The Influence of Liner on the Ultimate Load Carrying Capacity of PCPV

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A three-dimensional inelastic analysis of the liner is presented in which a provision is made for vessel concrete interaction, liner anchorages and lugs. The vessel concrete is represented by solid isoparametric elements. The liner assembly is a mixture of line, panel and spring elements. An existing vessel with and without liner assembly is analysed under increasing gas pressure. Three-dimensional crack patterns are developed for both cases, together with the yielding of the liner. The load-deflection curves are drawn for both cases. The final ultimate load carrying capacity of the vessel with liner is much higher than without it. The analytical results have also been compared with some of the known experimental and analytical results. Conclusions are drawn.

INTRODUCTION

The comparative study of liners and their limitations demands more sophisticated analysis in order to assess their integrity under vessel operational cycles, the pre-operational period, the elasto-plastic and the ultimate load conditions. Any analysis must also examine the stability of the liner before it is considered along with the prestressing steel, conventional steel and concrete as a contributor to the overall strength of the vessel.

The theoretical model developed herein envelopes two separate analyses for the liner. They are as follows:-

(a) The elastic and elastoplastic analysis of the liner based on a mixed type variational and finite element technique.

(b) The elastic, elastoplastic and stability analyses of the liner, including the effect of the anchors or studs, treated as a thin shell using the finite element technique.

In the above mixed type analysis it is assumed that the structural strength of the liner is evaluated when it is keyed back to the concrete by an extensive array of studs and spanning over any cracks.

A brief development of the analysis is given in which temperature, creep, loading and unloading criteria have been included for both isotropic and kinematic hardening cases. Various stability equations have been obtained with and without the influence of the studs and rings or ribs and are solved using the finite element method.

MODES OF FAILURE

The modes of failure are -

(a) Liner instability when the liner is not anchored, resulting in the collapse of a large portion of shell. When the liner is anchored to concrete three types of buckling occur, namely circumferential panel buckling (ring mode), axial panel buckling (strip mode) and combined lobar mode.

(b) Fatigue failure of anchors and the liner due to gas pressure fluctuations and other cyclic loads.

(c) Pulling out of the anchor and rupturing of the liner under ultimate vessel conditions.

THEORETICAL MODEL

Liner as a continuum using a variational principle

A variational principle, together with a criterion on bifurcation and stability, are used to develop incremental constitutive equations. The behaviour of the liner under pre-buckling and inelastic asymmetric buckling conditions is studied. Non-isothermal von Mises yield function is utilised for the liner shell made of work-hardening material.

Throughout the development of equations, a Legrangian displacement formulation is adopted which considers both isotropic and kinematic hardening rules. Using Fig (1) from Stage 1 (undeformed) and Stage II (deformed) with the liner own body force per unit mass $f^{(1)}$, $f^{(2)}$ and its surface traction due to adjacent concrete $\hat{T}^{(2)}$ and $\hat{T}^{(2)}$ respectively and using Gauss Theorem, the variational equation becomes

$$\int_{V_d} \dot{\sigma}'_{ij}\partial\dot{\varepsilon}_{ij} + \sigma'_{ij}u'_{k,i}\dot{u}_{k,j} \quad dV_d = \int_{V_d} \rho_1 \frac{\partial fi'}{\partial t}\dot{u}_i \; dV_d + \int_{A_o} \hat{T}_i \partial\dot{u}_i \; dA \qquad (1)$$

where '.' refers to a "quasi-static" rate . ε_{ij} The Lagrangian tensor is

$$= \tfrac{1}{2}(u^s_{i,j} + u^s_{j,i} + u^s_{k,i},\ u^s_{k,j});\ s = I, II \qquad (2)$$

s represents the Stage I or II. The terms "ρ" and "A" are density and elemental area. Again, a comma denotes differentiation with respect to the coordinates in Stage I. Equations 1 and 2 are valid for stable and unstable equilibrium.

