An Approximate Method of Predicting Crack Growth in Concrete Structures

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SUMMARY

This paper presents ideas to predict the cracking direction and the bond-slip occurrence in reinforced concrete structures that are unavoidable problems in application of the finite element method to failure analysis of the structures. Three approaches have been employed to analyze the failure of the concrete structures. Smeared crack model, discrete individual crack model, and fracture mechanics model. The third model is used in this paper.

Crack initiation and extension depend upon the strength of concrete. The total potential energy difference prior and after the crack advance should be maximum for the real crack in comparison with the virtual cracks under the identical condition. The potential energy release rate is a suitable index for the comparison. Comparing the potential energy release rates for a given loading condition together with making reference of a stress state in concrete to the failure criterion, direction of cracking can be numerically estimated. When the crack propagates to inner concrete across reinforcing bar, the bond-slip failure is taken into account.

The progressive crack failure analysis is performed by the finite element method of triangular elements. The total deformation is determined by linear elasticity using the secant modulus of the material. The result gives, therefore, the lower bound of load carrying capacity of the structure.
1. INTRODUCTION

One of the main obstacles to the failure analysis of reinforced concrete structures is that they are constructed with composite material. Properties of the individual materials are well known, while some problems is still unclear, such as crack propagation and crushing phenomenon in plain concrete. On the other hand the behavior of the composite material after local yielding and failure of a constituent, or concrete, is not sufficiently investigated.

Finite element models for these problems presented in the last decade give us simplicity in implementing the failure analysis of reinforced and prestressed concrete structures, but are not always realistic. The aim of this paper is to present the more realistic model. In section 2 a concept for prediction of the crack direction in concrete is related, in section 3 three assumptions for analyzing the bond-slip failure are introduced, and in section 4 some problems in applying our model to the finite element method are treated. Section 5 is conclusion.

2. MAXIMUM TOTAL POTENTIAL ENERGY RELEASE RATE

Consider a cracked reinforced concrete structure that is loaded by surface tractions $T_i$ on the boundary $s$ and body forces $F_i$ (Fig. 1 (a)). Let $U_{ij}$, $\varepsilon_{ij}$, and $U_i$ denote the stress, strain, and displacement at this state (prior the crack advance). Consider now a virtual crack extension due to the increase of loads, like $T_i$ from $T_i^0$ and $F_i$ from $F_i^0$. Let $U_{ij}$, $\varepsilon_{ij}$, and $U_i$ denote new values of stress, strain, and displacement after the crack advance (Fig. 1 (b)).

The total potential energy of the body at the first state is

$$U^* = \int_V u(\varepsilon_{ij}) dV - \int_S T_i u_i dS - \int_V F_i u_i^0 dV$$

(1)

where $u(\varepsilon_{ij})$ is the strain energy density $\frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl}$. If the material is linear elastic, $u(\varepsilon_{ij})$ is given by $\frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl}$. At the second state the total potential energy becomes

$$U = \int_V u(\varepsilon_{ij}) dV - \int_S T_i u_i dS - \int_V F_i u_i dV$$

(2)

where $\Delta V$ is loss in volume of the body due to the virtual crack bridging and crushing. For the tensile crack advance the volume loss of concrete is negligible, while the crushing failure develops with a certain amount of the volume loss.

Since concrete is a nonlinear material, the strain energy density should be determined by the integral formula. It is admissible and convenient, however, to evaluate it by the elastic formula using the secant rodulus.

The total potential energy release rate is approximately given by

$$G = \frac{U - U^*}{\Delta a} = \frac{\Delta U}{\Delta a}$$

(3)
, where $\Delta a$ is the area increment of the crack surface.

Just after the crack advance, the body would get a transient stable state. If the loading condition remains constant, the body would become completely stable. It is considered as a difficult problem to predict the direction of cracking and the amount of volume loss at crushing in concrete. It has been allowed as a reasonable assumption in the progressive failure analysis of the concrete structures that the crack extends in the direction normal to the maximum principal stress. We choose with good results the possible crack direction out of several virtual crack directions which satisfy approximately the above assumption by maximizing the total potential energy release rate $G$ at each step of the crack advance.

3. BOND-SLIP FAILURE

Cracks in concrete initiate approximately normal to the free surface of the tension side of the structural member. Then they extend towards the inside as load increases and reach the position of tension reinforcement. They cross the reinforcing bar simultaneously with the bond-slip failure and propagate to the inner concrete. Before the crack propagation, covering concrete has been usually damaged in some measure by hair cracks (Fig. 2). Thus stresses of the covering are released, and stresses of the inner section are intensified.

