Leakage of Pressurized Gases Through Unlined Concrete Containment Structures

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ABSTRACT

Eight reinforced concrete specimens were fabricated and subjected to tensile membrane forces and air pressure to study the air leakage characteristics in cracked reinforced concrete members. A mathematical expression for the rate of pressurized air flowing through an idealized crack is presented. The mathematical expression is refined by using the experimental data to describe the air flow rate through any given crack pattern. Graphical charts are also presented for the calculation of the air leakage rate through concrete cracks. The concept of equivalent crack width for a given crack pattern is introduced. The mathematical expression and graphical charts are modified to include this equivalent crack width concept. The proposed technique is applicable for the prediction of the leakage from concrete containment structures or any similar structures due to high internal pressure sufficient to initiate cracking.

1. Introduction

When concrete containment structures are subjected to high internal pressure, the membrane stresses induced may exceed the tensile strength of concrete, producing cracks passing through the thickness of the concrete and resulting in leakage from the containment. This paper provides a dependable technique to predict the rate of air leakage as related to any given crack pattern. The proposed technique is applicable to the prediction of the leakage from concrete containment structures or any similar structures due to high internal pressure.

The first attempt to examine the leakage of air through concrete cracks was undertaken by W. Buss (1). He examined the rate of flow of air through a pre-existing crack in a slab under air pressure. Later, researchers at the University of Alberta (6) initiated an experimental program by building prestressed concrete segments loaded uniaxially and biaxially to simulate the loading condition of containment structures under internal pressure. Neither of the above studies was in a general form to predict air leakage through any given crack pattern in a reinforced concrete structure. The first dealt with a pre-existing crack pattern, while the later study was based on a single specimen under a limited range of internal pressures. Thus, the data were insufficient for the development of a generalized predictive technique.

The main objective of this paper is to explore a general expression which could predict the rate of flow of pressurized air through cracked concrete. The results of the experimental program were used to refine and calibrate the proposed mathematical expression, and test its predictive ability.

2. Mathematical Formulation of Leakage Rate

In general, the geometric configuration of a particular crack extending through a cracked concrete section is extremely complex. It is almost impossible to model accurately the crack path. Since each crack is unique, modelling a particular crack would be of limited application. However, for pure membrane loads, the width of any given crack should be reasonably uniform through the concrete thickness. Thus, the flow through a concrete crack may be idealized as a flow through a gap between two parallel plates as shown in Fig. 1. Applying the momentum-balance theorem, which states that the summation of forces should equal the change of momentum (2):

$$PA - (P+dP)A - 2\tau_o B dx = \rho AV(V+dV) - \rho AV(V)$$

or

$$dP + \rho V dV + 2\tau_o \frac{B}{A} dx = 0 \qquad \text{eq. (1)}$$

where

A = BW, ft^2. B = extent of crack, ft.

W = gap between two parallel plates (crack width), ft.

P = absolute air pressure at any section, lb/ft^2.

V = velocity of air at any section, $lb./ft^2$.

τ_o = shear stress due to wall friction, $lb./ft.^2$

ρ = mass air density, $slug/ft^3$.

The mass flow rate, $\dot{m}$, and the friction coefficient factor, f, are assumed to be constant along the flow path. The air flow is also assumed to be isothermal. Thus, in a steady and uniform flow in a conduit of constant cross-section, shear stress can be expressed in terms of velocity (V), mass air density (ρ), and dimensionless friction coefficient factor (f) as follows:

$$\tau_o = \frac{f}{4} \frac{\rho V^2}{2} \qquad \text{eq. (2)}$$

Using this expression and integrating Eq. 1 along the length of the crack in the direction of flow, L, gives:

$$P_1^2 - P_2^2 = \frac{fL}{2W} \frac{P_2^2}{RT} \frac{q_2^2}{B^2W^2} \{\frac{4W}{fL} \ln (\frac{P_1}{P_2}) + 1\} \qquad \text{eq. (3)}$$

where subscripts 1 and 2 represent conditions at the beginning and at the end of the crack respectively. R and T are the gas constant and the absolute temperature respectively. q_2 is the rate of flow of air at the end of crack. The first term of Eq. 3 in the bracket is very small compared to unity and can be neglected. Hence

$$P_1^2 - P_2^2 = \frac{fL}{2W} \frac{P_2^2}{RT} \frac{q_2^2}{B^2W^2} \qquad \text{eq. (4)}$$