The surface traction incremental component $\bar{T}_i$ in terms of unit area and coordinates of reference state, and $\hat{n}_{2j}$ the unit normal, are related to the current surface traction as

$$\hat{T}_i = \bar{T}_i + \bar{T}_{1k}u_{i,k}\bar{T}_i = \dot{\sigma}'_{ij}\hat{n}_j \qquad (3)$$

Equation 1 can be used to derive the elemental force displacement relationship in a direct incremental approach. Where the terms are $\hat{n}$ the unit normal and u displacement.

Table 1 gives, briefly, equations for isotropic and kinematic hardening cases, derived for the finite element analysis. For both isotropic and kinematic hardening cases, the respective equations for F_L will determine the loading and unloading criteria. For example, for a perfectly plastic condition $F_L > 0$, and where unloading is done from the plastic stage to the elastic stage, then $F_L < 0$. The terms for F_L when equated to zero will define the neutral stage. The following is the finite element governing equation:-

$$\int_V (\partial\varepsilon^{T''}\hat{D}\varepsilon)\,dV = \int_V (\varepsilon^{T''}\hat{D}\varepsilon_o + \Delta u^{T''} f)\,dV + \int_{A_d} \Delta u^T T\,dA \qquad (4)$$

ε_o initial strain ; T'' Transpose ; f - body force/unit area

Equation 4, when differentiated, leads to a set of differential equations.

When the liner element is divided into volume elements having finite dimensions with interior and exterior boundary surfaces including studs, the liner shell three-dimensional stress analysis gives three displacement components u, v, w. These are given briefly below for n elements after transformation: using and as total global and elemental stiffness matrices.

$$\{\Delta\delta^{T''}_{E,n}\}\{K_{E,n}\}\{\delta_{E,n}\} = \Sigma^{s_n}_{i=1}\{\Delta u_{ni}\}^{T''}_g \quad \{\Sigma^{s_n}_{j=1}(k_{n,ij})_g(u_{nj})_g\}$$

$\delta_{E,n}$ - nodal displacement matrix

$$\Delta\delta^T_{E,n}F_{E,n} = \Sigma^{s_n}_{i=1}\{\Delta u_{ni}\}^{T''}_g \quad \{F_{n,i}\}_g \qquad (5)$$

s_n - particular stage I, II etc

$$\text{where} \quad \{k_{n,ij}\}_g = \{C^n\}^{T''}_g\{k_{n,ij}\}\{C^n\}_g \qquad (6)$$

Where

C^n - direction cosines (local coordinates)

$$\{F_{n,i}\}^s_g = \{C^n\}^{T''}_g\{F_{n,i}\}^s \qquad (7)$$

g - global formulation

Direction courness for the local coordination.
nodule replacement matrix
particular stage
global formulation

The influence of stud and ring on the liner

Design analysis of the liners with anchors involves consideration of both stability and stress problems. As the liner is rigidly attached to the surrounding concrete by means of studs or rings, or both, the stability consideration of the liner is important. Table 2 gives, briefly, the important steps for the inclusion of studs. Reference is made (1,2) for producing cracks behind the liner. The stability determinant given in Table 3 gives the lowest value of λ, and hence the critical instability load. Successive values of λ' are evaluated and are substituted in the same determinant using standard iteration procedure given (3). The bifurcation stage is reached when an element has just about yielded, i.e. $\sigma = \bar{Y}$. Program F-Liner calculates the critical load F_{cr} based on the available λ' value. If $F_{cr} > F_A$ the calculation is repeated at a higher value of F_A until $F_{cr} = F_A$. Knowing F_{cr}, the corresponding stability mode can be determined by calculating strain and stress rates. To accomplish this the flow rule is used (3).

Analysis of results

Detailed tests have been carried out on a cylindrical steel shell encased and restrained at certain points by a concrete jacket. The shell is subject to compressive strains induced by axial loads. Surface strains and buckling deflections have been measured. Fig. 2 shows the load-displacement relationship for studs. Results obtained from the tests are compared with those obtained from Nelson (4), Imperial College (5), Chapman, Carter, Doyle et al and Parker(6,7,8). These results are compared with finite element analysis of the liner carried out using the above model.

