

Modal Spectrum Analysis of Piping Systems Under Water-Hammer Loading: Spectra Examination

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Summary

In the last few years the dynamic calculation with spectra of piping systems under fluid-hammer has been developed. In comparison with the time-history solution method the spectra method has important advantages because it can calculate a bounded solution. In this bounded solution, the inevitable uncertainties of the time-dependent forces and the uncertainties in the modeling of the piping system are taken into account. The spectra also give valuable information about the frequency content of the time-dependent forces, which is important too for correct time-step selection when using the time-history-method.

Using the spectra method, the dynamic calculation is divided into stages. First and most essential is the calculation of the spectra. Secondly, a form of superposition is used for combining the results from each eigenmode analysis. For the last stage, reference /3/ provides examples of the most effective superposition forms of the many currently available. In this paper the first stage, calculation of the spectra due to fluid hammer loading, will be examined. An approximate method for load calculation is shown, whereby the results from a change of fluid-dynamic parameters can be quickly determined without making a full numerical analysis.

When changes are made in fluiddynamic parameters, the normal result is a change of shift in the frequency content of the spectra. However, for changes in certain parameters, only the force amplitudes are changed. Both types of changes will be discussed.

1. Introduction

The dynamic calculation with spectra of piping systems under fluid-hammer loading has, in comparison with the time-history solution, important advantages. It is a bounded solution, which takes into account the inevitable uncertainties of the fluid-forces and the piping system model. One further important advantage is, that knowing the eigenfrequencies of the system, the spectra quickly give the information as to which eigenmode has a relevant contribution to the resulting deformations, forces and stresses. Therefore, one can easily estimate the influence of a shift of the eigenfrequencies caused by changes of the layout of piping elements or supports without going through a new dynamic calculation of the system.

The spectra are the maximum dynamical responses of a single-degree-of-freedom (SDOF) system due to the fluid-hammer forces; hence, they are governed by the fluid-dynamic parameters. In this paper, the influence of the fluid-dynamic parameters upon the spectra will be examined.

2. Basic dynamic equations of a fluid-hammer calculation using spectra

The differential equations governing the dynamic behaviour of piping systems, can be split by modal analysis into n differential equations of the SDOF-System:

$$\ddot{q}_m(t) + 2D_m \omega_m \dot{q}_m(t) + \omega_m^2 q_m(t) = \sum_{j=1}^{n_K} F_j f_j(t) \phi_{mj} / M_j \quad (1)$$

The general solution ($\dot{q}_m(0) = q_m(0) = 0$) is /1/, /2/:

$$q_m(t) = \sum_{j=1}^{n_K} F_j DL_j(t) \phi_{mj} / (M_j \omega_j^2) \quad (2)$$

$$\text{with } DL_j(t) = \frac{v_m}{1-D_m^2} \int_0^t [f_j(\tau) e^{-D_m \omega_m(t-\tau)} \sin v_m(t-\tau)] d\tau; v_m^2 = \omega_m^2(1-D_m^2) \quad (3)$$

The final solution is obtained by the superposition of the results from all eigenmodes. In contrast to the time-history-solution (2), the spectra method provides the maximum value of $DL_j(t)$, taken in a band around the eigenfrequency f_j (f.e.: $0.9 f_j - 1.1 f_j$). That is,

$$DLF_j = \text{MAX}[DL_j(t)] \text{ for all } t \quad (4)$$

The spectra results for every eigenmode can be superimposed quadratically ($\mu=2$) or absolutely ($\mu=1$)

$$\begin{aligned} [q_m]_{\text{Max}}^\mu &= \left(\sum_{j=1}^{n_K} \phi_{mj}^2 F_j^2 DLF_j^2 / (M_j \omega_j^2) \right)^{\mu/2} \\ [x_i]_{\text{Max}}^\mu &= \sum_{m=1}^{n_R} [q_m]_{\text{Max}}^\mu \phi_{mi}^{\mu+R/\mu} \end{aligned} \quad (5)$$

All spectra results for frequencies above those of the rigid body frequency will be taken into account by a residual mode R. For details see /3/.

It clearly can be seen that, by using the spectra method, the calculation is separated into stages:

1.) The fluid-dynamic calculation of forces $\hat{F}_j f_j(t)$ and their spectra (The spectra (DLF) can be broadend and smoothed in order to get the bounded solution).

2.) The real structure analysis given by the rule of superposition.

As a consequence of this partition, the dynamic behavior of the piping system already can be evaluated by analysis of the spectra.

3. Fluid-dynamic calculation of the fluid-hammer forces

In general one has to integrate numerically the partial differential equations of fluid-dynamics. However, for a simple run piping system with a linear valve closing rate an analytical approximation /4/ of the fluid-hammer forces is possible.

