A Comparison of Experimental and Theoretical Vibration Results for Fluid-Coupled, Co-Axial Cylinder

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Summary :

A fluid-coupled cylinder under seismic loading reveals a much more complex behavior than was implied by design assumptions. The problem becomes more complex when two coaxial cylinders were coupled by water in their adjacent annular region such as pressure-suppression pools in Boiling Water Reactors.

Theoretical and experimental investigations of the vibrational behavior of a cylindrical shell and coaxial cylinder were conducted to seek possible improvements in the design.

The study is carried out in two steps.

1. Test on the shaking table
   Two models are used in this test. One is a cylindrical shell, and other is coaxial cylinder which has the fluid filled annular space.

The model was set up on the shaking table and excited with sinusoidal wave. The natural frequencies, mode shapes, and hydrodynamic pressure distributions were measured. Tests were conducted at several water level.

2. A theoretical treatment of the liquid-shell systems
   This study includes the development of a general finite element theory for axisymmetric shell, and computer code to implement finite element theory. This program compute the natural frequencies, mode shapes, and hydrodynamic pressure distributions including the initial hoop stress due to the hydrostatic pressure.

From above tests and analyses, this paper provides the following results ;

1) Experimental and analytical results regarding natural frequencies, mode shapes, and hydrodynamic pressure distributions were compared and a good agreements were found for both cases of the coaxial cylinder and the cylindrical shell.
2) The damping ratio of the cylindrical shell is about 5% and the damping ratio of the coaxial cylinder varied from 5% to 11% according to the water level.
   The small difference of the natural frequency of inner and outer cylinder are associated with higher damping ratios than those measured in the cylindrical shell.
3) The foundamental natural frequency ($n=1$) of the coaxial cylinder becomes lower than that of the cylindrical shell because the fluid has a considerable inertical effect on the shell.
   The natural frequencies of the coaxial cylinder were closer to these of the cylindrical shell for higher circumferential mode number ($n>5$).

1. Introduction

Many cylindrical structures which contain the fluid are used in nuclear power plants and chemical plants. Fluid-coupled cylindrical structures such as oil storage tanks reveal complex behavior than those without fluid under the seismic loading. (1),(2) The problem becomes more complex when two coaxial cylinders were coupled by water in their adjacent annular region such as pressure-suppression pools in Boiling Water Reactors.

Many studies of vibrational characteristics of fluid-coupled coaxial cylinder with the narrow gap and a fluid-coupled cylindrical shell have been published. (3),(4) However these studies were restricted to the development of the analytical method and the comparison of the experimental and theoretical results. Theoretical and experimental investigations of the vibrational behavior were conducted to seek possible improvements in the design.

This study includes calculations and experimental evaluations of hydrodynamic pressure distributions, mode shapes, modal dampings, and the comparisons of the vibrational characteristics of cylindrical shell and coaxial cylinder.

2. Analysis

A general finite element analysis was development for axisymmetric shell and a computer program was written to obtain the natural frequencies, mode shapes, and hydrodynamic pressure distributions including the initial hoop stress due to the hydrostatic pressure.

Since the finite element method is generally well known, only a discussion of the liquid element is provided in this paper. The liquid is idealized as a variable 4-to-8 node isoparametric element shown in Fig. 1. The distribution of the velocity potential within an element m are approximated as follows :

$$\phi_m\,(r,s,\theta) = \sum_{n=1}^{\infty} \sum_{i=1}^{8} h_i\,(r,s) \cos n\theta\, \phi_{im}$$

in which ϕ_{im} is the value of the velocity potential at node i of element m.

h_i is the interpolation functions.

3. Experiment

3.1 Method

Two models were used in this test. One is a cylindrical shell, and other is coaxial cylinder which has the fluid filled annular space between inner and outer cylinder as shown in Fig. 2.

The acrylic cylinders used in this study has the following dimensions and properties.

Outer Cylinder : 800 mm diameter, 850 mm height and 2 mm thick
Inner Cylinder : 300 mm diameter, 850 mm height and 10 mm thick
E (Young's Modulus) : 3.38×10^4 Kgf/cm²
ν (Poisson's Ratio) : 0.3
ρ (Density) : 1.21×10^{-6} Kg.sec²/cm⁴

The model was set up on the shaking table and excited with sinusoidal wave. The natural frequencies, mode shapes, and hydrodynamic pressure distributions were measured. Test were not only conducted at the water level of 800 mm, but also at 600 mm, 400 mm and 0 mm.