Fig. 3. shows the stresses, strains and displacements at three locations for the 1/8th segment of the Oldbury vessel. As evident, these stresses vary for various locations along the height and nodal circles of the vessel. As shown in Fig. 3 , the stress-strain curve for the nodal circle near the bottom cap is inconsistent. This is due to the assumed boundary conditions. For other points along the height, the results cover an idealised case for Oldbury and HTGCR vessels (Fig. 4.). Here the results are compared for the same liners with and without imperfections e_o. Using single layer and multi-layer solutions, data for three vessels, namely Oldbury , Dungeness B and HTGCR the idealised stress-strain results for the central line are plotted. These results are compared with those obtained by Gulf Atomic (9) and Doyle et al. The two-dimensional analysis given by Gulf Atomic gives very close results to those obtained by using the above theoretical model. There is a marked difference from the results obtained using Doyle et al (7) one dimensional approach as can be expected.

Conclusion

A theoretical model has been developed to test the stress-strain behaviour of liners chosen for the Dungeness B, HTGCR and Oldbury vessels. Both perfect and imperfect liner systems are considered. The final results are compared with some of the known theoretical and experimental results and are in good agreement. Table I gives the safety margins with and without the influence of the anchored liners, and idealised anchor models.

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TABLE 1 Hardening Analysis

Safety Margins

(a) Isotropic Hardening - The load F_L is given by

$$F_L = \bar{\sigma} - S_H(\varepsilon_p^{(1)} T^{(1)}) = 0 \qquad (A)$$

where $\bar{\sigma}$ is an average plastic stress and in terms of traction = $(\frac{3}{2}\hat{T}_{ij}\hat{T}_{ij}^{(1)})^2$

$$\hat{T}_{ij} = \sigma'_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$$

S_H = measurement of strain hardening $\bar{\gamma}(T_1)$

ε_p = equivalent plastic strain at Stage I = $\int d\varepsilon^P$

T = temperature

$$d\varepsilon^P_{ij} = \lambda_1(\partial F_L/\partial\sigma'_{ij})^{(1)*} \qquad (B)$$

*(1) denotes Stage I.

Naghdi (10) suggest the value of the positive scalar λ_1 as

$$\lambda_1 = -[(\partial F_L/\partial\sigma'_{k\ell})\sigma'_{k\ell} + (\partial F_L/\partial T(u)T]^{(1)} / [(\partial F_L/\partial\varepsilon^P_{k\ell})(\partial F_L/\partial\sigma'_{k\ell})]^{(1)} \qquad (C)$$

(b) Kinematic Hardening - The load function F_L is written as:

$$F_L = \frac{3}{2}[(\hat{T}^{(1)}_{ij} - \alpha^{(1)}_{ij})(\hat{T}^{(1)}_{ij} - \alpha^{(1)}_{ij})]^{\frac{1}{2}} - S_H T^{(1)} = 0 \qquad (D)$$

where α_{ij} is a tensor representing the complete translation of the yield surface in the deviatoric stress space.

$$\lambda_1 = \frac{\frac{\partial F_L}{\partial\sigma'^{(1)}_{k\ell}}\sigma'_{k\ell} + (\frac{\partial F_L}{\partial_T}(1))T - \frac{2}{3}H'\frac{(E^{(2)} - E^{(1)})}{E^{(1)}}\frac{\partial F_L}{\partial\sigma'^{(1)}_{ij}}d\epsilon^P_{ij}}{\frac{2}{3}H'(\frac{\partial F_L}{\partial\sigma'_{k\ell}})^2}$$

$$d\varepsilon^P_{ij} = T[\frac{3}{2}\frac{\partial\varepsilon_P}{\partial T}\bar{\gamma}].(\hat{T}^{(1)}_{ij} - \alpha^{(1)}_{ij} + \frac{1}{H'}(\frac{3}{2}\bar{\gamma})^2(\hat{T}^{(1)}_{ij} - \alpha^{(1)}_{ij})(\hat{T}^{(1)}_{k\ell} - \alpha^{(1)}_{k\ell}) \qquad (F)$$

