Calculations Methodology of the Instabilities for Thin Shell Structures

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This paper presents a calculation methodology of the instabilities applicable for the design of axisymmetric thin structures against buckling risk. The application case discussed here is a tank loaded by external pressure and seismic effects. The calculations have been made by the Finite Element Method with the CASTEM System Codes.

In a first step, the structure is supposed perfect and the calculations are made with an axisymmetric thin shell model. The analysis is made in the following way :

- research of the Euler critical load in the aim of determining the most sensitive parts against buckling risk,
- then, taking into account the plasticity by the research of the bifurcation loads for every parts previously determined.

The axisymmetric model gives good results for high circumferential modes. In the case of earthquake, simulated by lateral loads, complementary results about shear buckling modes are obtained using a three dimensional model.

However, the real structure is not perfect but includes necessarily initial imperfections. Consequently, in a second step, the study consists in determining the critical load reduction due to the introduction of an initial imperfection. For this, we use :

- even, analytical formulae, in analogy with the beam theory, supposing that the defect is parallel to the buckling mode of the perfect structure,
- or, directly, three dimensional calculations on a sector of structure including an initial defect.

However, calculations show that the defect parallel to the buckling mode of the perfect structure, is not always conservative in a stand point of design. This puts the stress on the problem of the determination of the most penalizing initial defect and limits, consequently, the use of the first method only for a coarse evaluation of the reduction.

1. INTRODUCTION

The purpose of this paper is to describe the different steps of the analysis of structure instabilities in order to elaborate a design review of buckling risk.

The studied structure is a great thin vessel, the geometry of which is presented on figure 1. The different applied loads, presented on figure 2, are the following :

- internal structures weight,
- internal hydrostatic pressure,
- gas external pressure,
- seïsmic load (vertical and horizontal forces and moment).

Calculations are performed with the CASTEM system codes : code INCA for axisymmetric models and codes TRICO or BILBO for three dimensional ones.

2. DETERMINATION OF POTENTIAL ZONES OF INSTABILITY

The first step is a research of the Euler loads of the structure, in order to determine the most sensible parts to the buckling risk. This elastic analysis has been made with an axisymmetric thin shell model (figure 3).

The resolution method is the following :

on the deformed body, the codes calculates :

- the stiffness matrix K,
- the geometric stiffness matrix K (σ),
- the matrix of pressures K(P).

Then, it calculates the mode X, and the multiplicative factor $\lambda = \omega^2$ so that :

(1) $\quad | K + \omega^2 (K(\sigma) - K(P)) | X = 0$

The buckling modes considered are X = a cos m θ h n(z), where m is the order of the circumferential mode imposed at the beginning.

On figure 4, are the curves giving the calculted multiplicative factor λ versus the circumferential mode m, for the three parts where instabilities may appear : top, bottom and cylindrical shell. The lost point of these curves gives the Euler load for the considered part. This first step is a quick and cheap approach of the structure behaviour.

3. DETERMINATION OF THE MOST PENALIZING LOADS

For our problem, the different working configurations don't carry to the same loads. So, the first selection of the most penalizing loads is made according to the following criteria :

- the greatest compression membrane stress,
- the greatest Von Mises Stress (yield criterion).

This second point is very important because the drop of critical loads is directly linked with the drop of the tangential modulus of the material.

4. TAKING INTO ACCOUNT OF THE PLASTICITY : BIFURCATION LOADS

For the multiplicative factor corresponding to the Euler loads, the considered structure is already plastified. So, the elastic analysis is not sufficient, and it is necessary to male elastoplastic study in order to determine the real critical load of the structure.

The elastoplastic analysis is made in two steps :

- First, we make an elastoplastic calculation, increasing the loads step by step and taking into account the nonlinear comportment of the material,

- Then, at different steps of the elastoplastic calculation, we make an instabilities research.

The resolution method is the same as that used in the elastic analysis but the code takes into account the plastification of the structure.

Then, we can draw curves giving the multiplicative factor ω^2 versus the circumferential mode m for different steps of the evolution. The structure instability is obtained when the multiplicative factor ω^2 reaches the value 1 (figure 5). So, the circumferential buckling mode is characterized and the meridian mode is defined by drawing the modal shape (figure 6).

5. TAKING INTO ACCOUNT OF SHEAR : THREE DIMENSIONAL MODEL

Until to now, we just considered the vertical effects of earthquake. In fact, shear induced by mode 1 contribution cannot be taken into account with an axisymmetric model. But, in this case, shear is important, and we must verify if shear buckling mode can eventually appear.

In order to introduce the lateral load, we have built a three dimensional model of a half vessel (figure 7). Then, on this model, we have made an elastoplastic instabilities research with code TRICO. The way of driving this analysis is the same as we made for the precedent one. Among the axisymmetric modes that we find necessarily, we can distinguish a shear buckling mode characterised by sliding lines (figure 8).

6. TAKING INTO ACCOUNT OF DEFECTS

6.1 - Necessity of the study and methods used

Until to now, the structure is supposed perfect. But shape measures made on the real structure show that the imperfections are not negligible and that their amplitude is nearly equal to the vessel thickness. So it is important to know their influence on critical load reduction.

