A New Approach to the Analysis of the Confinement Role in Regularly Cracked Concrete Elements

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SUMMARY

The most typical mechanism for shear transmission in a cracked planar concrete element is Aggregate Interlock between the crack faces, which are roughened by aggregate particles protruding from the cracked mortar.

A comprehensive approach to the analysis of shear transfer via Aggregate Interlock should be based on the analysis of the random distribution of interface asperities, but a simpler approach based on simplified micromechanic models and on test results can more effectively lead to the formulation of general equations between the static and kinematic parameters characterizing a crack (shear and confinement stresses, relative displacements at the interface, namely slip and opening).

In this paper a comparative analysis of the theoretical results at constant crack opening (as obtained by Bažant and Gambarova in the so-called Rough Crack Model) and of some very recent test results at constant confinement stress (Daschner, Munich, 1980-82) makes it possible to give a new formulation to the confinement equation along a crack. The reliability of the new formulation is checked against test results at constant crack dilatancy and at imposed displacement history.

Moreover, with reference to the relation between the interface shear and the crack displacements, some improvements are introduced into the previous formulation (Rough Crack Model), for a better description of the effects of aggregate size, and for a better formulation of the tangent shear modulus G^{CR} whenever cracking occurs. This is particularly important for updating the concrete stiffness characteristics in F.E.M. programs dealing with r.c. containment shells.

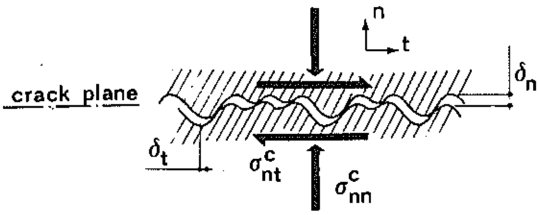
NATURE OF THE PROBLEM

The analysis of the static behaviour of nuclear reactor structures requires the evaluation of the strength and stiffness characteristics of the concrete subject to cracking. As a matter of fact, widespread cracking has to be expected in the extreme accident situations that are usually considered in the analysis of nuclear reactor structures, with particular reference to primary and secondary containment shells. Steel reinforcement generally prevents the formation of blunt continuous cracks, on condition that the bars are closely distributed: in this situation many thin cracks form, and these cracks tend to be locally parallel and evenly spaced because the gradients of the internal forces are relatively small in a bidimensional structure subjected to distributed loads.

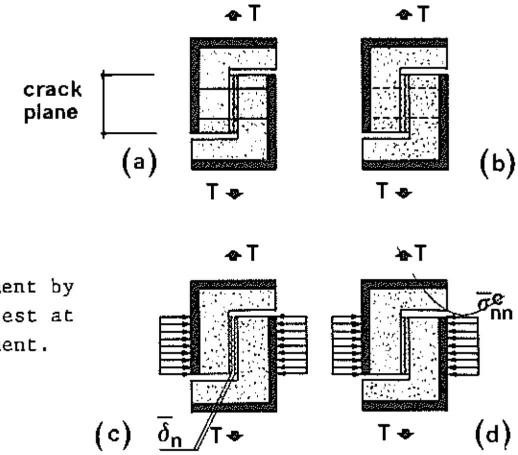
In the above mentioned situation, characterized by a relatively regular mesh of cracks, the stress-displacement relations at the crack interface may be considered as constitutive laws of the material, on condition that the finite relative displacements along the cracks (crack opening and slip, Fig. 1) are replaced by equivalent strains obtained by smearing the displacements over a length equal to crack spacing (Bažant and Gambarova /1/). The formulation of adequate stress displacement relations must be based on the results of Aggregate Interlock tests which show that considerable amounts of shear can be transferred across a crack provided that a sufficient confinement action is exerted by the reinforcement crossing the cracks or by external restraints (Fig. 1).

Various stress-displacement relations have been formulated, even very recently: some relations are incremental (see for instance Fardis and Buyukozturk /5/) and some are of total stress-total displacement type (Bažant and Gambarova /1/, Walraven and Reinhardt /3/), some are based on mechanical micromodels /3, 5/, some are empirical (Paulay and Loeber /2/, Houde and Mirza /6/, White, Gergely and others /7, 8/, Mattock /9/ and some are of mixed formulation /1/, being partly based on speculative properties of the cracks and partly on test data.