Safety Margins

	Vessel without liner	Vessel with anchored liner	
		Liner Intact	Liner leakage anchor failure
Dungeness B	2.50	3.95	2.70
Oldbury	2.80	3.60	2.88
HTGCR	3.00	4.80	3.20

TABLE 2: Analytical formulation of the steel liner

$[K_{TOT}]\{\delta\}^* + \{F_T\} - \{R_T\} = 0$ where $[K_{TOT}] = [K_\ell] + [K_a]$; $\{\delta\}^* = \begin{Bmatrix}\delta_{un}\\ \delta_b\end{Bmatrix}$; $\{F_T\} = \begin{Bmatrix}F_{un}\\ F_b\end{Bmatrix}$;

$\{R_T\} = \begin{Bmatrix}R_{un}\\ R_b\end{Bmatrix}$

K_{TOT} = Total stiffness matrix

K = Liner stiffness matrix

K_a = A stud stiffness matrix

F_T = Total initial load vector

R_T = Total external load vector

$[K_\ell]\{\delta_{un}\} + \{F_{un}\} = 0$

$\{\varepsilon\} = [B]\{\delta\}$

$\{\sigma\} = [D](\{\varepsilon\} - \{\varepsilon_0\})$

$\{\delta\}$ = anchor shear forces = $[K_a]\cdot\{\delta_{un}\}$

$\{F_{un}\} = \int_V [B]^T[D]\{\varepsilon_0\}dV$

$= \int_V [B]^T[D]\{\varepsilon_0\}\det[J]\,d\xi\, d\eta\, d\zeta$

Subscript un = quantities corresponding to unknown displacement

Subscript b = quantities corresponding to restrained boundaries

The Plastic Buckling Matrix is given by $(K + \lambda K_G)F_T = 0$,
where K = elastoplastic stiffness matrix as a function of the current state of plastic deformation; K_G = initial stress geometric stiffness matrix.

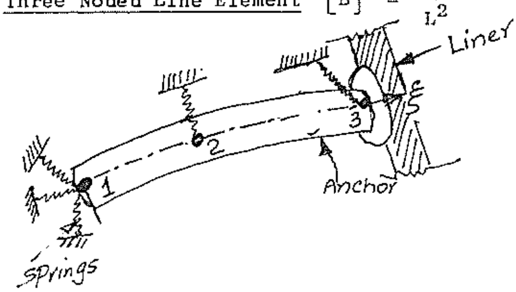
The determinant $[K + \lambda K_G] = 0$

The essential equation is characteristically triangularised for the ith loading step as
$(K^i + \lambda_c K_G^i)F_T^i = 0$; $\lambda_c = 1 + E_{ps}$; E_{ps} = accuracy parameter; $[K_\ell] = \int B^T DB$ dvol.

Material Matrix $[D]_{6x6} = \frac{E(1-v)}{(1+v)(1-2v)} \times \begin{bmatrix} 1 & a & a & 0 & 0 & 0\\ a & 1 & a & 0 & 0 & 0\\ a & a & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & b & 0 & 0\\ 0 & 0 & 0 & 0 & b & 0\\ 0 & 0 & 0 & 0 & 0 & b \end{bmatrix}$

where $a = \frac{v}{1-v}$; $b = \frac{1-2v}{2(1-v)}$

Three Noded Line Element $[B]^T = \frac{1}{L^2}$ $\begin{bmatrix} (\xi-\frac{1}{2})^2 x_1 & +(\xi^2-\frac{1}{4})x_2 & -2\xi(\xi-\frac{1}{2})x_3\\ (\xi-\frac{1}{2})^2 y_1 & +(\xi^2-\frac{1}{4})y_2 & -2\xi(\xi-\frac{1}{2})y_3\\ (\xi-\frac{1}{2})^2 z_1 & +(\xi^2-\frac{1}{4})z_2 & -2\xi(\xi-\frac{1}{2})z_3\\ (\xi^2-\frac{1}{4})x_1 & +(\xi+\frac{1}{2})x_2 & -2\xi(\xi+\frac{1}{2})x_3\\ (\xi^2-\frac{1}{4})y_1 & +(\xi+\frac{1}{2})y_2 & -2\xi(\xi+\frac{1}{2})y_3\\ (\xi^2-\frac{1}{4})z_1 & +(\xi+\frac{1}{2})z_2 & -2\xi(\xi+\frac{1}{2})z_3\\ -2\xi(\xi-\frac{1}{2})x_1 & -2\xi(\xi+\frac{1}{2})x_2 & +4\xi^2 x_3\\ -2\xi(\xi-\frac{1}{2})y_1 & -2\xi(\xi+\frac{1}{2})y_2 & +4\xi^2 y_3\\ -2\xi(\xi-\frac{1}{2})z_1 & -2\xi(\xi+\frac{1}{2})z_2 & +4\xi^2 z_3 \end{bmatrix}$