The test consists of two phases. First, natural frequencies were obtained by frequency sweep excitation. Second, vertical and circumferential mode shpes for each natural frequencies were obtained by harmonic excitation. The test cases are shown in Table 1.

3.2 Results

Fig. 3, 4, 5 and 6 show transfer functions of response acceleration of the outer cylinder in the case of 800 mm water level. Fig. 3 and Fig. 5 show the tangential direction acceleration at the top and the radial direction acceleration at the middle in the case of the cylindrical shell. Fig. 4 and Fig. 6 show those of the coaxial cylinder.

From the maximum peak of the curves in Fig. 3 and Fig. 4, foundamental natural frequencies (n=1, m=1) are obtained 28.3 Hz for the cylindrical shell and 24.3 Hz for the coaxial cylinder, where n is harmonic number of circumferential mode and m is number of vertical mode.

From Fig. 5 and Fig. 6, several resonance frequencies besides n=1 mode's natural frequencies are obtained. From Fig. 4, a small resonance peak having 35.6 Hz is obtained. This is a beam mode, of which the inner cylinder is mainly vibrating. In Fig. 7 and Fig. 8, the relation between the resonance frequencies and the harmonic number n are shown in the case of cylindrical shell and coaxial shell, respectively.

The foundamental natural frequencies and it's critical damping ratio for each water level are shown in Table 2.

4. Discussion

4.1 Natural Frequencies and Mode Shapes

The calculated and the experimental frequencies at 400 and 800 mm water level are presented in Fig. 7 and Fig. 8. As seen from these figures, the calculated values are in good agreement with the experimental results. This confirms the accuracy of the analysis.

In the cylindrical shell, the experimental value is about 5% higher than calculated one for the foundamental mode (n=1). And in the coaxial cylinder, the experimental value is only 1% higher than calculated one. Initial irregularities of the shell excited another modes and make the experimental value higher.

A comparison between the experimental value and the calculated value for the foundamental modes of the outer cylinder was made and the good agreement is obtained as shown in Fig. 9 and Fig. 10. The good agreement is obtained for higher modes, too. In coaxial cylinder, the inner cylinder vibrates in beam mode and out-of-phase to the outer cylinder. The higher harmonic number modes superpose of the foundamental mode. This tendancy is more evident at low water level.

The hydrodynamic pressure distributions acting on the outer cylinder are shown in Fig. 11. The experimental values was similar to the calculated value, although the former is a little larger than the latter.

4.2 Comparion between Cylindrical Shell and Coaxial Cylinder

The founamental frequencies of the coaxial cylinder are lower than those of the cylindrical shell. The higher the water level is, the larger the difference of the frequencies between each model is. Then, being accompanied with the increase of the harmonic number n, the difference of the of the frequencies between each model is decreasing.

This suggests that added water mass for each mode varies by the water level and inner cylinder's size.

The abscissas in Fig. 11 shows the nondimensional value of the added water mass. This figure show that the added water mass of the coaxial cylinder is larger than that of the cylindrical shell. The mode shapes and the pressure distributions of both model are almost equal.

The damping ratios of the cylindrical shell evaluated from the transfer function is about 5% for each water level. On the countrary, the damping ratios of the coaxial cylinder vary from 5% to 10% according to the water level. It will be supposed that the small difference of the natural frequencies between inner and outer cylinder are associated with higher damping ratios than those measured in the cylindrical shell.

4.3 Effect of Initial Hoop Stress

Fig. 12 shows the initial hoop stress due to the hydrostatic pressure has a signifficant influence upon the vibrational characteristics of the cylindrical shell and coaxial cylinder.

For the lower harmonic number (n=0, 1), there are little influence. In contrast, for the higher harmonic number modes (n≥2), the vibrational modes may be significantly influenced by the initial hoop stress. It should be noted that the effect of the initial hoop stress due to the hydrostatic pressure must be considered in the case of higher harmonic number vibration.

4.4 Effect of Inner cylinder size

Fig. 13 shows the variation of the natural frequencies (n=1, m=1, 2) versus the variation of the radius ratio between inner cylinder (b) and outer cylinder (a).

In the case of m=1, the natural frequency decreases monotonously as the inner shell radius increases. This shows that the added water mass increases as the inner cylinder radius increases. As the radius (b) comes to zero, the natural frequency converages to that of the cylindrical shell.

In the case os m=2, the natural frequency has the peak. At the small inner cylinder radius region, the vibrational mode which the inner cylinder mainly vibrate, dominates because of the decreasing the inner cylinder's bending stiffness.