With reference to the experimental results, most of the tests carried out sofar refer to precracked concrete specimens with reinforcing bars (or stirrups) passing through the crack plane (Fig. 2a, b) /3, 7, 8, 9/. Some tests refer to precracked specimens tested at constant crack opening, Fig. 2c (/2, 6/ and recently Daschner and Kupfer /4, 10/): typical response curves are shown in Fig. 3a. In a very limited number of tests the confinement has been kept constant throughout each test (Fig. 2d) /4, 10/: typical response curves are plotted in Fig. 3b. These last tests throw new light on crack behaviour and make it possible to give a better formulation to the relation between the confinement stress (i.e. the interface normal stress σ_{nn}^{c}) and the crack displacements δ_n and δ_t. This is chosen as the objective of this work, although a revision may be expected as soon as further test data at constant confinement become available.

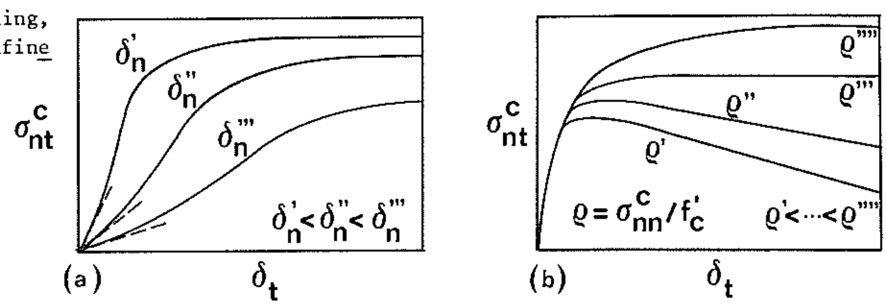


▲Fig.1 - Crack morphology.

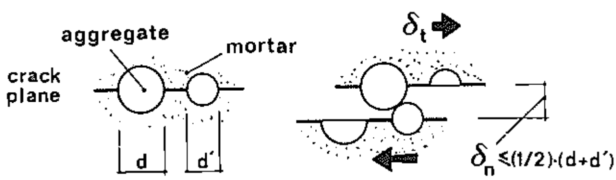


▶Fig.2 - Typical experimental set-ups : (a) confinement by external bars; (b) confinement by embedded bars; (c) test at constant crack opening; (d) test at constant confinement.

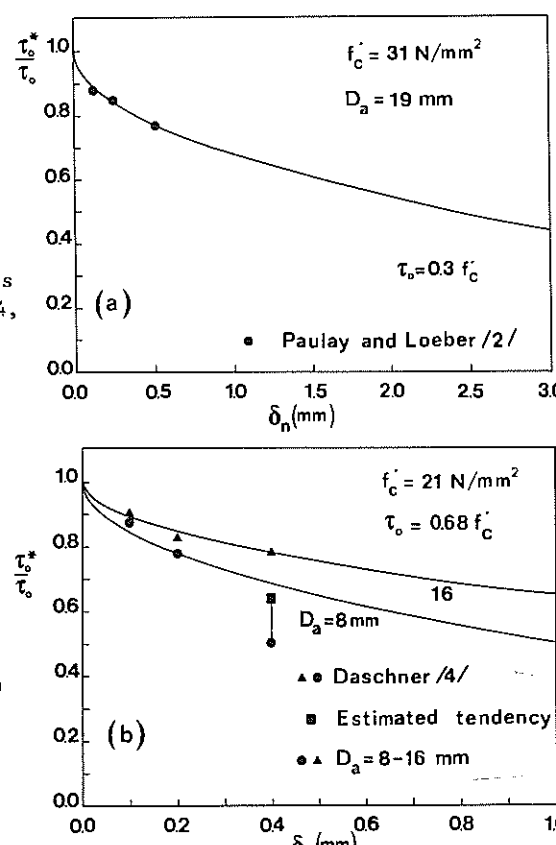
▶Fig.3 - Response curves at (a) constant crack opening, and (b) at constant confinement.

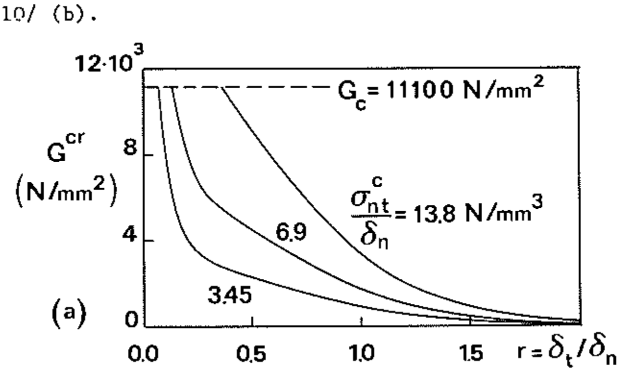


▼Fig.4 - Simplified structure of the crack plane. d = aggregate diameter.