where $L = \sqrt{(\frac{\partial x}{\partial \xi})^2 + (\frac{\partial y}{\partial \xi})^2 + (\frac{\partial z}{\partial \xi})^2}$

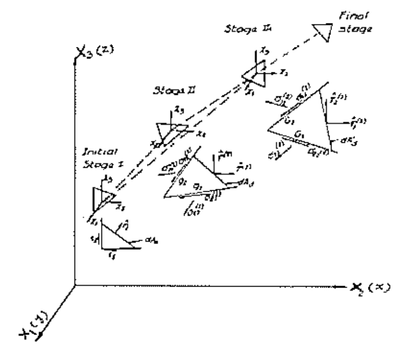


FIG.(I) Stress resultants and stages of deformation

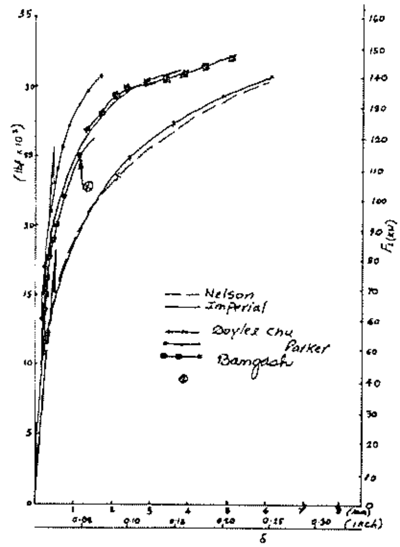


FIG Load-displacemt For studs

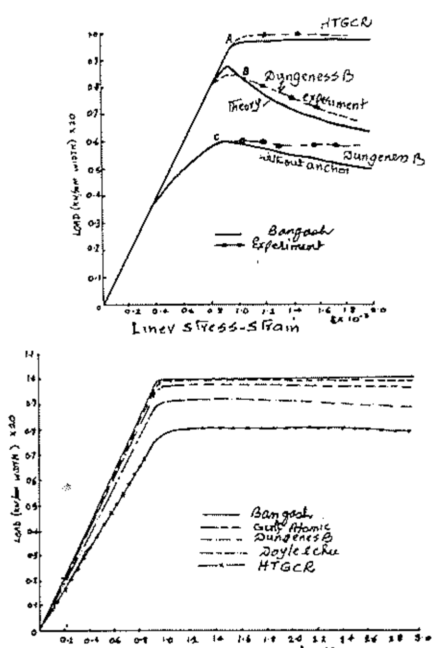
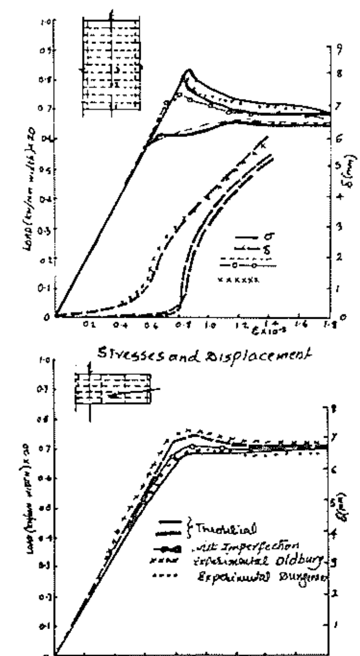


FIG.3 + Liner stress-strain Relation



FIGA Stress-Strain (with imperfection)