fig. 4 - Eigenmode n° 26 (10 x 10 bundle in water).

Fig. 5 - Eigenmode n° 84 (10 x 10 bundle in water).

Fig. 6 - Transfer function for the cavity with tubes and air (arrows show computed eigenfrequencies).

Fig. 7 - Measured spectral density of tube movement (from [7]).
Table I
Eigenfrequencies of air-bundle system \( (C = 340 \text{ m/s}) \)

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Table II
Eigenfrequencies water-bundle \( (C = 1,200 \text{ m/s}) \)

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