Comparison of Analytical Response of Piping Systems Supported by Various Hydraulic Restraint Devices

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ABSTRACT

Restraint devices, commonly known as snubbers, are installed in piping systems to limit stresses due to potentially damaging transients. A compressible fluid snubber model is developed in this study based on the inherent velocity sensitive and hydraulic characteristics of one class of these restraint devices.

For piping systems with these restraint devices installed, the steady-state response of the system can be obtained by employing a transfer function concept. Utilizing this method, the Laplace transform with respect to time is employed in order to obtain an interaction equation which combines the piping system's equations of motion with the snubber's fluid pressure equations. With this interaction equation, the model for the piping system supported by a compressible fluid hydraulic snubber is developed and piping system response is predicted for various excitation conditions.

For these piping systems, time history analysis is performed in a piecewise linear fashion. This enables the analysis to characterize the snubber in each of the two conditions (locked or unlocked) that occur as a result of forced motion. The value of fluid pressure in the hydraulic snubber is used as a test criterion in the solution algorithm of the compressible fluid model to determine the locked/unlocked condition of the snubber.

The results of the analysis performed in this study with a compressible fluid hydraulic snubber model are compared to results obtained previously with an incompressible fluid model. For the velocity sensitive hydraulic snubber, the incompressible fluid model was computationally simpler (second order differential equations) than the compressible fluid model (third order equations). Yet with this simpler model, results were obtained that were conservative (compared to results obtained with the more complex compressible fluid model) for parameters in the range of interest for seismic events.
1. INTRODUCTION

In previous studies, [1,2], nonlinear models have been developed for velocity sensitive piping system restraint devices, snubbers. In particular, the model presented in Reference [2] pertains to the poppet and orifice class of hydraulic snubbers. This model has been shown to agree well with experimental data in the form of force vs. displacement curves [2] and also with experimentally determined snubber equivalent stiffness [3]. The model developed in [2] accounts for the affect of the compressibility of the hydraulic fluid in the snubber. The model developed in Reference [1] assumes the snubber fluid is incompressible. This study examines the affect that snubber fluid compressibility has on piping system response. Additionally, the affect that variations in snubber fluid bulk modulus have on piping system response will also be examined.

2. COMPRESSIBLE FLUID MODEL DEVELOPMENT

In order to perform these evaluations, the Laplace transform with respect to time is employed in order to obtain an interaction equation which combines the piping system's equation of motion with the snubber's fluid pressure equations as developed in [2]. For the purposes of this study, the piping system is visualized as a straight run of pipe anchored at both ends. Using a separation of variables technique, this distributed parameter system can be reduced to a single degree of freedom system (SDOF) as shown in Reference [1].

For a SDOF system subjected to harmonic base excitation, \( u \), the equation of motion can be written in terms of the relative displacement \( z \), velocity, and acceleration between the base and the mass, \( m \), as

\[
\dddot{z}(t) + f(\dot{z}(t)) + kz(t) = -m\ddot{u} \tag{1a}
\]

where \( f(\dot{z}(t)) \) represents an undetermined function of velocity which exerts a force on the mass. This function can also be written in terms of pressure as

\[
f(\dot{z}(t)) = g(A \cdot P(t))
\]

where

\[
A = \text{cross sectional area of snubber piston}
\]

\[
P(t) = \text{an undetermined fluid pressure function}
\]
Thus Eq. 1a can be expressed as

\[ \ddot{z}(t) + \frac{(A/m)P(t)}{\tau} + \omega_n^2 z(t) = -\ddot{u}(t) \]  

(1b)

Utilizing the model developed in Reference [2], a relation between pressure and relative velocity can be obtained as

\[ \frac{dP(t)}{dt} + \frac{1}{\tau} P(t) = \frac{\beta}{L} \frac{dz(t)}{dt} \]

(2)

where

\[ \beta = \text{fluid bulk modulus} \quad R = \text{fluid flow resistance} \]

\[ L = \text{snubber stroke length} \quad V_0 = \text{original fluid volume snubber} \]

\[ \tau = RV_0/\beta \]

In general, the response of the SDOF system can be expressed in terms of the steady-state and transient response of the system. The steady-state response of the system can be obtained by employing a transfer function concept as presented by Harrison and Bollinger [4]. Utilizing this method, the Laplace transform with respect to time is obtained for the hydraulic snubber interacting equations, Eqs. 1 and 2. The transforms of Eqs. 1 and 2 are combined algebraically to obtain

\[ [s^3 + a_2 s^2 + a_1 s + a_0]z(s) = s^2(s + b_2)u(s) \]