▶Fig.5 - Fitting of Paulay and Loeber'test results /2/ (a), and of Daschner and Kupfer'test results /4, 10/ (b).





▲Fig.6 - Curves of the tangent shear modulus G^{CR} of cracked concrete : (a) curves at constant crack dilatancy.
f'_c = 31 N/mm^2, τ_o = 0.30 f'_c, p = 200 mm, D_a = 19 mm.

Continuous reference will be made to the so-called Rough Crack Model presented in /1/ (see also /11, 12/): within this model the confinement-displacement relation is based on a very limited number of tests at constant crack opening. A secondary objective of this work is the analysis of the role of the maximum aggregate size D_a, which certainly affects both the strength and the stiffness of the Aggregate Interlock Mechanism: although of relatively limited importance, the role of aggregate size will be discussed first, because of its effects on the confinement action.

1. EFFECTS OF MAXIMUM AGGREGATE SIZE ON CRACK BEHAVIOUR

Simple micromechanical models /1/ and many test results at constant crack opening definitively show that the interface shear stress σ^c_{nt} is basically related to the ratio $r = \delta_t/\delta_n$ (plane stresses only are taken into consideration here). That is, the relation between the stress σ^c_{nt} and the ratio r has been formulated as follows, within the Rough Crack Model /1/:

$$\sigma^c_{nt} = \tau^*_o \, r \, \frac{a_3 + a_4 |r|^3}{1 + a_4 r^4} \qquad (1)$$

where the coefficients a_3 and a_4 are a function of the standard cylindrical strength f'_c /1/. The parameter τ^*_o represents the asymptotic value of σ^c_{nt}, or in other words the ultimate shear strength of a crack when r tends to infinite. With reference to eq. (1) it should be noted that this equation agrees also with some basic properties of the cracks ($\partial\sigma^c_{nt}/\partial\delta_t > 0$ always, $\partial|\sigma^c_{nt}|/\partial\delta_n < 0$ always). The asymptote τ^*_o should reasonably depend on the crack opening δ_n: as a matter of fact, should crack opening increase beyond the size of aggregate particles, the contact at the interface of a crack would vanish.

Let us assume that the interface asperities have the same grading as the aggregates (which is a first approximation because the cracks form at the interface of the largest aggregate particles and propagate through the cement paste, thus avoiding the smallest aggregate particles: as a consequence the cracks are somewhat smoother than required by the aggregate grading) and let us simplify the aggregate particles (protruding from each face) to spheres (as in /3/), having diameter d, and embedded in the matrix of cement paste (Fig. 4)

Due to the random distribution of the aggregate particles, the probability for the local contact to be between two particles with different diameters (d > d') is higher than the probability for the contact to be between two particles with the same diameter (d = d'). As a consequence, the local contact is possible only if $\delta_n \leqslant (1/2)(d+d')$, which is always verified if the more strict condition $\delta_n \leqslant (1/2)d$ is complied with. (2)

As a general rule, the more the asperities which comply with condition (2), the larger the contact area at the crack interface, and the greater the crack shear strength τ^*_o.

Summing up, the larger the percent p of crack asperities satisfying the condition (2), the greater the crack shear strength τ_o^*. Let us assume that aggregate distribution matches the Fuller grading curve. In this case the percentage p_i of asperities having diameter smaller than a given value d_i is expressed by the well known relation: $p_i = 100\sqrt{d_i/D_a}$ (3)

As a consequence the percentage p_n of the asperities belonging to the crack faces and no longer in mutual contact ($d<2\delta_n$) is represented by $p_n = 100\sqrt{2\delta_n/D_a}$ (3')

and the percentage p' of the asperities still in mutual contact ($d\geq 2\delta_n$) is:

$$p' = 100-p_n = 100\,(1-\sqrt{2\delta_n/D_a}) \qquad (3")$$

and the value τ_o^* may be defined as follows:

$$\tau_o^* = \frac{p'\tau_o}{100} = \tau_o(1-\sqrt{2\delta_n/D_a}) \qquad (4)$$

where τ_o represents the crack shear strength in the limit case of zero crack opening($\delta_n=0$): the value of τ_o is (0.25-0.30) f'_c according to Paulay and Loeber' test results /2/, but may be as large as (0.6-0.7) f'_c according to other authors /4/. In Fig. 5a the theoretical curve of τ_o^*/τ_o is plotted versus crack opening δ_n, with the same concrete characteristics as in /2/: the agreement with test results is quite acceptable, although limited to relatively small values of δ_n, because of insufficient test data. In Fig. 5b some of the test data shown in /4, 10/ are compared with the theoretical curve: the agreement is acceptable for D_a = 16 mm, while for D_a = 8 mm the theoretical results show too stiff a behaviour. Obviously equation (4) is somewhat crude, because the random distribution of the asperities is far from adequately introduced, but the benefit of eq. (4) lays in its simplicity.