Two methods have been used to evaluate this reduction :

1. Analytical method

An analogy is made between the structure and a beam model, in order to use the reduction diagram etablished for this model.

2. Direct method

An instability study is directly computed on a three dimensional model of the structure including an initial defect.

6.2 - Analytical method

This method is presented in notes (1) and (2) and in the paper (3). Under some hypothesis, it is possible to make an analogy with the beam theory. Stresses in the imperfect structure are writen as a limited development on the fondamental mode and the first buckling mode :

$$\text{imperfect structure} : \sigma = \lambda\sigma_0 + \varepsilon\,\sigma_E \text{ with } \varepsilon = \frac{\varepsilon_0\,\lambda/\lambda_E}{1-\lambda/\lambda_E} \quad (2)$$

$$\text{beam theory} : \sigma = \lambda \left|\, \sigma_0 + 6\,\sigma_0\,\varepsilon_0 \frac{\lambda_E}{\lambda_E - \lambda} \right| \quad (3)$$

By identification between the two formulae, we obtain the equivalent defect ξ_0 :

(4) $$\xi_0 = \frac{\varepsilon_0 \,|\, \sigma_E \,|}{6\,|\sigma_0\,|\,\lambda_E}$$

with : λ_E : load factor corresponding to the Euler load

σ_E : stresses corresponding to the first buckling mode

ε_0 : amplification of the first buckling mode

$|\sigma_0|$: stresses for the basic load

$|\sigma|$ represents a norm of σ, here we take the Von Mises stress.

We use the method proposed in note (2) to evaluate the load reduction. We applied it to the case without any seismic load :

- Euler load for the perfect structure : $\lambda_E = 37.8$
- Plastic bifurcation load for the perfect structure : $\lambda_B = 2.9$
- Equivalent defect computation for a shape imperfection equal to the vessel thickness.

  $|\sigma_0| = 379$ daN/cm^2

  $|\sigma_E| = 1073$ daN/cm^2 $\qquad \xi_0 = 0.03$

  $\lambda_E = 37.8$

  $\varepsilon_0 = 2.5$

- Determination of the critical load λ_c for the imperfect structure by using the reduction diagram etablished for the beam model with λ_B, λ_E and ξ_0 (figure 9).

We notice that the reduction induced by the defect is small : about 10 %, so that $\lambda_c = 2.6$. A direct calculation gave a critical load equal to 2.8. For this load case, the structure is not very sensible to the imperfections.

6.3 - Direct method

The analytical method can only give a quick evaluation fo the critical load reduction. More precise results can be obtained by computing a stability calculation on an imperfect three dimensional model.

For these three dimensional models, we choose a defect parallel to the buckling mode of the perfect structure, this one being supposed the most critical. By symmetry, this amounts to study a sector of vessel, the angle of which is Π/m, where m is the order of the circumferential mode.

This method has been applied to determine the load reduction in the case of external pressure increase. The stability study computed on the perfect structure shows that the top buckles on the circumferential mode 22. The meridian mode is presented on figure 10. The imperfect three dimensional mesh has been built on the buckling mode of the perfect structure, the imperfection amplitude being equal to the structure thickness. Only the cylindrical part of the vessel has been modelised, and low part effect has been taken into account by putting equivalent forces and moment at the bottom of the sector. The mesh, which modelises a $\Pi/22$ angle sector, is presented on figure 11, and defects introduced in meridian and circumferential directions are also indicated.

Stability computations are performed with code TRICO, as previously described :

- incremental elastoplastic calculation taking into account non linear comportment of geometry and material.
- bifurcation calculations at different steps of the evolution.

Critical differential pressure obtained are the following :

- on the perfect structure : $(P_{ext})_B = 800$ mbars
- on the imperfect structure : $(P_{ext})_C = 250$ mbars

For this load case, shape imperfections reduce considerably critical loads of the structure.

7. CONCLUSIONS

Results obtained in this analysis show that, for this type of structure shape imperfections can reduce considerably the critical loads. Therefore, it is important to take them into account in stability computations.

But the hypothesis made on the imperfection shape, defined on the buckling mode of the perfect structure, is not necessarily the most penalizing. Otherwise, the choice of the defect is not easy when the structure buckles on multiples modes.

Experimental tests will be achieved on this type of cylinder. Their analysis can give us some precisions on these points.

