

## Adaptive Techniques for Diagnostics of Vibrating Structures

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### ABSTRACT

An adaptive diagnostic procedure for vibrating structures based on correspondence between current estimates of stiffness matrix and structure status is proposed. Procedure employs adaptive mathematical description of the vibrating structure in frequency domain, statistical techniques for detection and location of changes of structure properties, "recognition" and prediction of defects.

## 1. Introduction

The normal operational conditions of the most industrial piping systems, offshore drilling platforms, etc. exhibit steady state vibrations. Influence of different random factors, such as corrosion, overstress, fatigue, and crack development cause continuous structure deterioration and finally failure of its units. Vibration monitoring can properly assure the system integrity, evaluate system performance, estimate current structure status.

Estimates of vibration spectrum obtained via Fast Fourier Transform (FFT) routines contain vast information on current structure status, however, the explicit estimates of structural characteristics in the form of mass, viscous damping and stiffness matrices (M,C,K) still have to be retrieved from the spectrum. M,C,K - parameters reflect static and dynamic properties of all corresponding structural units, and hence are very sensitive to actual physical status of particular units and the entire structure. Problem of structure diagnostics which is understood as current estimation of structure status, detection, location and prediction of failures of particular structural units can be solved on the basis of current estimation of M,C,K - parameters, i.e., by identification of the vibrating structure. In the situation when system properties are time dependent an adaptive approach for identification which implies development of "base" model and periodic updating of model parameters can be recommended.

Analysis of current parameter values of the adaptive model allows detection of sufficient changes of the structure status, trend analysis and prediction of further structure deterioration. Existing correspondence between elements of M,C,K matrices and structural units (locations) allows to associate changes of certain model parameters with changes of properties of certain structural units.

When elaborate development of the "base" model is completed, the proposed procedure is to be implemented in the form of a microcomputer based diagnostic system.

## 2. Base Model

The configuration of the structure model is usually known and can be written in the form of force balance equation:

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F(t) \quad (1)$$

where M,C,K are N x N matrices

$X(t)$ ,  $\dot{X}(t)$ ,  $\ddot{X}(t)$  are displacement vector and its derivatives,

$F(t)$  - is force vector,

$t$  - is continuous time

Frequency domain approach is based on representation of this equation in frequency domain by substitution in (1)

$$X(t) \rightarrow X(j\omega), \dot{X}(t) \rightarrow j\omega X(j\omega), \ddot{X}(t) \rightarrow -\omega^2 X(j\omega), F(t) \rightarrow F(j\omega)$$

where

$$x(j\omega) = 2 \int_0^T X(t) e^{-j\omega t} dt \quad - \quad \text{FFT of time function } X(t),$$

$\omega$  - is angular velocity, related to frequency  $f$  by  $\omega = 2\pi f$

$i$  - is imaginary unit,

$T$  - is time interval sufficient for FFT

Equivalent of (1) in frequency domain can be written as:

$$\{P(\omega) + j Q(\omega)\} X(j\omega) = F(j\omega) \quad (2)$$

where

$$P(\omega) = K - \omega^2 M \quad (3)$$

$$Q(\omega) = \omega C$$

Procedure for initial estimation of the system dynamic parameters is based on steady state excitation of all system locations of interest (one at a time) and monitoring of system responses.

Equation (4) can be transformed in the form

$$\{R(\omega) + jS(\omega)\} F(j\omega) = X(j\omega) \quad (4)$$

where

$$R(\omega) + jS(\omega) = \{P(\omega) + j Q(\omega)\}^{-1} \quad (5)$$

when force of excitation is applied at  $s$ -th location only

$$F(j\omega) = \{0, \dots, 0, F_s(j\omega), 0, \dots, 0\}' \quad (6)$$

(' is symbol of transpose),

the frequency response of the system measured at all locations will be represented by column vector

$$X^S(j\omega) = \{X_{1s}(j\omega), \dots, X_{Ns}(j\omega)\}' \quad (7)$$

Thus, the elements in the  $s$ -th column of  $R(\omega)$  and  $S(\omega)$  matrices can be expressed through the  $F_s(j\omega)$  and  $X_s(j\omega)$  functions:

$$R_{is}(\omega) = \frac{a_{is}(\omega)c_s(\omega) + b_{is}(\omega)d_s(\omega)}{c_s(\omega)^2 + d_s(\omega)^2}$$

$$S_{is}(\omega) = \frac{b_{is}(\omega)c_s(\omega) - a_{is}(\omega)d_s(\omega)}{c_s(\omega)^2 + d_s(\omega)^2}$$

where:

$i, s=1, \dots, N$  - index of locations,

$c_s(\omega)$ ,  $d_s(\omega)$  and  $a_{is}(\omega)$ ,  $b_{is}(\omega)$  are FFT of excitation (6) and system response (7):

$$X_{is}(j\omega) = a_{is}(\omega) + j b_{is}(\omega)$$

$$F_s(j\omega) = c_s(\omega) + j d_s(\omega) \quad \omega = \omega_1, \omega_2, \dots, \omega_v, \dots$$

The forcing function when applied to other locations of the system will accumulate enough information to estimate all columns in the  $R(\omega)$  and  $S(\omega)$  matrices for all values of interest of angular velocity  $\omega$  which are obtained from the excitation spectrum of the structure.