In Fig. 6 the curves of the tangent shear modulus $G^{CR} = \partial\sigma_{nt}^c/\partial\gamma_{nt}^{CR}$ are plotted versus the ratio r of the crack displacements, for different values of crack dilatancy (δ_n/σ_{nt}^c, Fig. 6a), aggregate size(D_a, Fig. 6b), and crack opening (δ_n, Fig. 6c). Now introduce eq.(4) into eq.(1):

$$\sigma_{nt}^c = \tau_o(1-\sqrt{\frac{2p}{D_a}\varepsilon_{nn}^{CR}})\; r\,\frac{a_3+a_4|r|^3}{1+a_4 r^4} \qquad (5)$$

where $r = \gamma_{nt}^{CR}/\varepsilon_{nn}^{CR}$, with $\gamma_{nt}^{CR} = \delta_t/p$ and $\varepsilon_{nn}^{CR} = \delta_n/p$ (p is the crack spacing, the suffix CR means "cracked concrete"). The expression of the tangent shear modulus G^{CR} can be immediately obtained by derivation of eq. (5):

$$G^{CR} = \frac{\tau_o}{\varepsilon_{nn}^{CR}}\,(1-\sqrt{2(p/D_a)\varepsilon_{nn}^{CR}})\,\frac{a_3+4a_4|r|^3-3a_3a_4r^4}{(1+a_4r^4)^2} \quad \text{or} \quad G^{CR} = \frac{\sigma_{nt}^c}{\varepsilon_{nn}^{CR}}\,\frac{a_3+4a_4|r|^3-3a_3a_4r^4}{r(1+a_4r^4)(a_3+a_4|r|^3)} \qquad (6',6")$$

In eqs. (6') and (6") the strains ε_{nn}^{CR} and γ_{nt}^{CR} may be replaced by ε_{nn} and γ_{nt} provided that the strains in the solid concrete between the cracks are neglected (which is almost invariably the case). The curves of Fig. 6a show a marked decreasing in the modulus G^{CR} at constant crack dilatancy (crack opening linearly increases at increasing shear). The limited but not negligible role of aggregate size is shown in Fig. 6b, where crack opening is imposed. Of course the effects of crack opening are remarkable (Fig. 6c): should crack opening be small (δ_n =

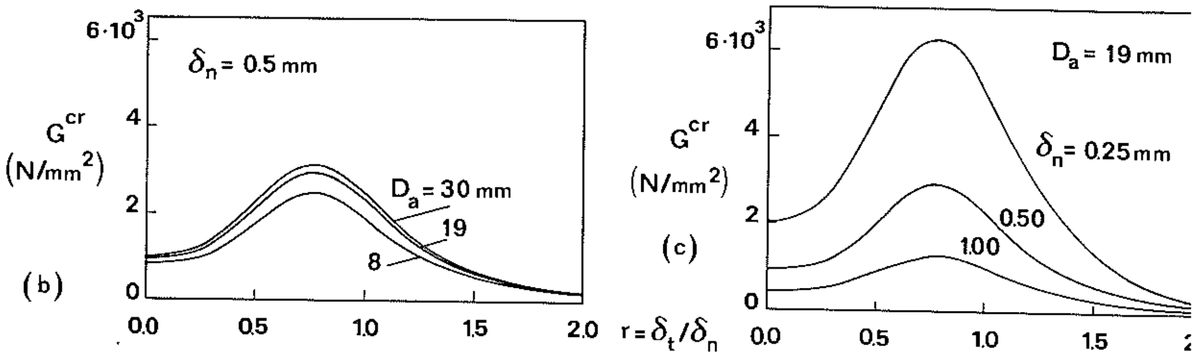


Fig.6 - Curves of the tangent shear modulus G^{CR} of cracked concrete : (b) effects of the aggrega size; (c) effects of the crack opening.
f'_c= 31 N/mm^2, τ_o= 0.30 f'_c, p = 200 mm, D_a = 19 mm .

