

A Constitutive Model for Concrete Under Dynamic Loading

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The rational analysis of a structure subjected to impact requires the constitutive relations of the structural material over a wide range of strain-rates. The dynamic test results available for concrete are few and no satisfactory constitutive model exists for predicting the dynamic behaviour of concrete. A continuous damage theory for the quasistatic and dynamic behaviour of concrete is presented here. The continuous damage theory is a rational choice for use in predicting the dynamic behaviour of concrete as the strain-rate effects that have been observed for concrete can to a large extent be attributed to the rate-sensitivity of the microcracking process. A vectorial representation is adopted for the damage to account for the planar nature of the microcracks in concrete. Damage is treated as an internal state variable influencing the free energy of the material and the constitutive equations and the damage evolution equations are derived consistently using thermodynamic considerations. The developed constitutive model is then calibrated by using test results in flexure and compression over a range of strain-rates. The constitutive model is also shown to be capable of predicting certain other experimentally observed characteristics of the dynamic response of concrete.

1. Introduction

There is a likelihood that many concrete structures would be subjected to dynamic loads during their lifespan. These dynamic loads may arise from impact of ballistic or tornado generated missiles, impulses due to air blasts or wind gusts as well as from earthquakes. While the effect of such dynamic loads should be accounted for in the design of all structures, they become quite critical when dealing with special purpose structures such as water-retaining structures, fall-out shelters and nuclear containment structures.

The customary design approach, for impact and impulsive loading has been mostly empirical in nature because of the complexity of the structural response. For instance, various semi-empirical formulae have been proposed for the behaviour of the contact zone under impact, using which the stress pulse generated at the contact zone can be evaluated. The overall structure can then be designed to carry this stress pulse. The design is usually carried out by using idealized material parameters. A more rational analysis of the structure can, however, be undertaken with the knowledge of the constitutive properties of concrete, steel reinforcement and their interface over a range of strain-rates.

Along with the experimental results under dynamic loading, satisfactory constitutive models for the materials, are also necessary for such an analysis to be feasible. The behaviour of the steel-concrete interface has been found to be rate sensitive only where deformed bars are used (1). This apparent strain-rate sensitivity can therefore be attributed to concrete itself as deformed bars produce extensive cracking. The properties of steel under dynamic loading have been well documented and various constitutive theories incorporating rate effects are available. For concrete, however, only a limited number of experimental results are available and no satisfactory constitutive model exists.

Most of the available dynamic test results for concrete are on its strength increase with increasing strain-rate. Several researchers (e.g., H. Mihashi and F. H. Wittman (2)) have also developed expressions to predict the strain-rate ($\dot{\epsilon}$) sensitivity of the fracture strength (σ_f) of the form.

$$\sigma_f \propto \dot{\epsilon}^{1/(p+1)} \quad (1)$$

A scrutiny of the available experimental results reveals, however, that the above expression is not very satisfactory as the value of p appears to decrease with the strain-rate. For instance, results obtained by the authors (3) over a strain-rate range of 10^{-6} - 10^{-1} Sec^{-1} indicate a decrease in the value of p from about 47 to 16. At higher rates an even lower value of p equal to 2 had been obtained by F. M. Mellinger and D. L. Birkimer (4). These models are also incapable of predicting the influence of strain-rate on the stress-strain curves and the higher strain-rate sensitivity of the tensile strength as compared to the compressive strength (5). This paper presents a constitutive model which is capable of predicting such effects. Details of its calibration using the results obtained by the authors (3) are also presented.

2. Formulation of the Constitutive Model

It is widely accepted that the failure of concrete is brought about by the nucleation and growth of a number of microcracks. A review of the dynamic test results available for concrete also indicate that the strain-rate effects observed for concrete can be to a large extent attributed to the rate dependence of this microcracking process (5). It appears, therefore, that the continuous damage theory is a rational choice for predicting the mechanical behaviour of concrete under both quasistatic and dynamic loading.