Now when transfer functions matrix (5) is numerically estimated, matrix (3) which explicitly contains M,C,K parameters can be also numerically estimated for each particular frequency by the inversion of matrices (5) calculated for the same frequencies.

Inversion of a square complex matrix  $R(\omega) + jS(\omega)$

$$P(\omega) = Q(\omega) R(\omega) S^{-1}(\omega)$$

$$Q(\omega) = -\{R(\omega) S^{-1}(\omega) R(\omega) + S(\omega)\}^{-1}$$

allows to obtain elements of P(ω) and Q(ω) and M,C,K system parameters:

$$K_{is} - \omega_v^2 M_{is} = P_{is}(\omega_v), \quad (8)$$

$$\omega_v C_{is} = Q_{is}(\omega_v), \quad v=1,2,3,\dots,L \quad (9)$$

where:

- v - is a frequency index.
- L - is total number of considered frequencies,
- i,s are location indices.

System (8,9) can be solved for each particular combination of two frequencies or for the entire set of frequencies via the Least Squares technique. In the first case the total number of estimates of the same system parameter is equal to  $\binom{L}{2}$ , where L is total number of contributive frequencies in the excitation spectra. Due to the measurement and calculation errors (errors of methodology) parameters estimates in this case will obey most likely normal distribution, so confidence intervals of the system parameters can be defined (for elements of stiffness matrix) as

$$K_{is} - \delta_{is} \leq K_{is} \leq K_{is} + \delta_{is}$$

where

$\delta_{is}$  - width of confidence interval

$$\delta_{is} = T(P^0) \sigma_{is} / \binom{L}{2}^{1/2} \quad (10)$$

$K_{is}$  - estimate of is-th parameter,

$\sigma_{is}$  - standard deviation of parameter estimates,

$T(P^0)$  - T-distribution corresponding to confidence probability  $P^0$ .

### 3. Model Adaptation

As mentioned above, structure deterioration process causes continuous random drift of structure parameters, most likely elements of stiffness matrix. In this context adaptation procedure can be interpreted as current statistical estimation of the structure stiffness matrix. Adaptation procedure implies derivation of "instant" stiffness estimates from each recently received portion of statistical data, and calculation of "current" estimates on the moving average basis. "Instant" estimates derived at each particular step of adaptation are affected by random errors and obviously are random, however, the series of sequential estimates reflects the existing tendency of the drift of structure properties. This drift can be easily detected by "moving average" procedure.

Convergence and effectiveness of the adaptation algorithm heavily depends on the accuracy of the initial (base) estimates of system parameters what emphasizes the role of the identification procedure described in Section 2. Adaptation procedure can be developed in the form of relatively simple repetitive routines which allow minimization of necessary computer resources and can be implemented on microcomputer basis.

Assume  $X(t, j\omega)$  and  $F(t, j\omega)$  are current (associated with  $t$ -th time interval) estimates of the system displacement and forcing function spectra. While  $X(t, j\omega)$  can be obtained by FFT of current system displacements,  $F(t, j\omega)$  in some cases cannot be estimated directly.

Case 1 - Vibration monitoring is performed under normal operational conditions and forcing functions assumed to be stationary. In this situation "instant" estimates of system parameters are to be derived from equations written for all "meaningful" frequency components of the displacement spectra:

$$W(t-1, j\omega) X(t-1, j\omega) = W^*(t, j\omega) X(t, j\omega), \quad \omega = \omega_1, \omega_2, \dots, \omega_L \quad (11)$$

where

$$W(t-1, j\omega) = K(t-1) - \omega^2 M + j\omega C$$

$$W^*(t, j\omega) = K^*(t) - \omega^2 M + j\omega C$$

$K(t-1)$  - is "current" estimate of the stiffness matrix at the  $t-1$  -st step of adaptation,  
 $K^*(t)$  - is "instant" estimate at  $t$ -th step.

Normally stiffness matrix is not fully populated and up to  $2L$  nonzero elements can be estimated at each row based on matrix equation (11).