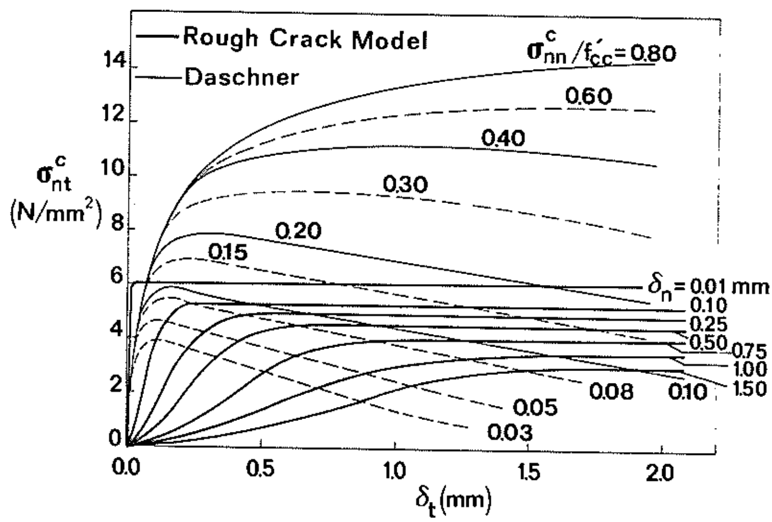


Fig.7 - Curves at constant crack opening (Rough Crack Model /1/,eq.(5)), and at constant confinement (Daschner /4/).
f'_{cc} = 25 N/mm^2, f'_c= 21 N/mm^2, D_a = 8 mm, τ_o = 0.30 f'_c .

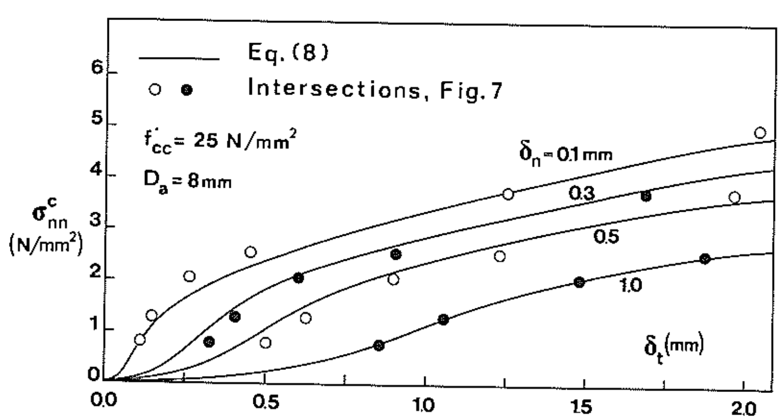


Fig.8 - Fitting of the points representing the intersections of the two families of curves shown in Fig.7 .
f'_{cc}= 25 N/mm^2, f'_c= 21 N/mm^2, D_a= 8 mm, τ_o = 0.30 f'_c .

= 0.25 mm) and should crack spacing be - say - 10 times larger than the aggregate size (p = 200 mm), the tangent shear modulus G^{CR} would be as large as 50% of the elastic modulus.

2. CRACK BEHAVIOUR AT CONSTANT OPENING AND AT CONSTANT CONFINEMENT

The few tests carried out at constant confinement show that in this case crack behaviour is very different from the behaviour observed in the tests at constant crack opening. In the latter case the response curves $\sigma^c_{nt}(\delta_t,\bar{\delta}_n)$-Fig. 3a are characterized by a well defined plastic plateau, which comes after an ascending branch strongly affected by the value of the crack opening. In the former case the response curves $\sigma^c_{nt}(\delta_t,\bar{\sigma}^c_{nn})$-Figs. 3b and 7- show a very steep ascending branch, practically unaffected by the amount of confinement, and a subsequent softening branch, which is well defined for low confinement values (σ^c_{nn}/f'_{cc} = 0.1 - 0.2), becomes close to a plastic plateau for moderate values (σ^c_{nn}/f'_{cc}=0.4) and is replaced by a moderately increasing branch for very large values (σ^c_{nn}/f'_{cc}=0.8). Obviously very large confinement values are of no interest here, because in an actual r.c. element the confinement exerted by the reinforcement can never be very large due to the limited amount of steel and to the yielding threshold. Of course large confinement values may be induced by specific restraints or by particular load situations, but in no case value as large as those experienced in /4/ are likely to occur.

A possible explanation as to the difference between the two limit behaviours of a crack is that at constant opening the smallest and stiffest asperities never contribute to shear transmission (because of the non-zero initial opening) so that the initial stiffness is relatively low; at large slip values the shear response tends to become uniform because of the engagement of the largest and somewhat more deformable asperities, and because of micro cracking. At constant confinement the initial stiffness is very high because contact takes place all over the interface (due to the non-zero initial confinement), but the crack tendency to open at increasing slip values together with interface deterioration and micro-cracking causes a sudden loss of stiffness and gives way to a softening behaviour.