A vectorial representation is adopted for the damage which is motivated by the planar nature of the microcracks in concrete. The damage vector is chosen to be normal to the plane of the crack field and having a magnitude equal to the area density of the cracks. A vectorial damage variable, in contrast to a scalar damage variable, is also capable of modelling the crack induced anisotropy observed in concrete. Damage is treated as an internal state variable which influences the free energy of the material. For the

present formulation the Helmholtz free energy function (χ) is defined in terms of the coupled invariants of damage and strain as,

$$\begin{aligned} \rho\chi = & \frac{1}{2}(\lambda+2\mu)\varepsilon_{kk}\varepsilon_{kk} - \mu(\varepsilon_{kk}\varepsilon_{kk} - \varepsilon_{kl}\varepsilon_{lk}) \\ & + \gamma_1(w_i^\alpha w_i^\alpha)^{-1/2} |\varepsilon_{kk}| w_k^\alpha \varepsilon_{kl} w_l^\alpha \\ & + \gamma_2(w_i^\alpha w_i^\alpha)^{-1/2} (w_k^\alpha \varepsilon_{kl} w_l^\alpha)^n + \gamma_3(w_i^\alpha w_i^\alpha)^{-1/2} (w_k^\alpha \varepsilon_{kl} \varepsilon_{lm} w_m^\alpha) \end{aligned} \quad (2)$$

where λ , μ are the lame parameters, w_i is the damage vector, ε_{ij} is the strain tensor and ρ the density of the material. The three parameters γ_i which define the influence of the microcracks on the state of the material, are considered to be constants in the present formulation. The superscript α indicates the possibility of the occurrence of more than one independent damage field.

This particular form of the free energy function differed from the form chosen by L. Davison and A. L. Stevens (6) by virtue of the power n associated with the fourth term. This term can thus be of a different order in strain and damage, and provides for the obtainment of an equilibrium damage configuration from the free energy function.

A consistent thermodynamic approach yields the constitutive equation,

$$\sigma_{ij} = \frac{\partial \chi}{\partial \varepsilon_{ij}} \quad (3)$$

and the damage evolution equation,

$$\rho k \dot{w}_i = g_i^D(\varepsilon_{ij}, w_i, \dot{w}_i) - \rho \frac{\partial \chi}{\partial w_i} \quad (4)$$

where σ_{ij} is the stress tensor and k is the inertia associated with the microcrack growth (7).

g_i^D should satisfy the entropy production inequality resulting from the thermodynamic formalism and is chosen as,

$$g_i^D = -\gamma_4 (w_o \dot{w}_o)^{1/m} \frac{\dot{w}_i}{w_o} \quad (5)$$

where w_o is the magnitude of w_i and γ_4 and n are constants.

Eqs. 2-5 can be combined to yield the stress-strain relations for a particular state of the material. Due to the assumption of the planar nature of the microcracks, however, the theory appears to yield reliable results only for planar problems where cleavage strains are small enough to keep the crack openings small. For a realistic prediction of the biaxial behaviour and the strain-softening of concrete the friction at the crack surfaces has also to be accounted.

3. Calibration of the Constitutive Model and Numerical Results

The calibration of the constitutive model can be conducted by the use of experimental data in simple response modes such as in uniaxial tension and compression, under both quasistatic and dynamic loading.

3.1 Uniaxial Tension

In a uniaxial tensile test a tensile strain is applied in one direction while the external surfaces normal to the other two coordinate directions are maintained stress free. If it is assumed that the material has not undergone any prior loading, the damage can be represented by a vector which is parallel to the axis of application of the tensile strain. The reduced axial stress-strain relation under these conditions can be obtained by combining equations (2) and (3). The value of the damage to be used in the stress-strain relation is obtained by combining equations (4) and (5). If the rate of straining is small enough it can be assumed that the damage reaches its stationary value after each increment of strain. This can be considered as quasistatic loading for which the value of damage can be obtained by setting $\frac{\partial X}{\partial w} = 0$ (see equations (4) and (5)). For higher strain rates the differential equation given by equations (4) and (5) has to be solved for each strain increment. Setting the transverse stress equal to zero yields a value for the apparent Poisson's ratio of the material. This value decreases with the increase of the applied strain and therefore the effect of lateral strain on the strain-strain response gradually decreases. A good approximation of the uniaxial stress (σ_{11}) vs. strain (ϵ_{11}) curve can therefore be obtained by neglecting the Poisson effects and for quasistatic loading is given by:

$$\sigma_{11} = E \left[1 - \frac{2(n-1)}{n} \left(\frac{\epsilon_{11}}{\epsilon_{tm}} \right)^{\frac{2-n}{2(n-1)}} \right] \epsilon_{11} \quad (6)$$

where the strain corresponding to the peak stress is given by:

$$\epsilon_{tm} = \left\{ \frac{(2n-1)}{D} \left[\frac{2(n-1)(2n-1)}{n(3n-2)} \right] 2(n-1) \right\}^{\frac{1}{2-n}} \quad (7)$$

$$D = \frac{(-C_1 - C_3)^{2n-1}}{C_2} \quad (8)$$

and

$$C_i = \frac{\gamma_i}{DK} \quad (i = 1, \dots, 4) \quad (9)$$

where E, K are, respectively, the modulus of elasticity and the bulk modulus of the undamaged material (4).