The dimensionless friction coefficient factor, f, depends on the Reynolds number, Re, and the wall roughness, k, and it can be expressed in general for both laminar and turbulent flow as

$$f = (\frac{k}{Re})^n \qquad \text{eq. (5)}$$

where n is the flow coefficient, which can be determined experimentally.

By introducing,

$$Re = \frac{\rho_2 V_2 D}{\mu_2} \qquad \text{eq. (6)}$$

$$f = \frac{k^n \mu_2^n}{(\rho_2 V_2 2W)^n} \qquad \text{eq. (7)}$$

Substituting Eq. 7 into Eq. 4 and simplifying yields

$$\frac{P_1^2 - P_2^2}{L} = (\frac{k^n}{2}) (\frac{\mu}{2})^n (RT)^{n-1} \left|\frac{P_2 q_2}{B}\right|^{2-n} \frac{1}{W^3} \qquad \text{eq. (8)}$$

The total flow rate, Q_2, through a wall having j cracks can be expressed as the sum of the flows through the individual cracks or

$$\frac{P_1^2 - P_2^2}{L} = (\frac{k^n}{2}) (\frac{\mu}{2})^n (RT)^{n-1} \left|\frac{P_2 Q_2}{B}\right|^{2-n} \frac{1}{\sum_{i=1}^{j} W_i^3} \qquad \text{eq. (9)}$$

Eq. 9 can be expressed as

$$P'' = C\left|\frac{P_2 Q_2}{B}\right|^m \qquad \text{eq.(10)}$$

where $$P'' = \frac{P_1^2 - P_2^2}{L}, \text{ "modified pressure gradient"}$$

$$C = \frac{\left(\frac{k}{2}\right)^n \left(\frac{\mu}{2}\right)^n (RT)^{n-1}}{\sum_{i=1}^{j} W_i^3}, \text{ "a constant"}$$

$$m = 2-n$$

$$\frac{P_2 Q_2}{B} = \text{"rate of flow per unit extent of crack"}$$

It should be noted that in the above derivation W_i is the width of the i^{th} crack which must be determined from measurements. The width of other cracks, i.e., i+1 crack, may be different.

3. Test Specimens

In containments and similar structures subjected to internal pressure, the magnitude of the membrane stresses, and hence the crack width, is a function of the internal pressure. However, in leakage tests, the tensile forces applied to the specimen are independent of the applied air pressure, which permits an evaluation of the leakage rate for different pressure levels. Moreover, planar specimens were used to simplify the fabrication, testing, and interpretation of the results.

All specimens were rectangular in shape and reinforced in two directions with deformed bars. Transverse reinforcement was provided by M10 bars spaced at 3 inches (76 mm) centre-to-centre on both faces. Longitudinal reinforcement was also spaced at 3 inches (76 mm) centre-to-centre with concrete cover of 1/2-inch (12 mm). The three quantities varied from one specimen to another were: longitudinal bar size, concrete thickness and steel reinforcement ratio. The overall dimensions and reinforcements of the specimens are shown in Table 1.