By cross-examining the curves at constant crack opening (as obtained from eq.(5)) and those at constant confinement (shown in /4/) a set of points (at the intersections of the two sets of curves) may be obtained where the four parameters σ^c_{nt}, σ^c_{nn}, δ_t, δ_n are known (Fig. 7). These points can be represented in the plane (σ^c_{nn}, δ_t) - Fig. 8 -, and through a suitable fitting at constant crack opening a relation between σ^c_{nn} and δ_t, δ_n can be worked out.

3. CONSTITUTIVE LAWS FOR THE CONFINEMENT STRESS

Within the Rough Crack Model /1/ the following relation between the confinement stress and the crack displacements has been proposed:

$$\sigma^c_{nn} = -\frac{a_1}{\delta_n} (a_2 |\sigma^c_{nt}|)^q \qquad (7)$$

where a_1, a_2 are constants and q depends on crack opening. According to eq. (7) the confinement stress is both a linear function of the interface shear ($q = 1-1.30$) and of the inverse of crack opening: the latter property is a reasonable interpretation of some limited experimental results /2/, which are not very clear. A drawback of eq. (7) is that the confinement stress required at small values of the crack opening tends to be too large.

A more realistic formulation of the relation $\sigma^c_{nn}(\delta_t, \delta_n)$ may obtained through the approach mentioned in Sec.2. The points common to the curves at constant crack opening and at constant confinement (Fig. 7 and 8) are adequately fitted by the following equation (Fig. 8, full lines):

$$\sigma^c_{nn} = -a_1 a_2 \frac{\delta_t \sigma^c_{nt}}{(\delta_n^2 + \delta_t^2)^q} \qquad (8)$$

with $a_1 \cdot a_2 = 0.62$, $q = 0.25$.

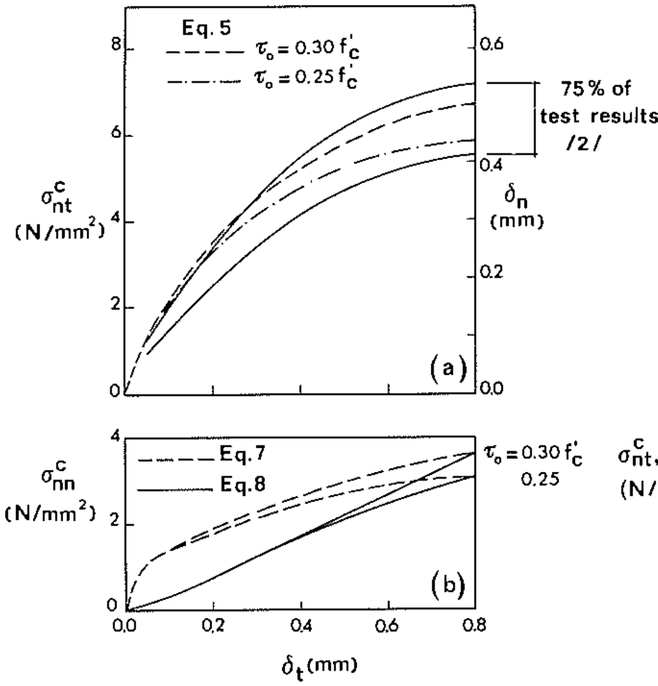
Note that in eq. (8) the product $\delta_t \cdot \sigma^c_{nt}$ is always positive, because δ_t and σ^c_{nt} have always the same sign, for obvious physical reasons.

With the new formulation - eq. (8) - the confinement required to maintain the crack at a given opening is less sensitive to the value of the opening, and goes down to zero much more rapidly with δ_t than required by the old formulation - eq. (7). This tendency is confirmed by many experimental results which show that a very limited amount of confinement is required at small values of δ_t. For δ_t tending to infinite, eq. (8) does not provide any asymptote for σ^c_{nn}, although the gradient tends to become very small (Fig. 8). Eq. (8) satisfies also the following conditions: $\partial|\sigma^c_{nn}|/\partial\delta_n < 0$ always, $\partial\sigma^c_{nn}/\partial\delta_t \lessgtr 0$ for $\delta_t \gtrless 0$. These conditions are based on the fact that any positive increment of the crack opening (which is always positive) decreases the number of contact points at the crack interface (and the same is true for the absolute values of the stresses), while any increment of the slip increases the number of contact points only if the increment and the total slip have the same sign.