(3)

where

\[ a_0 = \omega_n^2/\tau \quad a_2 = b_2 = 1/\tau \]

\[ a_1 = \frac{\omega_n^2 [A/(mL) + 1]}{\tau} \quad \omega_n = \text{natural frequency} \]

The left hand side of Eq. 3 can be regarded as an output function, 0(s), and the right hand side as an input function, 1(s). The transfer function relation can be written as

\[ T(s) = \frac{0(s)}{1(s)} = \frac{z(s)}{u(s)} = -\frac{s^2(s + b_2)}{s^3 + a_2 s^2 + a_1 s + a_0} \]

(4)
The steady-state frequency response can be obtained by letting \( s = j \omega_f \) in Eqs. 4. Thus a magnitude ratio, \( MR \), is obtained as

\[
MR = \frac{Z(\omega_f)}{u} = \omega_f^2 \sqrt{\frac{\omega_f^2 + \omega_f^2}{(a_0 - a_2 \omega_f^2)^2 + \omega_f^2(a_1 - \omega_f^2)^2}}
\]

where

\[
\omega_f = \text{forcing frequency} \quad j = \sqrt{-1}
\]

In general, the steady-state response can be written in terms of the magnitude ratio obtained from the transfer function as

\[
Z_{ss}(t) = MR \cdot \sin(\omega_f t + \phi_2)
\]

Consequently, for harmonic excitation, \( \ddot{u} \), the steady state response can be written as,

\[
Z_{ss}(t) = Z_1 \sin(\omega_f t - \phi_2) + Z_2 \cos(\omega_f t - \phi_2)
\]

where

\[
Z_1 = A_1 p^{MR} \quad Z_2 = A_2 p^{MR}
\]

\[
\phi_2 = \arctan\left(\frac{\omega_f(a_1 - \omega_f) / (a_0 - a_2 \omega_f)}{1 - \omega_f^2}ight) - \arctan\left(\frac{\omega_f}{b_2}\right)
\]

\[
\ddot{u}(t) = A_1 p \sin(\omega_f t) + A_2 p \cos(\omega_f t)
\]

\[
A_1 p = \text{magnitude of Sin term in forcing function} \\
A_2 p = \text{magnitude of Cos term in forcing function}
\]

The system transient response can be evaluated from Eq. 7, which is the characteristic equation obtained from Eq. 3.

\[
s^3 + a_2 s^2 + a_1 s + a_0 = 0
\]

Upon solving for the roots of Eq. 7 using the appropriate initial conditions, the transient response is obtained. Combining the transient response with the steady state response of Eq. 6, the total system response can be written as

\[
Z(t) = Z_{ss}(t) + Z_{trans}(t)
\]
After the system response, \( z(t) \), is obtained from Eq. 8, the fluid pressure in the snubber can be determined by solving Eq. 1b for pressure.

\[
P(t) = (m/a) [\ddot{u}(t) + \ddot{z}(t) + \omega_n^2 z(t)]
\]

Substituting for \( \ddot{u}(t) \) as given in Eq. 6, pressure is obtained as

\[
P(t) = - (k/A) [\ddot{z}(t)/\omega_n^2 + z(t) - (\omega_p^2/\omega_n^2)(A_{1p}\sin(\omega_p t) + A_{2p}\cos(\omega_p t))] \quad \text{(9)}
\]

where \( k = m \omega_n^2 \) = stiffness of the SDOF system

The restraining force that the snubber applies to the system is obtained directly from Eq. 9.

System operation is modeled using a locking and unlocking computational logic. Snubber fluid pressure, as determined from Eq. 9, is used as the snubber lock/unlock criterion in the computational logic. The values of various system parameters in Eqs. 8 and 9, which are dependent on the locked/unlocked state of the system or upon initial conditions, are appropriately transferred when lock/unlock occurs.

3. FORMAT OF RESPONSE PRESENTATION

In this compressible fluid model, the snubber's rated load, \( F_R \), and its fluid column stiffness (as evaluated from bulk modulus, piston head area, and stroke length) are parameters of interest in addition to the locking velocity, unlocking velocity, and base acceleration. Since rated load, piston area, and stroke length are dependent on the size and the manufacturer of the snubber (and thus are essentially scale factors to the restraining force developed by the snubber), these parameters are held constant at typical values in order to study the effect that variations in bulk modulus and input excitation have on system response.