From the foregoing it can be observed that the stress strain curve in uniaxial tension can be predicted by using two constants n, D in addition to the Young's modulus E and the Poisson's ratio ν of the undamaged material. For predicting the dynamic response of concrete the three additional constants in equations (4) and (5) have also got to be defined.

Although the calibration can be conducted using uniaxial tensile test results for concrete, such results are quite limited because of the numerous problems associated with the tensile testing of concrete. It is common practice, therefore, to measure the tensile properties of concrete through a flexural test.

3.2 Flexure

Instrumented impact tests have been conducted on many metals in the past to yield their dynamic behaviour. The same degree of success has not been attained for concrete, however,

because of its inherent brittleness and low strength/weight ratio. They manifest as inertial effects in the recorded load-time history and in many instances have resulted in the overshadowing of the material response. The authors were recently able to obtain reliable mechanical properties of concrete through analyzing and controlling these inertial effects (3). The moment vs. extreme tensile-fiber strain curves obtained from these tests were used for the calibration of the present model.

The moment vs. extreme tensile fiber strain curve at the midspan of the beam can be evaluated by considering the equilibrium of the beam section. A few assumptions can also be made to simplify the analysis; i.e., (a) the damage in the compression region is negligible; (b) the strain distribution across the depth is linear; and (c) the effect of shearing stresses is small. If the shearing stresses were considered they would result in a rotation of the damage vector away from the longitudinal direction. This would result in decreasing the effect of the damage and a higher strength than that predicted can be expected. This effect may be partly responsible for the higher MOR generally obtained from 3 point bend tests as compared to 4 point bend tests.

The analytical and experimental moment vs. strain curves at two strain-rates are given in Figure 1. It can be noticed that increasing the strain-rate has no significant effect on the initial tangent modulus but results in decreasing the nonlinearity of the curves.

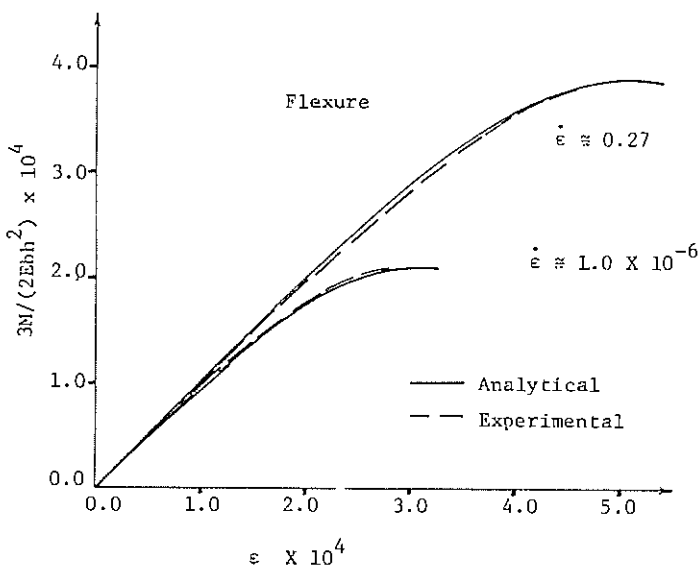


Figure 1

Analytical and experimental non-dimensionalized moment vs. strain curves in flexure
 (E = modulus of elasticity of the undamaged material, b = width of the beam, h = depth of the beam,
 M = moment at mid span, ϵ = extreme tensile-fiber strain)

3.3 Uniaxial Compression

It is a well known fact that in an uniaxial compression test the microcracks develop parallel to the axis of the loading. Therefore two damage fields which are mutually independent and oriented in two perpendicular transverse directions have to be introduced. For evaluating

the stress-strain curve in uniaxial compression a constant c_1/c_3 has to be defined in addition to the constants n and D which are required for predicting the uniaxial tensile response. This additional constant was determined from uniaxial compression test results. It can be noticed that only three constants in the strain energy function have to be defined for predicting the quasistatic response of concrete. The fourth constant appearing in the strain energy function can be uniquely defined only if numerical values of the damage are available.