Fig. 14 shows the effect by the inner cylinder thickness for the natural frequencies in the case of b/a=0.875. As the thickness decreases, the natural frequencies decrease. This reason is the inner cylinder's bending stiffness decrease and the inner cylinder mainly vibrate. It is remarkable that the natural frequency is converged to the certain value. The foundamental natural frequency is independent for the inner cylinder's stiffness when the stiffness is considerable large.

5. Conclusions

1) Tests and analytical results regarding natural frequencies, mode shapes, and hydrodynamic pressure distributions were compared and good agreement was found for both cases of the coaxial cylinder and the cylindrical shell. The computer program gave satisfactory results.
2) The cos nθ-type modes which have been attributed to initial irregularities of the shell radius were excited by the shaking table.
3) The foundamental natural frequency (n=1) of the coaxial cylinder becomes lower than that of the cylindrical shell because the fluid has a considerable inertia effect on the shell. The natural frequencies of the coaxial cylinder were closer to these of the cylindrical shell for higher circumferential mode number (n>5)
4) In the vibration of the coaxial cylinder, the thicker the inner shell is, the higher natural frequency is, and the smaller the inner shell is, the lower natural frequency is.

5) The damping ratio of the cylindrical shell is about 5% and the damping ratio of the coaxial cylinder varied from 5% to 11% according to the water level. The small difference of the natural frequency of inner and outer cylinder are associated with higher damping ratios.

Acknowledgement

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Reference

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(3) H. Chung et.al., "Analysis of a Cylindrical Shell Vibration in Cylindrical Fluid Region", Nuclear Eng. and Design, No. 63, 1981.
(4) M. Livolant et.al., "Fluid-Structure Interaction : A General Method Used in the CEASMT Computer Programs", Computer & Structures Vol. 12, 1980.

Table 1 Experimental Case

Water Level (mm)	400		600		800	
Cylindrical Shell	○	△	○	–	○	△
Coaxial Cylinder	○	△	○	–	○	△

Remark : ○ : Sweep Excitation
△ : Stationary Excitation

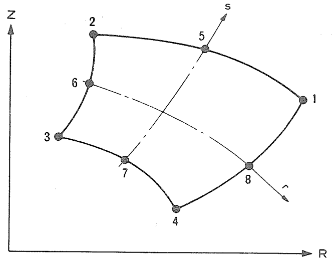


Fig. 1 Liquid Element

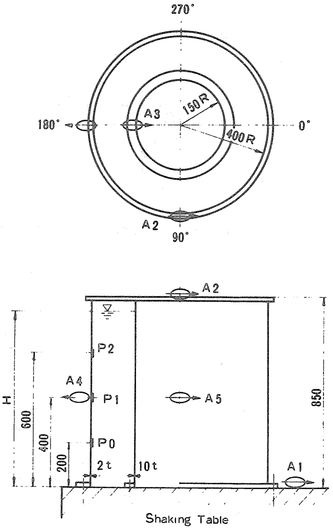


Fig. 2 Model Tank

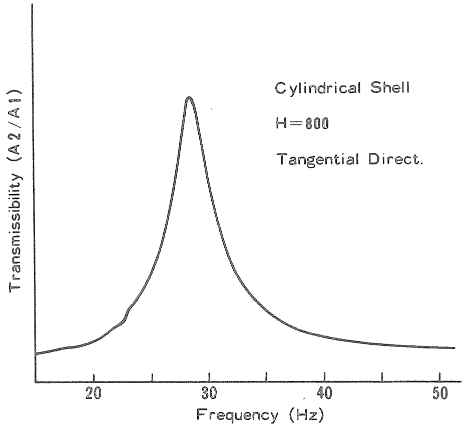


Fig. 3 Transfer Function (1)

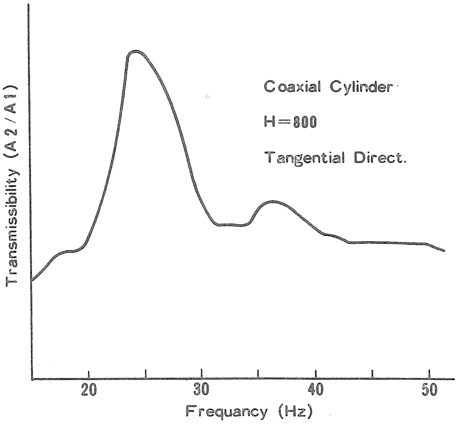


Fig. 4 Transfer Function (2)