- REFERENCES -

/ 1 / A. HOFFMANN, A. COMBESCURE
Quelques considérations simples pour l'étude de l'effet des défauts de formes sur les charges critiques de flambage
Rapport DEMT/SMTS/BAMS/80-28 (Septembre 1980)

/ 2 / C. CHAVANT, A. HOFFMANN
Influence des défauts de forme. Application au cas des coques axisymétriques
Rapport DEMT/SMTS/BAMS/81-47 (1981)

/ 3 / C. CHAVANT, J. DEVOS, A. HOFFMANN
On the influence of geometric imperfections and thermal gradients on elastic-plastic buckling of shells
Paper E 5/6 - SMiRT 6 - PARIS 1981

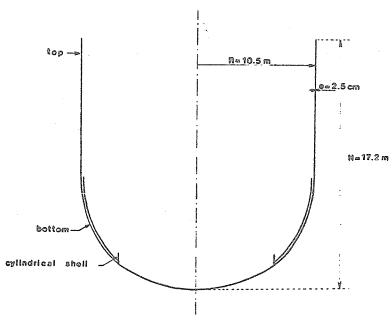

FIGURE 1 : Geometry of the structure

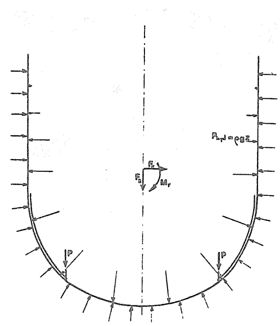

FIGURE 2 : Applied loads

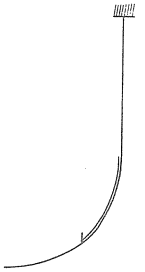

FIGURE 3 : Axisymmetric thin shell model

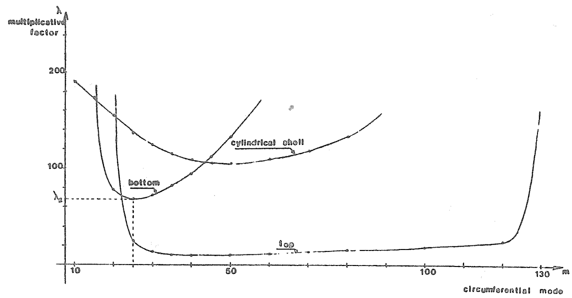

FIGURE 4 : Euler loads research

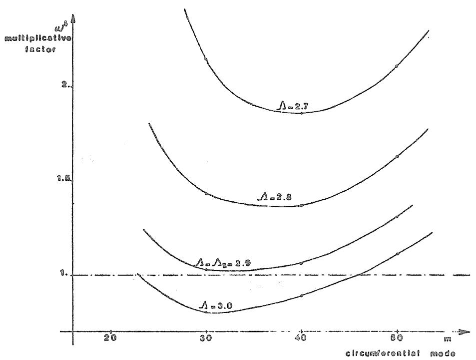

FIGURE 5 : Bifurcation load research

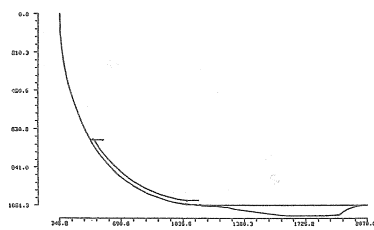

FIGURE 6 : Modal shape (buckling of the top)

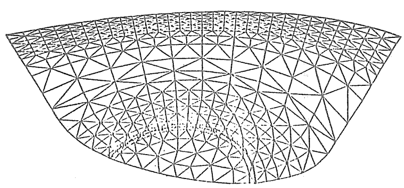

FIGURE 7 : Three dimensional modal of a half vessel

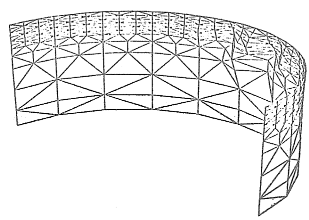

FIGURE 8 : Deformed shape of the shear buckling mode

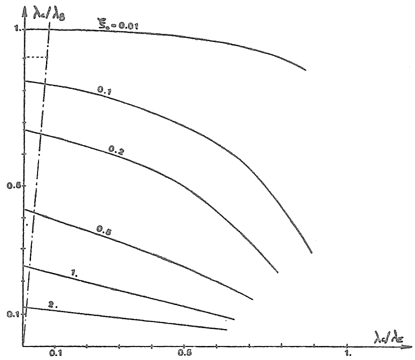

FIGURE 9 : Critical load reduction diagram

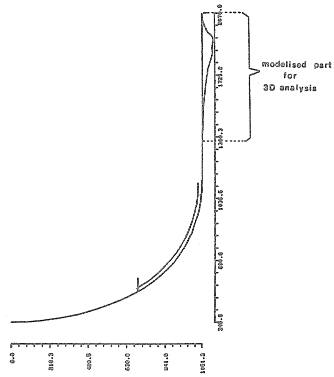

FIGURE 10 : Buckling of the perfect structure -
visualisation of the meridian mode

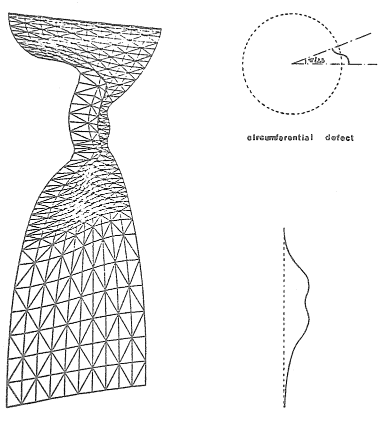

FIGURE 11 : Three dimensional imperfect model.