The analytical predictions for the strength increase in tension, flexure and compression are compared with results obtained by the authors and other available results in Figure 2. The constitutive model can be observed to be successful in predicting the higher strain-rate sensitivity in the tensile mode as compared to the compression mode.

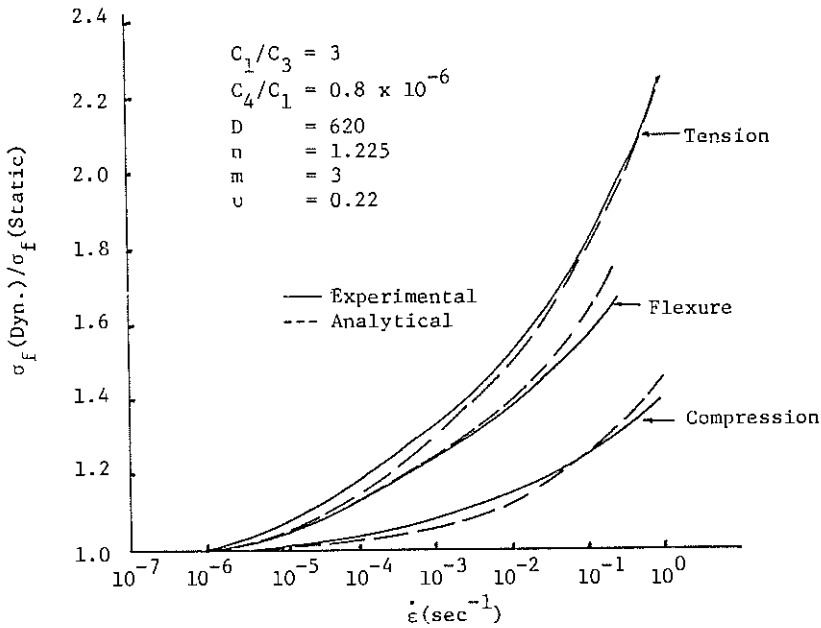


Figure 2
Analytical and Experimental Curves for the Effect of Strain-Rate on the Ultimate Strength

Figure 3 gives the variation of the apparent Poisson's ratio (calculated at a strain corresponding to the peak stress under quasistatic loading) with strain-rate. This values exhibits an increasing trend in tension and a decreasing trend in compression, as the strain-rate is increased. Such trends can be attributed to a decrease in the microcrack accumulation (in the direction of the principal tensile-strain) as the strain-rate is increased.

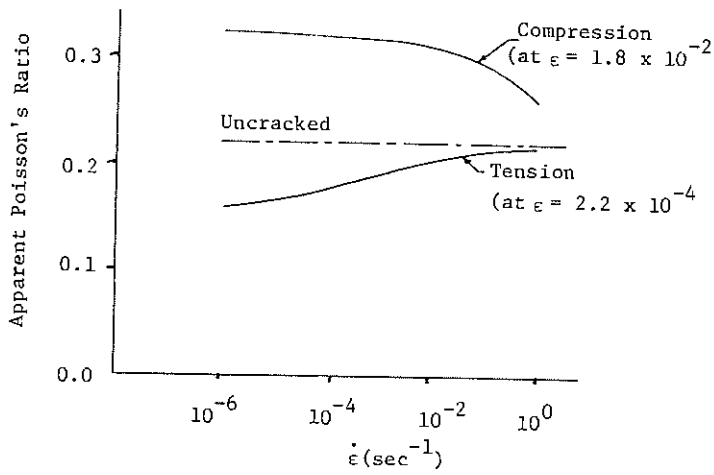


Figure 3
The Effect of Strain-Rate on the Apparent Poisson's Ratio

4. Conclusions

A constitutive model was developed for the quasistatic and dynamic behaviour of concrete, based on the continuous damage approach. The constitutive equations as well as the damage evolution equations were derived consistently from thermodynamic considerations. The model was calibrated by the use of flexural and compression test results over a range of strain-rates. The model was also found to be capable of predicting certain other phenomena of the dynamic response of concrete.

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