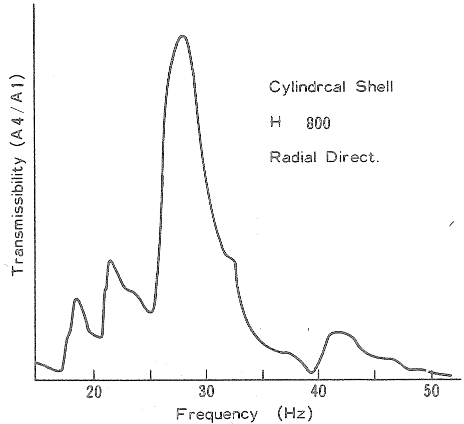


Fig. 5 Transfer Function (3)

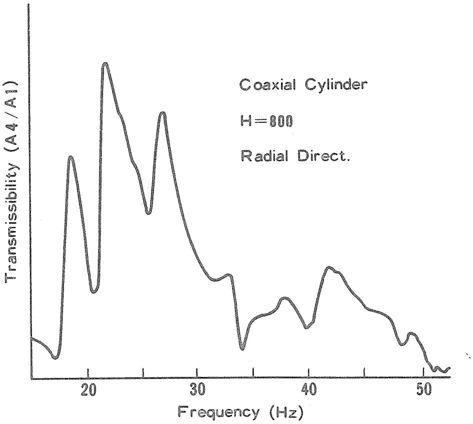


Fig. 6 Transfer Function (4)

Table 2 Natural Frequencies and Damping Factor of Foundamental Mode

Type	Water Level (mm)	Natural Frequency (Hz)	Damping Factor (%)
Cylindrical Shell	800	28.4	5.1
	600	39.2	5.8
	400	57.0	5.1
Coaxial Cylinder	800	24.3	11.3
	600	35.3	9.1
	400	54.4	4.8

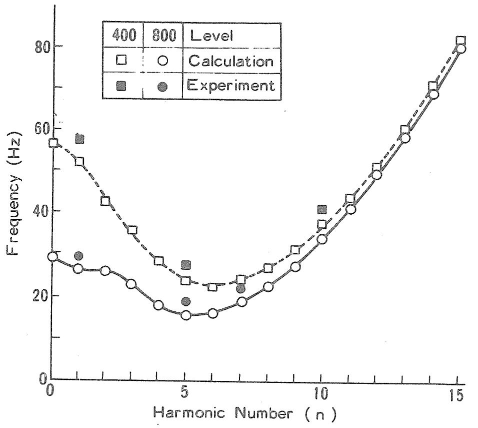


Fig. 7 Natural Frequencies of Cylindrical Shell

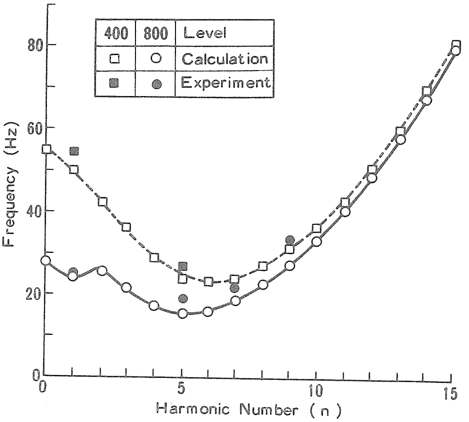
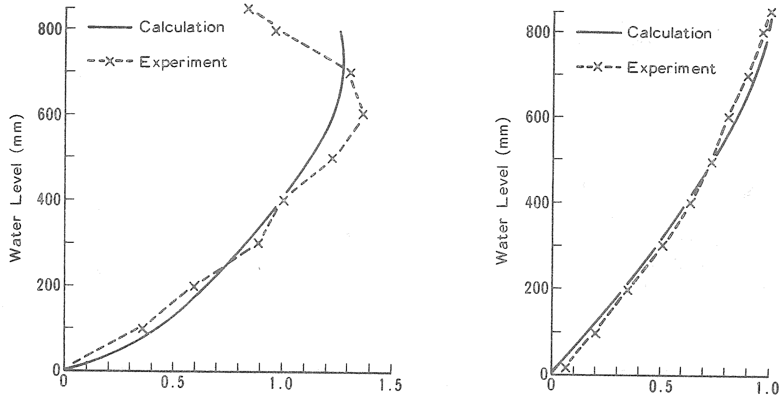