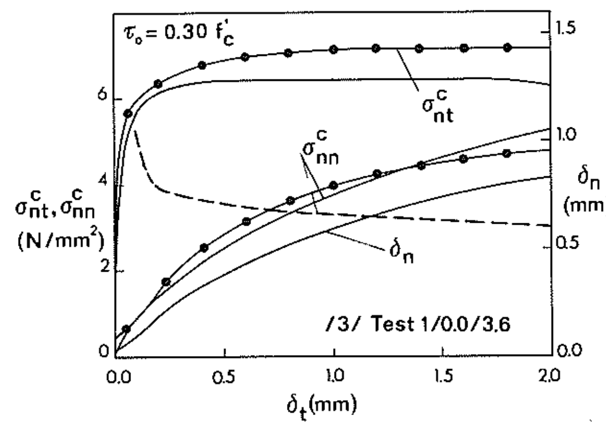
Eqs.(5) and (8) have been tested in several cases, with the same input data as in the tests carried out by Paulay and Loeber /2/ and by Walraven and Reinhardt /3/ respectively.

Fig. 9 shows the experimental and theoretical results for a test at constant crack dilatancy: $\delta_n/\sigma^c_{nt} = 0.074\ mm^3/N$. Two values of τ_o are introduced, and in both cases the theoretical curves of σ^c_{nt} (Fig. 9a) and the two experimental curves (bounding 75% of the test results) show a remarkable agreement. In Fig. 9b the theoretical curves of σ^c_{nn} are plotted according to the previous formulation - eq. (7) - and to the new formulation - eq. (8): no test data are available in this case.

Fig. 10 to 13 refer to 5 of the 7 tests run by Walraven and Reinhardt /3/ (the experimental curves are identified by solid circles): the curves of the crack opening provided the input data for the theoretical evaluation of both σ^c_{nt} and σ^c_{nn}. The Test 1/0.0/3.6 (Fig. 10)

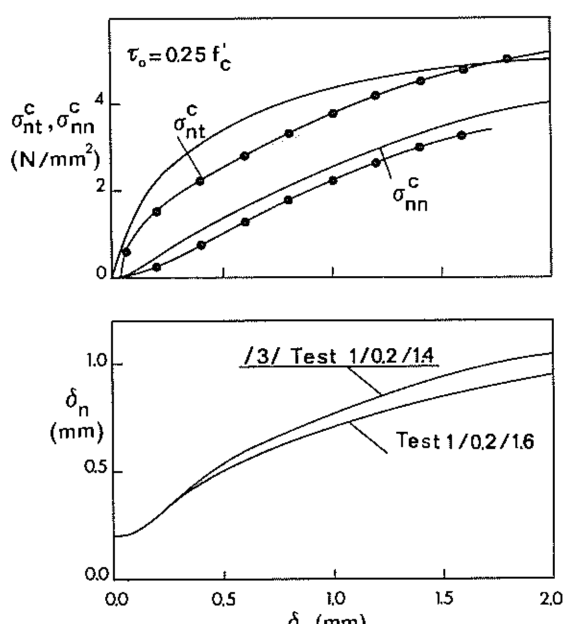


◀ Fig.9 - Test at constant crack dilatancy (δ_n/σ^c_{nt} = 0.074 mm³/N) : (a) fitting of Paulay and Loeber test results /2/; (b) curves of the confinement stress according to the old formulation (eq.(7)) and to the new formulation (eq.(8)). f'_c = 31 N/mm², D_a = 19 mm .

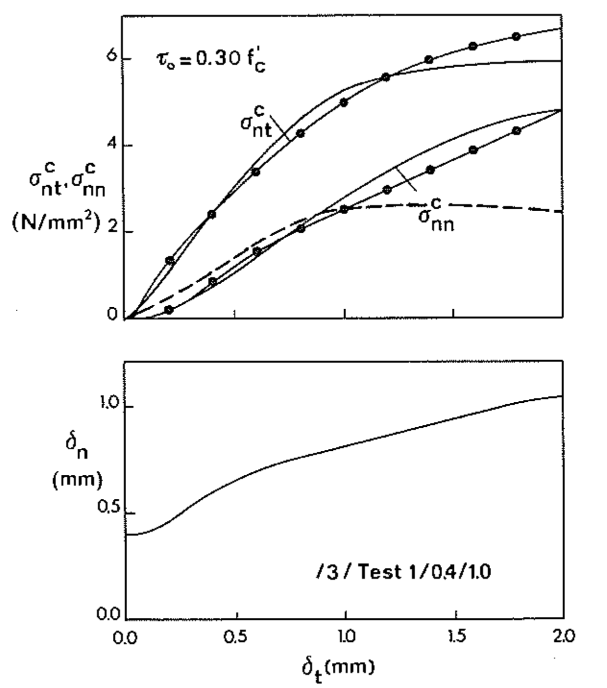


▲ Fig.10 - Test 1/0.0/3.6 : the initial crack opening is (practically) zero. ---- Eq.(7)