Two air chambers were provided one on each face of the specimen. The upstream chamber was filled with pressurized air and the downstream chamber was used to collect air that had leaked through the cracks of the specimen. To provide air-proofing along the edges of the specimen, a 1/2-inch (12 mm) reinforced rubber liner was cast around the edges of the specimen. The shape of the rubber liner was specially cast so that a 3/8-inch (10 mm) steel plate could be bolted to the rubber liner at the upstream face to form a chamber. Likewise, the downstream chamber was formed by bolting a 1/2-inch (12 mm) plexiglass plate on to the rubber liner. Details of the air chambers and rubber liner are shown in Fig. 2. The rubber was bonded to the concrete surface by using steel wires which embedded into the concrete (Fig. 3). More detailed information is given in Ref. 5. To further reinforce the rubber liner during testing, four angles were provided at each corner and held together by six long bolts on each side. These can be seen in Fig. 4(a) and 4(b), which show the specimen ready for testing.

4. Test Procedure.

When a concrete containment structure is subjected to high internal pressure, the membrane stresses induced may exceed the cracking strength of concrete, resulting in cracks passing through the wall of the containment. In the testing program, the situation is simulated by applying the tensile membrane forces to the longitudinal reinforcing bars which protrude along the ends of the specimen and providing pressurized air to the upstream chamber of the specimen.

Before performing the test, the specimen was aligned vertically and the load cells were adjusted to provide equal distribution of load on each longitudinal reinforcement. The load was increased in increments of 5 or 10 kips (22 kN or 44 kN) as was deemed appropriate to produce a sufficient quantity of data before yielding of the reinforcement occurred. At each load increment, the load was held constant and air pressure was applied on the upstream chamber while the downstream chamber was kept at atmospheric. The air pressure in the upstream chamber was controlled by a pressure regulator. A pressure tranducer attached to the upstream chamber was connected to the Data Acquisition System to automatically record the pressure in the chamber.

Once leakage was detected, it was obvious that through-the-concrete cracks had been formed. The air pressure was increased in increments of 3 or 5 psi (22 kPa or 35 kPa). The amount of air flowing through cracks in a 30-second period was measured by two dry test meters connected in series. Two readings were taken for each increment of air pressure to derive an average value of rate of flow for each increment. The maximum air pressure applied was 30 psi (207 kPa).

The cracking patterns were marked and numbered on the surface of the plexiglass at the downstream chamber. Crack widths were measured by using a movable microscope along the centre line of the specimen. Widths of all subsequent cracks were measured through the plexiglass for each load increment.

This procedure was repeated for each load increment until the load applied approached the predicted yield load of the longitudinal reinforcing bars. Testing was terminated when the reinforcing bars reached yielding point to avoid damage to the load cells which were reused in the other test.

5. Test Results.

Eq. 10 implies a linear relationship between the "modified pressure gradient", P", and the "rate of flow of air per unit extent of crack" using a log-log scale, as is clear in the following expression.

$$\text{Log } P'' = \log C + m \log \left| \frac{P_2 Q_2}{B} \right| \qquad \text{eq. (11)}$$

Based on the experimental results, a graphical representation of the relationship for a typical leakage specimen (L4) is shown in Fig. 5. The test results for the other specimens are given in detail in Ref. 5. The slope of each regression line represents the exponent value of m from which the turbulent coefficient, n, can be determined.

The relationship between the values of k and $\Sigma W_i^{\,3}$ for all the leakage specimens, was plotted and shown in Fig. 6. The relation between k and $\Sigma W_i^{\,3}$ was found to have an exponential form:

$$k = 2.907 \times 10^7 \, (\Sigma W_i^{\,3})^{0.428} \qquad \text{eq. (12)}$$

Likewise, the relationship between the flow coefficient, n, and $\Sigma W_i^{\,3}$ is shown in Fig. 7 and was found to be

$$n = \frac{0.133}{(\Sigma W_i^{\,3})^{0.081}} \qquad \text{eq. (13)}$$

Thus, for a given crack pattern, the wall roughness, (k) and flow coefficient, (n), can be found by Eq. 12 and Eq. 13 respectively and the constant (C) can be calculated. With the known value of the "modified pressure gradient", (P"), and the "constant", (C), the "rate of flow of air per unit extent of crack" can be found, from which the rate of flow of air at the end of cracks can be found subsequently.

