Calculation of Fluid (Steam) Hammer Loading to Piping Systems 
by the Response Spectrum Method

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SUMMARY

Today computations of fluid and steam hammer loading to piping systems are usually performed as a time-history analysis in which the transient pressure forces act as external excitations.

For practical purposes it is desirable to be able to treat fluid hammer loading using the response spectrum method similarly as loads from external events.

Two advantages arise from the use of spectra in the analysis of piping systems subjected to dynamic force excitations. Firstly, the response spectrum method is much less sensitive to model idealization than the time-history method. Secondly, computational efforts are reduced.

In this paper the algorithm for the treatment of force excitations through the modal response spectrum method is briefly presented. The effect of the residuum accounting for higher modes which are not part of the modal decomposition is considered. In particular various methods of superposition of the responses of the dynamic forces and of the modes are investigated. Results and comparisons are presented of several response spectrum analyses and time-history analyses.
1. INTRODUCTION

Fluid or steam filled piping systems are dynamically excited by high pressure pulses running through the entire system due to sudden closure of stop valves or due to pipe rupture. It is common practice to consider deformations and stresses of these load cases in order to comply with the design code requirements. Design calculations are usually carried out by time-history methods, i.e. by direct integration methods or by the modal time-history method, in which the pressure forces are applied as external loads.

For conservative structural design the maxima of deformations and stresses are of major interest rather than the time histories themselves. This favors the application of the response spectrum method which has been used successfully for several years in piping analysis of vibrations induced by external events.

In contrast to these load cases in which the piping system is subjected to support excitations the fluid hammer loading or pipe rupture cause excitations by forces. The algorithm of the response spectrum method in case of support excitation is well known /1/ and can be adapted for dealing with force excitation.

The response spectrum method in practical analysis proves some advantages. Using a few enveloped spectra instead of many detailed time histories can decrease the amount of design calculations. The possibility of smoothing and broadening leads to less sensitivity caused by slight changes of model boundary conditions (e.g. support stiffness).

Treating force excitation with the response spectrum method requires superposition rules for the modes and for the different response spectra. The simplest rules are absolute summation or root mean square. The modes can also be combined by complete quadratic combination. The combinations of the spectra can be carried out in groups. Within each group the spectra are summed absolutely; the individual groups are combined by root mean square.

In this paper basic equations of modal analysis are briefly cited. Special emphasis is put on the residuum of the "higher" modes with are not part of the modal decomposition and therefore are unknown. Various superposition methods are proposed. Finally, the results of piping calculations carried out with time-history analysis and the response spectrum method are presented and compared.

2. THEORETICAL BACKGROUND

The equations of motion of multi-degree-of freedom systems are written in matrix notation as

\[ \mathbf{m} \ddot{\mathbf{x}} + \mathbf{c} \dot{\mathbf{x}} + \mathbf{k} \mathbf{x} = \mathbf{p}(t) \]  

(2.1)

where the matrices \( \mathbf{m} \), \( \mathbf{c} \) and \( \mathbf{k} \) represent system mass, damping and stiffness. The vectors \( \ddot{\mathbf{x}} \), \( \dot{\mathbf{x}} \) and \( \mathbf{x} \) are nodal acceleration, velocity and displacement. The forcing term on the right hand side of eq. (2.1) is expressed by

\[ \mathbf{p}(t) = \mathbf{e} (\dot{\mathbf{p}}(t)) \]  

(2.2)

In eq. (2.2) the \( N \times K \) excitation matrix is denoted by \( \mathbf{e} \) and the vector \( \dot{\mathbf{p}}(t) \) contains all the force time-histories which are different. The number \( N \) is equivalent to the number of degrees of freedom and the value of \( K \) corres-
ponds to the number of applied forces.