The fluid-hammer force \hat{F}_j between two elbows of the piping system (pipe-length l_j , inner diameter d , velocity of sound a , fluid-density ρ , change of fluid velocity Δc) for linear valve closing in T_0 seconds (Joukowsky) is

$$\hat{F}_j = \rho \Delta c \pi \frac{d^2}{4} \begin{cases} l_j / T_0 & l_j < a T_0 \\ a & l_j \geq a T_0 \end{cases} \quad (6)$$

Each force arrives later than the previous force because of the pressure-wave transit time

$$t_j = l_j / a. \quad (7)$$

With formular (6) and (7) the time-dependent fluid-hammer forces can be constructed, as Fig. 1 shows.

The influence of fluid-dynamic-parameter changes upon forces and spectra can easily be determined by this approximation. All results are transferable to more complicated systems.

4. Influence of a change of fluid-dynamic parameters upon fluid-hammer forces and spectra

The DLF_j-spectra are normalized by the maximum force amplitudes \hat{F}_j . Therefore, in the following section, each fluid parameter change will be evaluated for its effect on both force amplitudes and spectra.

4.1 Influence of a change in piping length

The fluiddynamic calculation of the fluid-hammer forces is based on the theory of stream lines. Therefore, only the length of the piping elements is considered. The layout of the piping system, supports etc. do not appear. However, equation (6) shows that, when a change in length is made, amplitudes are affected only when $l_j < a T_0$. On the other hand

every change in length has an important influence on the spectra. With decreasing length the time-dependent force curves have steeper slopes which causes a compression of wave peaks and valleys (Fig. 2). Therefore, the spectra frequencies increase as the length of the piping elements decrease (Fig. 3).

4.2 Influence of either a pressure change or a temperature change with water

The elasticity of water is given by the isothermic bulk modulus $\lambda = 1/E_{Fl}$. This modulus is nearly independent of pressure and temperature. The pressure effect of the density ρ is also relatively small. Both quantities affect the velocity of sound $a^2 = E_{Fl}/\rho$. However, for elastic pipes the velocity of sound is smaller because of its dependence on piping diameter d and wall thickness s :

$$a_{El}^2 = a^2 / [1 + dE_{Fl} / (sE_{Pipe})] \quad (8)$$

Therefore, for the medium water the increase of mass flow $m = \rho \Delta c \pi d^2 / 4$ and velocity of sound with increasing pressure is very small. Logically, a pressure change has little influence on fluid-hammer force amplitudes and spectra. But the spectra frequencies tend to increase as the pressure increases. For the temperature change the change of the velocity of sound is more significant. With increasing temperature the amplitudes of the forces become larger due to the increase of the velocity of sound. Again, the density change is very small. The first spectra maxima are shifted to higher frequencies because the influence of an increase of the velocity of sound is similar to that caused by a decrease in piping element length.

4.3 Influence of either a pressure change or a temperature change with gases

For perfect gases the velocity of sound a is only a function of the temperature ϑ (isentropic exponent κ)

$$a = a_0 (\vartheta / \vartheta_0)^{1/2} \quad a_0 = c_p (\kappa - 1) \vartheta_0^{1/2} \quad (9)$$

The density ρ changes according to

$$\rho = \rho_0 (\vartheta / \vartheta_0)^{1/(1-\kappa)} \quad (\kappa \approx 1,3)$$

With increasing temperature the decrease of fluid density ρ is larger than the increase of the velocity of sound. Therefore, the force amplitudes are reduced. For the spectra the effect of a temperature change is the same as with watermedia.

An isothermic pressure increase leads, according to the equation of state, to a larger density. The force amplitudes are also larger, but the spectra are unchanged (because of (9)). The same tendency is valid for saturated steam or superheated steam.

Due to the lower velocity of sound of gases, the spectra frequencies are lower with gases than those with water.

4.4 Influence of either a medium velocity change or a change of the piping diameter

For gases the force amplitudes are proportional to the change of the medium velocity Δc or the piping diameter. The spectra are not affected by a change of these two parameters.

However, for water the velocity of sound a_{E1} decreases with increasing d/s (see equation (8)). Therefore, the spectra maxima are shifted to lower frequencies. Also, the force amplitudes decrease with larger d/s .

4.5 Influence of a change of the valve closing rate T_0

With linear valve closing rate T_0 two cases have to be distinguished

- $l_j < aT_0$: the force amplitudes are inversely proportional to T_0
- $l_j > aT_0$: the force amplitudes are not affected by a change of T_0

However, a change of T_0 always leads to significant spectra changes because of the plateau change ($T_0 - t_j$) of the force-curves. Fig. 4 shows this effect very clearly, Fig. 5 shows how the spectra are affected by different T_0 values.