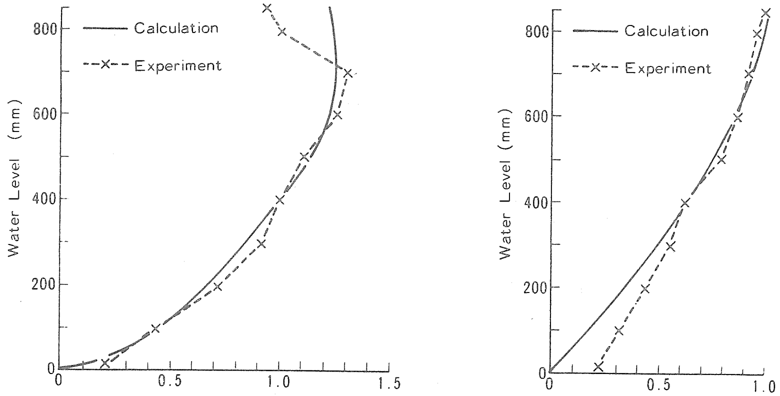
Fig. 8 Natural Frequencies of Coaxial Cylinder



(a) Radial Direct. Vib. Mode

(b) Tangential Direct. Vib. Mode

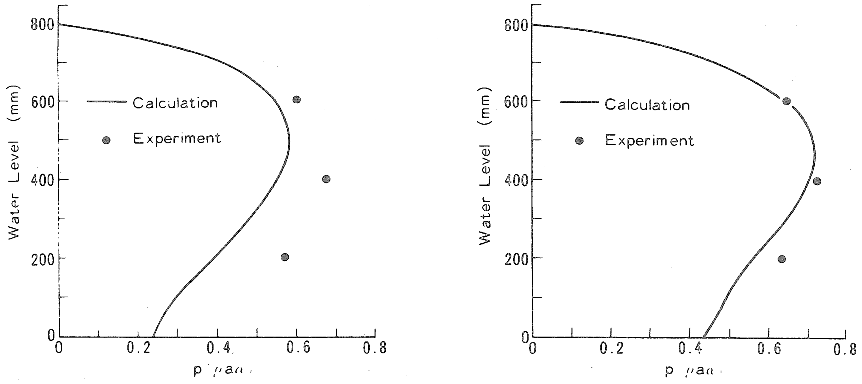
Fig. 9 Vibrational Mode Shapes of Outer Shell (Cylindrical Shell, H = 800)



(a) Radial Direct. Vib. Mode

(b) Tangential Direct. Vib. Mode

Fig. 10 Vibrational Mode Shapes of Outer Shell (Coaxial Cylinder, H 800)



(a) Cylindrical Shell

(b) Coaxial Cylinder

Fig. 11 Hydrodynamic Pressure Distribution

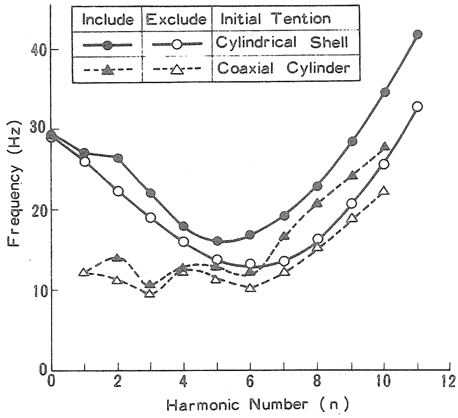


Fig. 12 Effect of Initial Tension

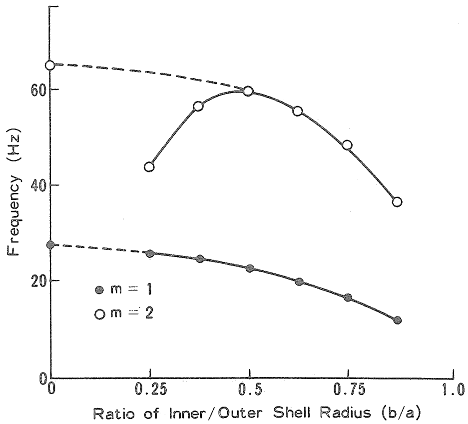


Fig. 13 Influence of Inner Shell Radius

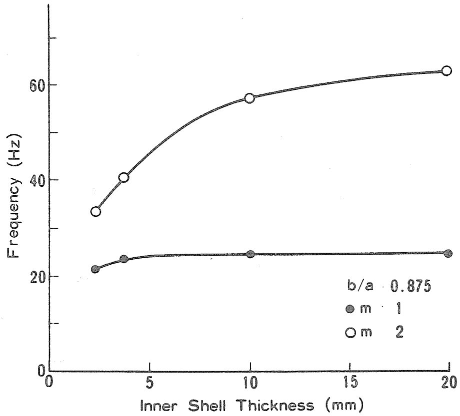


Fig. 14 Influence of Inner Shell Thickness