▼, ▶▼ Fig.10,11,12,13 - Fitting of Walraven and Reinhardt' test results /3/ : the confinement is provided by external bars. Solid circles identify the experimental curves. f'_c = 31 N/mm², D_a = 16 mm



▲ Fig.11 - Tests 1/0.2/1.4 and 1/0.2/1.6 : the initial crack opening is 0.2 mm. The experimental response curves are the average of the actual curves (very close) shown in /3/.



▲ Fig.12 - Test 1/0.4/1.0 : the initial crack opening is 0.4 mm .
---- Eq.(7)

has no initial crack opening, while the other tests (Fig. 11,12,13) have non zero initial crack opening ranging from 0.2 to 0.4 mm.

The best theoretical/experimental agreement has been obtained in some cases with τ_o=0.30 f'_c and in some others with τ_o=0.25 f'_c: this fact must not be overemphasized: due to the limited number of tests /3/ with full details on stresses and displacements, a statistical analysis of the results is hardly feasible and differences of 10-20% on τ_o may be due just to the scattering of the results.

For the tests 1/0.0/3.6 (Fig. 10),1/0.2/1.4-1.6 (Fig.11),1/0.4/1.0 (Fig. 12) the theoretical curves of the confinement are astonishingly close to the experimental curves (which confirms the soundess of eq. (8)); also the theoretical/experimental agreement for σ^c_{nt} is quite acceptable. With reference to the test 1/0.2/0.4 the general trend of the theoretical curves is acceptable, but the calculated response is always stiffer than the measured response. Probably something went wrong during the test(see the discontinuity in the $\sigma^c_{nt}(\delta_t)$curve), as suggested also by the very soft response compared (for instance) to the test 1/0.2/1.4 (Fig. 11): the displacement histories are only marginally different.

Concluding Remarks

The analytical description of the strength and stiffness characteristics of cracked concrete requires the knowledge of reliable relations among the stresses transferred across the cracks and the strains corresponding to the interface displacements.

Starting from the response curves obtained so far by different authors (at constant crack opening and at constant confinement), and within the already well known ROUGH CRACK MODEL, a new formulation is worked out for the relation between the confinement stress and the crack displacements, and some improvements are introduced into the shear-displacement relation. Although obtained through a cross examination of very different theoretical and experimental results, the new formulation of the confinement leads to a good fitting of the experimental curves at variable crack opening (as those obtained in Aggregate Interlock Tests with the confinement provided by external, unbonded stirrups or bars). Furthermore, the improved formulation of the interface shear stress is based on a more correct introduction of the maximum aggregate size and leads to a more realistic evaluation of the tangent shear modulus of cracked concrete. This is very important in the step-by-step updating of concrete stiffness as required by Finite Element Analysis.

ACKNOWLEDGEMENTS

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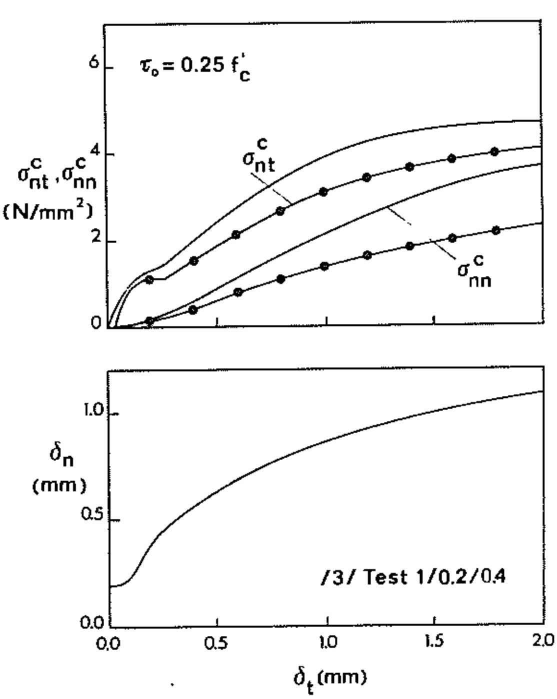

Fig.13 - Test 1/0.2/0.4 : the initial crack opening is 0.2 mm .