4.6 Spectra either for valve closing or pipe rupture

In case of valve closing the force-curves decay after a few seconds, assumed the fluiddynamic damping is not zero. In comparison to these curves the forces for pipe rupture are pulse-like. In the lower frequency range their spectra can be represented by $1 - \exp(f/f_0)$. The rigid body frequency (DLF=1) can be very high, especially in the case of a simple piping run. For a simple run, the rigid body frequency can take values larger than 100 Hz, and the spectra maxima have values around DLF=2. The pipe rupture spectra do not show the small range maxima that are typically seen for valve closing spectra, (Fig. 6).

Reference

- /1/ R.W.Clough, J. Penzien: Dynamics of structures, Mc. Graw Hill, Inc. New York 1975
- /2/ K. Gordis: Mathematical Basis of a Dynamic Structural Analysis Method for Piping Systems, ASME Joint Conference Proceeding, Denver, Colorado, June 1981
- /3/ G. Meder, K. Rick, Spectra Methods for Analysis of Piping Systems under Fluid-Hammer Loadings, 3R-International, 22. Jahrgang, Heft 9, Sept. 1983
- /4/ G.R. Küpers: Force on pipes due to unsteady fluid flow, 3R-International, 18. Jahrgang, Heft 1, Jan. 1979

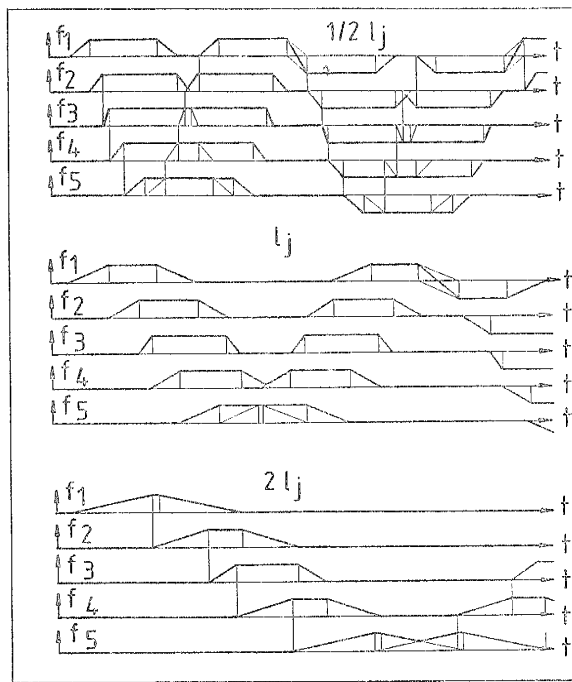


Fig. 2 Forces for different piping element length ($T_o = 50$ ms)

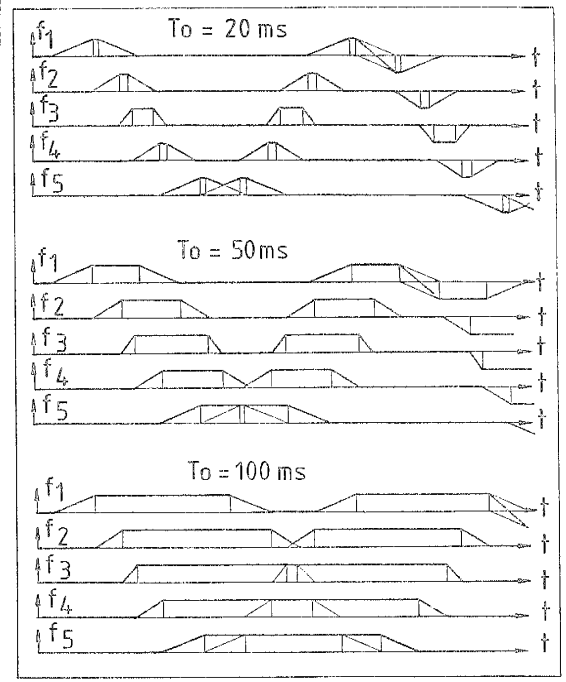


Fig. 4 Forces for different valve closing rates T_o ($l_j = \text{const.}$)