Additionally, a value for the stiffness of the SDOF system is assumed. The value of 24 kip/in [4.2 kN/mm] was chosen as a typical stiffness for a straight 30 foot (9.14 m) long, 8 inch (20.35 cm) diameter pipe. (The 30 foot pipe is a distributed parameter system and it can be reduced to a SDOF system by a separation of variables technique as shown in Reference 1). Since numerous investigations have been made of the effect of piping system stiffness on the system response, and since this study is devoted to the analysis of the snubber as it affects the overall piping system, the stiffness is held constant throughout this study. The displacement response obtained from this model (Eq. 8) is normalized with respect to the magnitude of the harmonic excitation.

\[
z_n = \frac{z}{A_3/\omega_n^2} \quad \text{where;} \quad A_3 = \sqrt{A_{1p}^2 + A_{2p}^2}
\]

\[-- 399 --\]
The results are presented in the form of curves wherein the maximum normalized displacement response, $z_n$, is plotted versus the ratio of forcing frequency to system natural frequency, $\omega_f/\omega_n$. Natural frequencies for piping systems in power plants typically range from 5 to 30 Hz. Typical values of the forcing frequency for design basis earthquakes range from about 0.25 to 10 Hz. Consequently frequency ratios range from about 0.1 to 2.0 for systems subjected to seismic excitations. Results are also presented for higher frequency ratios. These results may be used to analyze the response for low amplitude, high frequency excitations such as pump vibrations.

4. DISCUSSION OF RESULTS

The bulk modulus of the fluid in the snubber is a parameter that affects the behavioral response of the device and hence it is a parameter of analytical interest. The bulk modulus has a certain theoretical value. But in practical application, the fluid in the snubber may have a bulk modulus that differs from the theoretical value due to internal and environmental effects (air entrainment, high temperature, irradiation). Based on manufacturer's design specifications for hydraulic fluids [5] and discussions with a hydraulic snubber manufacturer, bulk modulus values for this study were chosen as

- bulk modulus = 100 ksi (689.5 MPa) significant air entrainment
- bulk modulus = 180 ksi (1241 MPa) some air entrainment, typical value
- bulk modulus = 250 ksi (1724 MPa) negligible air entrainment

Figure 1 shows the effect that these variations in fluid bulk modulus have on system response for various frequency ratios at an input excitation of 38.64 in/sec\(^2\) (0.1g). For frequency ratios in the range of interest for seismic considerations, the largest values are obtained for the lowest bulk modulus. This is due to the fact that a low bulk modulus causes the snubber to behave similar to a soft spring, thus allowing larger displacements than a snubber behaving as a stiffer spring due to a larger bulk modulus. Throughout the remainder of this study the bulk modulus was held constant at a value of 180 ksi (1241 MPa) which is considered typical for snubber hardware.

Figure 2 shows the effect that variations in base acceleration have on the response versus frequency plots. For a base acceleration of 0.3604 in/sec\(^2\) (0.001g) lock up does not occur. But for the other 3 base accelerations shown, lock up does occur and snubbing action occurs at lower frequency ratios as base acceleration increases and thus maximum response decreases. Since the response for these 3 excitations appears to form an upper bound to the response for higher excitations, the trend in the results is similar to that obtained for the incompressible model (1). Figure 3 shows the response versus frequency ratio curves for both the compressible fluid and incompressible fluid models of the velocity sensitive snubber. For most frequency ratios in the range of interest for seismic considerations (0.25 to 2.0), the incompressible fluid model predicts larger response than does the compressible fluid model, while for frequency ratios indicative of pump vibrations (greater than 2.0), the incompressible fluid model
predicts lower response. Thus Figure 3 shows that for seismic considerations, the incompressible model is conservative, while for the analysis of response due to pump vibrations, the compressible model predicts conservative results.

5. CONCLUSIONS

In this study a time-history analysis is performed for piping systems containing a velocity sensitive hydraulic snubber. This time-history analysis is performed in a piecewise linear fashion which enables the analysis to characterize the piping system in each of the two conditions (locked or unlocked) that occur as a result of forced motion. Using the models presented herein, system response is predicted for various excitation conditions and for the various hardware design parameters that exist for these snubbers. The analysis was performed on a relatively small computational device (64k bytes of memory), yet results were obtained that compared favorably with experimental data for the velocity sensitive snubber.

For the velocity sensitive hydraulic snubber, the incompressible fluid model was computationally simpler (second order differential equations) than the compressible fluid model (third order equations). Yet with this simpler model, results were obtained that were conservative (compared to results obtained with the more complex compressible fluid model) for parameters in the range of interest for seismic events. However, when analyzing the response due to low amplitude, high frequency excitations such as pump vibrations, it is recommended that the more complex compressible fluid model be used, since it gave higher response than the incompressible model for high frequency ratios.

6. REFERENCES


Figure 1  Compressible fluid model; displacement response vs frequency ratio for various bulk moduli

Figure 2  Compressible fluid model; displacement response vs frequency ratio for various base accelerations

Figure 3  Compressible and incompressible fluid models; displacement response vs frequency ratio for various base accelerations