It should be noted that the strength of concrete around reinforcing bars is usually lower because of bleeding effect and ill conditions for concrete work. Various types of pullout tests have been performed to investigate the bond shear strength between concrete and reinforcing bar. The bond-slip phenomenon in the real concrete structure is different from the material behaviors observed in the pullout tests and is rather similar to a single plane shear phenomenon.

From these considerations we adopt three following assumptions for analyzing the bond-slip failure.

1. Stresses in cracked zone are transmitted through the net sectional width between outer sides of reinforcing bars. Denoting the ratio of the total width to the net width by $\lambda$, stresses in elements adjoining to the reinforcing bars are evaluated by multiplication of the finite element solutions by $\lambda$.

2. The strength of bond concrete is lower than the standard value.

3. The strength of concrete for the bond-slip failure is determined by the modification of empirical shear strength formula which Endo and Aoyagi [1] derived from a series of single plane shear tests for concrete.

$$\frac{\tau_u}{r\sigma_t} = -0.0714 \left( \frac{\sigma_n}{r\sigma_t} \right)^4 - 0.475 \left( \frac{\sigma_n}{r\sigma_t} \right) + 0.469 \quad (\sigma_n < 0)$$

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\frac{\tau_u}{r_0 c} = -0.782 \left( \frac{c_n}{r_0 a} \right) \ast 0.469 \quad (c_n > 0)
\] (4-2)

where \( \tau_u \) is tangential stress on the shear plane, \( c_n \) is normal stress to the shear plane, \( c_t \) and \( c_c \) are uniaxial tensile and compressive strength of concrete, and \( r \) is the strength ratio between the bond concrete and the standard concrete.

4. FINITE ELEMENT MODELING AND COMPUTATIONAL TECHNIQUE

Various idealizations have been proposed by many investigators for solving the post failure behavior of reinforced concrete structures with the finite element method.

Let us consider a reinforced concrete beam, as shown in Fig. 3, which behaves as a two dimensional structure. Utilization of triangular elements is caused by the experience that changing the coordinates of nodes for the selection of crack direction can be made without much labor and time. Reinforcing bar is simulated by one dimensional element. These two kinds of elements are connected at nodes and are monolithic before the bond-slip occurs.

We use a sharp interelement crack to determine the direction of tension crack and the form of crushing zone. When the stress state in any concrete element reaches the failure criterion of concrete under loading of a certain magnitude, a sharp crack is assumed to develop along the boundary of the element. Then we duplicate nodes at the beginning point of the virtual crack. Consequently boundary lines of adjoining elements which contain the nodes overlap each other (Fig. 3 (b)). The same load is applied again. The duplicated nodes and boundaries separate. There act no external loads on the nodal points (Fig. 3 (c)).

The material nonlinearity is considered by using the secant modulus evaluated from the stress state of each element at every loading step. According to the incremental method, a part of stresses of the cracked elements should be released at the moment of crack opening. We adopt the secant modulus method so that an obscure calculation of the stress release may be omitted.

When stress state in an inner concrete element, which lies ahead of the existing crack and adjoins to a reinforcing bar, violates the failure criterion of concrete, we triplicate nodes at the crack tip (Fig. 4 (a)). It is necessary for the crack advance across the reinforcement in addition to the violation that the intensified stresses along the bar, as explained in section 3, violate the bond-slip failure criterion.

If the two conditions concur, the triplicated nodes separate each other. Thus the bond-slip and the crack advance to the inner concrete are simulated (Fig. 4 (b)).
§. CONCLUSION

Ideas for predicting the crack propagation direction in concrete and modeling the bond-slip phenomenon between concrete and steel bar are explained.

It is noteworthy that the result is controlled by the estimation of \( r \)-value in eq.(4). The next remarkable point is that the proposed method gives the lower bound of the load carrying capacity of structure, because it is based on the secant modulus method.

REFERENCE

/1/ Endo, T. and Aoyagi, Y., "Strength of Concrete under Combined Stress State of Normal and Shear Stresses", Annual Report of Cement Engineering in Japan '78.

Fig. 1 Change of Crack Surface before and after Crack Extension

Fig. 2 Structural Crack and Hair Crack
Fig. 3  Finite Element Modeling of 2-D Reinforced Concrete Beam

(a) Duplicate Nodes Before Crack Development
(b) Disconnected Nodes and Boundaries after Crack Development

Fig. 4  Modeling for Crack Advance over Reinforcement and Bond-slip