Modal analysis treats the matrix equations (2.1) as \(N\) uncoupled single-degree-of-freedom-systems by the aid of the eigenvectors \(\{\phi\}_n\) which are computed for the undamped free vibrations corresponding to eq. (2.1). This leads to

\[
\ddot{y}_n + \frac{c}{m_n} \dot{y}_n + \frac{\omega_n^2}{m_n} y_n = \frac{1}{m_n} \{\phi\}_n^T \{e\}_j \gamma_{nj} \{\dot{y}\}_j(t) \tag{2.3}
\]

where \(y_n\) is the normal coordinate and \(m_n\) represents generalized mass. We define the modal participation factor as

\[
\gamma_{nj} = \frac{1}{m_n} \{\phi\}_n^T \{e\}_j
\]

(2.4)

where the excitation vector \(\{e\}_j\) is the \(j\)th column of the excitation matrix \(e\) corresponding to the \(j\)th force time history \(\dot{p}_j(t)\).

The solution of eq. (2.3) is evaluated by the Duhamel integral or the dynamic load factor (DLF); thus

\[
y_n(t) = \frac{1}{\omega_n^2} \ddot{y}_n(t) = \frac{1}{\omega_n^2} \sum_{j=1}^{K} \gamma_{nj} DLF_{nj}(t) \tag{2.5}
\]

in which

\[
DLF_{nj}(t) = \frac{1}{\omega_n^2} \sum_{j=1}^{K} \gamma_{nj} \phi_{in} DLF_{nj}(t) \tag{2.6}
\]

The product of the modal matrix \(\phi\) and the vector of the normal coordinates \(\{y\}\) express the total response of eq. (2.1) in the form

\[
\{x\} = \phi \{y\} \tag{2.7}
\]

The \(i\)-th degree-of-freedom in eq. (2.7) is given by

\[
x_i(t) = \sum_{n=1}^{L} \gamma_{nj} \phi_{in} DLF_{nj}(t) + \sum_{n=L+1}^{N} \gamma_{nj} \phi_{in} DLF_{nj}(t) / \omega_n^2 \tag{2.8}
\]

or

\[
x_i(t) = \sum_{n=1}^{L} \phi_{in} \ddot{y}_n(t) / \omega_n^2 + \sum_{n=L+1}^{N} \phi_{in} \ddot{y}_n(t) / \omega_n^2 = x_i(t) + \Delta x_i(t) \tag{2.9}
\]

The underlined terms in eqs. (2.8) and (2.9) are unknown because the eigenvalue problem is normally not solved completely, i.e. instead of \(N\) eigenvectors only \(L\) (with \(1 \leq L \leq N\)) eigenvectors are computed. Nevertheless the contribution of the "higher" modes with eigenfrequencies \(\omega_n, n \geq L+1\) to the total response can be estimated. For this purpose we make use of the matrix equation

\[
\{\dot{y}\}_j = \phi^T (\{e\}_j \gamma_{nj} \{\dot{y}\}_j(t) \tag{2.10}
\]

which is the generalized form of eq. (2.4). Substituting in eq. (2.10) the modal matrix \(\phi\) by the expression \(\phi^T \phi^{-1}\) yields

\[
\phi^T (\{e\}_j \gamma_{nj} \{\dot{y}\}_j(t) \tag{2.11}
\]

The contribution of the higher modes can be evaluated if we change from displacements \(\{x\}\) to accelerations \(\{\dot{x}\}\). Thus response accelerations resulting from eq. (2.8) are

\[
\ddot{x}_i(t) = \sum_{n=1}^{L} \gamma_{nj} \phi_{in} DLF_{nj}(t) + \sum_{n=L+1}^{N} \gamma_{nj} \phi_{in} DLF_{nj}(t) \tag{2.12}
\]

The sequence of summation of the second term in eq. (2.12) may be altered only if the dynamic load factor \(DLF_{nj}\) remains constant for modes beyond the \(L\)-th mode.
In this case the products $\gamma_{nj}$ $\phi_{in}$ may be replaced by eq. (2.11) so that

$$\ddot{x}_i(t) = \sum_{n=1}^{L} \phi_{in} \ddot{v}(t) + \sum_{j=1}^{K} DL_{ij} \left( \ddot{e}_{ij} - \sum_{n=1}^{L} \gamma_{nj} \dot{\phi}_{in} \right) \ddot{x}_j(t) + \Delta \ddot{x}_{i,L+1}(t) \quad (2.13)$$

The second term on the right hand side of (2.13) yields a "pseudo"-mode which includes the contribution of all "higher" unknown modes and can be evaluated using the computed modes. In order to get displacements and stresses of this "pseudo"-mode the residual accelerations $\Delta \ddot{x}_{i,L+1}(t)$ have to be multiplied with the modal masses $m_i$ or inertias $\Theta_i$, respectively. This product results in an equivalent static load from which residual displacements and stresses are obtained.