"Current" estimate of stiffness matrix can be calculated as

$$K(t) = K(t-1) + \gamma \left[ K^*(t) - K(t-1) \right] \quad (12)$$

where

$K^*(t)$  - "instant" estimate of stiffness matrix, derived from (11),

$\gamma$  - is parameter which controls rate of convergency and noise reduction in the above procedure, and can be estimated as  $\Delta T / T_{st}$

$\Delta T$  - is period of execution of procedure (11),  $T_{st}$  is expected interval of stationarity of system characteristics.

Case 2 - Forcing function spectra can be estimated through spectral characteristics of other variables (for example - pressures) available for monitoring. In this situation left hand side of equations (11) is to be replaced by real and imaginary components of the forcing function spectra and the same approach is to be applied for derivation of current estimates of  $K$ -matrix.

#### 4. Diagnostic Analysis

Diagnostic analysis of vibrating structure is based on assumption that stiffness matrix reflects the entire structure status which is time dependent due to continuous deterioration process. Hence, deviations of the stiffness matrix elements indicate changes of the structure properties and subscripts of deviated elements can lead to the particular structure locations (units).

If  $K(o)$  is initial and  $K(t)$  is current stiffness matrices, matrix of deviations

$$\Delta K(t) = K(t) - K(o) = \{ \Delta K_{is}(t), \quad i, s = 1, \dots, N \}$$

is to be calculated. Elements of matrix  $\Delta K(t)$  are random variables with a distribution which can be assumed to be normal, so only the "sufficient" elements are to be taken into

account,

Sufficient elements of  $\Delta K(t)$  matrix can be selected by comparing against the width of confidence intervals (10) of corresponding parameters. Matrix of sufficient deviations

$$\delta K(t) = \{ \delta K_{is}(t), i,s=1,2,\dots,N \}$$

can be defined by expression

$$\delta K_{is}(t) = \begin{cases} \Delta K_{is}(t) & \text{if } \Delta K_{is}(t) \geq \delta_{is} \\ 0, & \text{otherwise} \end{cases} \quad i,s=1,2,\dots,N.$$

Detection of deteriorated structural units is based on analysis of  $\delta K(t)$  matrix. Presence of non-zero elements in the  $\nu$ -th row of this matrix indicates properties change of structural unit associated with  $\nu$ -th location.

More accurate diagnostic analysis requires development of "diagnostic" matrices corresponding to each particular defect of structure. Diagnostic matrix

$$D^z = \{ D_{is}^z, i,s=1,2,\dots,N \}$$

can be obtained by computer simulation of the  $z$ -th defect of the structure (bend, crack, replacement of support, etc.) at certain location and derivation of corresponding deteriorated stiffness matrix based on (1.1).  $Z$ -th diagnostic matrix can be defined as a matrix of sufficient deviations between deteriorated and initial stiffness matrices. In this case diagnostic problem can be interpreted as "Recognition of pattern" of defect represented by current matrix  $K(t)$ , and can be solved by calculation of the identity criteria

$$C \{ D^z, \delta K(t) \} \quad (12)$$

for all  $z=1,2,\dots$ , and selection of those " $z$ " which maximize criteria.

Trend analysis of matrices  $K(t)$ ,  $t=1,2,\dots$  allows prediction of unacceptable changes of structural properties.

Sequential values of  $K(t)$ ,  $t=1,2,\dots$  are to be used for development of autoregression equation

$$\tilde{K}_{is}(t+1) = E \{ K_{is}(t+1) / K_{is}(t), \dots, K_{is}(1) \} \quad (13)$$

via the Least Squares method technique (here  $E \{ ./.\}$  is a symbol of conditional mathematical expectation). Equation (13) supplemented with confidence analysis can be used for calculation of future values of stiffness parameters.

Occurrence of  $z$ -th structural defect can be predicted by comparing of matrices

$$\tilde{\delta K}(t) = \tilde{K}(t) - K(0)$$

against diagnostic matrices  $D^z$ ,  $z=1,2,\dots$  through the use of criteria (12).

Finally, the following diagnostic procedure can be proposed:

1. Calculation of a current estimate of stiffness matrix  $K(t)$
2. Detection of sufficient changes of structural properties
3. Detection of current structural defect
4. Prediction of stiffness matrix  $\tilde{K}(t+\Delta t)$
5. Prediction of structural defects for time  $t+\Delta t$

## 5. Verification of Results

The above methodology was verified by computer simulation of existing nuclear power plant piping system. During the simulation steady state forces were applied and structure responses were calculated at various locations, this information was used for calculation of base, "instant" and "current" estimates of system parameters for initial and artificially deteriorated systems. Simulation had shown one-to-one correspondence between deviations of stiffness matrix parameters and structural "defects", and hence applicability of this procedure for practical cases.

As a result of this study, reliable technique for diagnostics of industrial vibrating structures has been developed. Results are accepted for implementation in EBASCO projects.

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