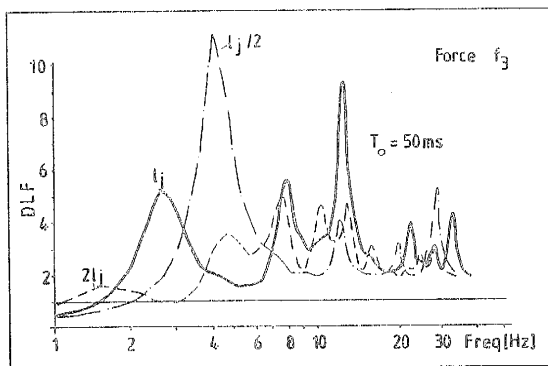


Fig. 3 Spectra of the forces Fig 2

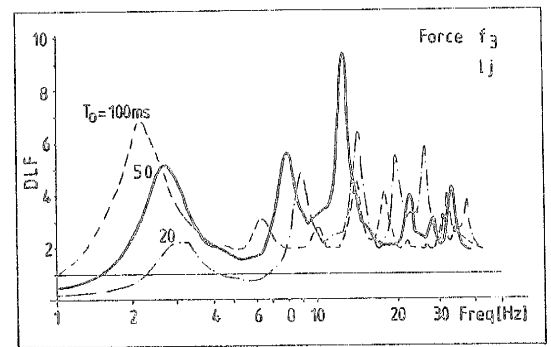


Fig. 5 Spectra of the forces Fig 4

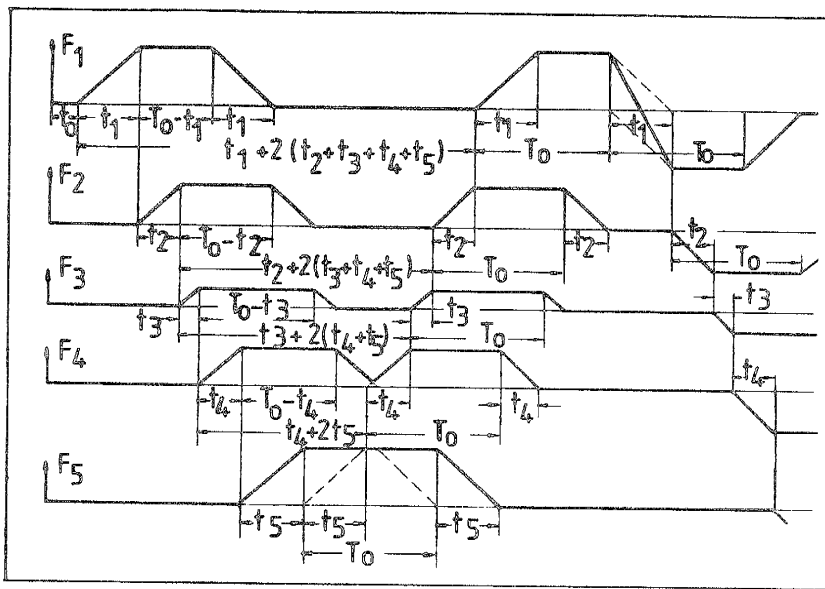


Fig. 1 Construction of fluid-dynamic forces due to fluid-hammer loading (single run piping; linear valve closing rate)

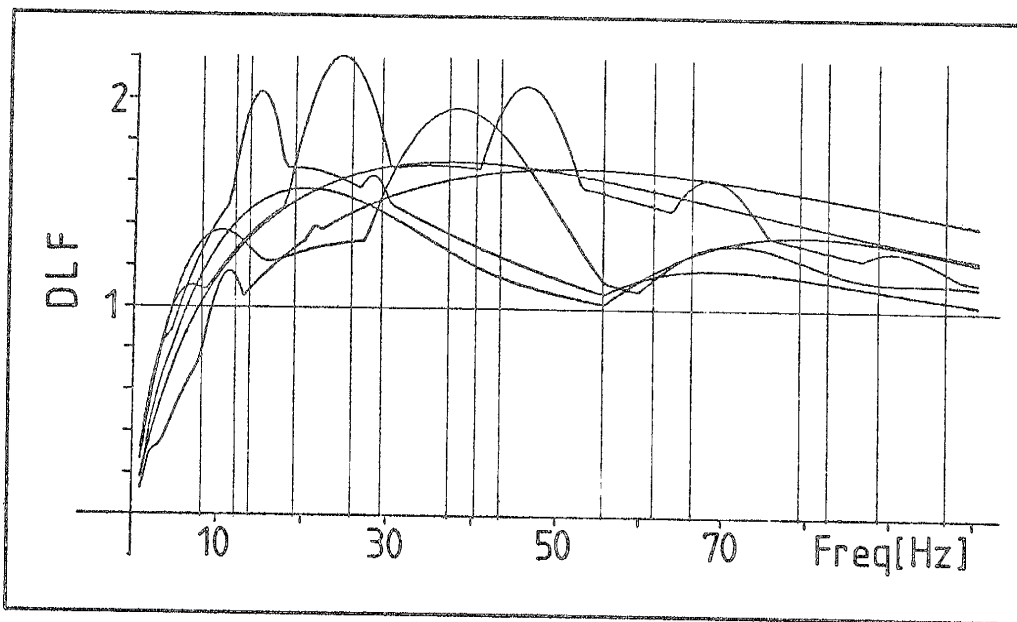


Fig. 6, Typical spectra of single run piping forces due to pipe rupture