3. RESPONSE SPECTRUM METHOD

As indicated earlier only the maxima of the responses are of interest. Therefore the first step of the analysis generates response spectra for different frequencies and damping values by evaluating eq. (2.6). This procedure is carried out for each external load time history $\ddot{v}_j(t)$ resulting in $K$ DLF-spectra.

According to eq. (2.8) the displacement $x_i$ is given by a double sum covering the mode shapes and the contributions from the different external forces. The sum over the spectra is independent from the summation of the modes.

Several methods for superposition of the modes are known as such:

- **absolut summation**
  \[ \bar{x}_i = \sum_{n=1}^{L} \phi_{in} \ddot{v}_n / \omega_n^2 + |\Delta \ddot{x}_{i,L+1}| \quad (3.1) \]

- **SRSS-method**
  \[ \bar{x}_i = \sum_{n=1}^{L} (\phi_{in} \ddot{v}_n / \omega_n^2)^{1/2} + (|\Delta \ddot{x}_{i,L+1}|)^{1/2} \quad (3.2) \]

- **CQC-method**
  \[ \bar{x}_i = \left[ \sum_{n=1}^{L} \phi_{in} \ddot{v}_n \phi_{im} \ddot{v}_m + (|\Delta \ddot{x}_{i,L+1}|)^{2} \right]^{1/2} \quad (3.3) \]

The absolut summation of the modal responses yields a very conservative upper bound. The SRSS-method guarantees the most probable maximum response in case of statistic independence but the results may be lower than the time-history responses. The CQC-method (complete quadratic combination) is a generalization of the SRSS-method. The cross-correlation factors $\rho_{nm}$ are determined from probability calculations based on stationary excitation. In Ref. /2/ the coefficients $\rho_{nm}$ are

\[ \rho_{nm} = \frac{(\zeta_n + \zeta_m)^2}{(\zeta_n + \zeta_m)^2 + (1 - r)^2} \quad (3.4) \]

where $\zeta_n$, $\zeta_m$ is modal damping and $r = \omega_m / \omega_n$.

Based on Ref. /3/, Ref. /4/ states for constant damping the formula

\[ \rho_{nm} = \frac{8\pi^2 (1 + r) \chi_{nm}^2}{(1 - r^2)^2 + 8\pi^2 \chi_{nm}} \quad (3.5) \]

The choice of the correlation factors (3.4) or (3.5) is not significant because both the expressions result in nearly the same $\rho_{nm}$ values. Typically the CQC-method gives responses in the range between those of the SRSS-method and the absolut summation. Its validity for deterministic excitations such as fluid (steam) hammer loading is still subject to debate.

The superposition of DLF-spectra in order to get the modal responses $Y_n$,
eq. (2.5) is considered next. It may be carried out according to eq. (3.1) or
(3.2) by absolute summation or by the SRRS-method, respectively.

These simple superposition rules can be improved by grouping the DLF-spectra.
This grouping should be adapted to the loading pattern and piping geometry at
hand. First, the DLF-spectra are divided into two groups according to
\[ \bar{Y}_n = \sum_{j=1}^{K_1} |Y_{nj}^{DLF}| + \left( \sum_{j=1}^{K_1} Y_{nj}^{DLF} \right)^{1/2} \]
where \( K = K_1 + K_2 \).

Further improvement is obtained by assigning the spectra to more than two
groups. In this case two or more superposition rules are possible, either
\[ \bar{Y}_n = \sum_{j=1}^{K_1} |Y_{nj}^{DLF}| + \left( \sum_{j=1}^{K_2} Y_{nj}^{DLF} \right)^{1/2} + \left( \sum_{j=1}^{K_3} Y_{nj}^{DLF} \right)^{1/2} + \ldots \]
\[ \bar{Y}_n = \left( \sum_{j=1}^{K_1} Y_{nj}^{DLF} \right)^{1/2} + \ldots + \left( \sum_{j=1}^{K_N} Y_{nj}^{DLF} \right)^{1/2} \]

4. PRACTICAL APPLICATION

The first example problem serves to qualify the various algorithm and super-
position rules. Fig. 1 displays the piping system and the external applied for-
ces which have been chosen arbitrarily.

Time-history calculations were carried out by direct integration with a time
step of t=0.001 [s]. The damping of critical during the direct integration varied
within the range of 3% to 5% for the first ten modes (14.7 Hz). The constant
damping in the response spectrum method was chosen 3%. Further test cal-
culations with 5% damping yielded results only from 10 to 15 percent lower than
those corresponding to 3% damping.

Tables 1 and 2 show results of displacements and support reactions for se-
lected nodes. The comparison of the time-history calculation with the response
spectrum method reveals the absolute summation of both the modes and the DLF-
spectra being too conservative. The SRRS-method applied to modes and spectra
can result in displacements and reactions which are lower than the THDI-values.
The combination of SRRS-superposition of the modes and absolute summation of
the spectra approaches the time-history results in a conservative sense.

The results of the response spectrum method imply 25 modes. The highest
eigenfrequency of 70.7 Hz is found to be in the rigid body range of all DLF-
spectra. For practical applications it is sufficient to compute only a limited
number of eigenfrequencies and modes lower than the frequency corresponding to
the residual cut-off point in this case the consideration of 12 modes up to
25.2 Hz is adequate. The contribution of modes with higher frequencies is ne-
gligible for this example problem, as it is evident from the residual load.

The resulting bending moments versus time for two selected nodes are drawn
in Fig. 2. The results at node 17 are representative for the complete system,
i.e. the SRRS-ABS-combination is slightly conservative with respect to the
time-history.
The second example refers to calculations of a part of the primary steam line of NPP Krümmel displayed in Fig. 3. It is a primary steam system of about 600 mm diameter starting from RPV at node 541, ruptured at node 351 outside the containment. Three relief systems and one aftercooling system of about 250 mm diameter are connected with the main line inside the containment. In this case the dynamic excitation is caused at first by pipe rupture (node 315) and - after a short time interval - by closing the isolation valve (Fig. 5). The pipe rupture force acts like an impulsive load and dominates the dynamic excitation of the main line. This can be concluded from the shape of the corresponding dynamic load factor. In contrast, the steam hammer loading of the isolation valve governs the excitation of piping systems attached to the main line. A characteristic shape of a force time history with dynamic load factor is shown in Fig. 4. Altogether the number of different steam hammer loads amounts to 38.

Table 3 contains some of the support reactions evaluated by direct integration and by the response spectrum method. A total of 39 modes up to 25.6 Hz have been superimposed. The frequency of 25.6 Hz is lower than the residual cut-off frequency of some of the spectra. The contributions of the unknown "higher" modes are however, either negligible or are well represented by the residual load. A look at the vertical reaction force Nz of node 551 confirms that the residual load adds the missing quantity of the higher modes to the computed 39 modes. Some bending moments for selected elements of the piping system are shown in Table 4. The results of the response spectrum method approach those of the time history calculations fairly well. It is observed again - see My of element 45 of My of element 230 - that the residual load contributes to the total response exactly where it is needed.

5. REFERENCES


FIG. 1: Piping System of Example Problem and External Forces

TABLE 1: Displacements of Example Problem; $u \text{ [mm]}$

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TABLE 2: Support Reactions of Example Problem; $[N]; [Nm]$

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FIG. 2: Comparison of Bending Moments

FIG. 3: Primary Steam Line of Nuclear Power Plant Kruemmel
FIG. 4: Load Time History and Dynamic Load Factor at Node 239

FIG. 5: Load Time History and Dynamic Load Factor at Node 315

FIG. 6: Relationship between Damping and Frequency for Primary Steam Line
### TABLE 3: Support Reactions

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### TABLE 4: Bending Moments

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