Based on the derived formulae, the solution is presented in graphical form as shown in the chart in Fig. 8. The reliability of these equations can be examined by comparing the rate of flow calculated using Eq. 10, with the measured values obtained from the leakage tests which are shown in Fig. 9. A linear regression analysis was performed on the test data and the results are represented by a solid line with a linear regression coefficient of 1.03. The scattering of the data results for low range flow rates can be attributed to the insensitivity of the instrument used to measure flow rate and crack width.

6. Equivalent Crack Width Concept

To eliminate the necessity of considering the width of each individual crack, the concept of equivalent crack width was introduced. This technique permits replacing ΣW_i^3 by an equivalent crack width for a given crack pattern with number of cracks, N, as follows

$$\sum_{i=1}^{j} W_i^3 = 1.42 \; N \; W_{ave}^3 \qquad \text{eq. (14)}$$

where 1.42 is the average value of all the linear regression coefficients between ΣW_i^3 and N W_{ave}^3 for each specimen as shown in Fig. 10 for a typical leakage specimen (L4). Thus Eq. 10 can be modified as follows:

$$P" = C_1 \left| \frac{P_2 Q_2}{B} \right|^m \qquad \text{eq. (15)}$$

where

$$C_1 = \frac{\left(\frac{k}{2}\right)^n \left(\frac{\mu}{2}\right)^n (RT)^{n-1}}{1.42 \; N \; W_{ave}^3}$$

Following the same arguments as before, the flow coefficient, n, and the wall roughness coefficient, k, can be related to the parameter N W_{ave}^3 from the test data. Based on the experimental results of all tests, these relationships were found from Fig. 11 and 12 to be

$$n = \frac{0.195}{(N \; W_{ave}^3)^{0.063}} \qquad \text{eq. (16)}$$

$$k = 8.702 \times 10^6 \; (N \; W_{ave}^3)^{0.367} \qquad \text{eq. (17)}$$

Thus, if the value of the average crack width and average crack spacing can be predicted, the flow coefficient, n, and wall roughness coefficient, k, can be determined by using Eq. 16 and Eq. 17, respectively. A methodology for predicting the average crack width and average crack spacing is given in Ref. 4.

The calculated rate of flow using Eq. 15 is compared with the measured values for all the leakage specimens, as shown in Fig. 13. The perfect relationship between the calculated and the measured rate of flow is presented by the dotted line in this figure. A linear regression analysis was performed on the test data and the results are presented by the solid line with a linear regression coefficient of 0.893. The discrepancy between the mean value and the perfect relationship is greater than the one presented in Fig. 9. This discrepancy can properly be attributed to the fact that the formation of additional cracks would lower the value of the average crack width which would reduce the calculated rate of flow in comparison with the measured values.

7. Prediction of Leakage Through Concrete Containment Structures

Following the technique proposed by researchers in Alberta (3), if the surface of the containment can be divided into zones of equal surface strain, it is possible to predict the number and width of cracks in each zone. The number and extent of cracks in each zone can be determined from the spacing of cracks and dimensions of the structure respectively. The crack length is taken as the thickness of the containment wall. Thus, Eq. 15 can be used to predict the leakage rate in each zone for any large internal pressures. The rates for each zone are then added to obtain the total leakage rate for the containment structure.

8. Conclusions

Two independent methods were introduced to predict the air leakage through cracked concrete. The mathematical formulation of the first method is designed to predict the air leakage through cracked concrete with known width of each individual crack in any given crack pattern. The concept of equivalent crack width for a given crack pattern is introduced in the second method. This concept removes the requirement of knowing every individual crack width of a given crack pattern, by introducing an equivalent crack width, provided that the number of cracks is known. Both methods were compared with the measured values and were found to be in good agreement.

To verify the reliability of the proposed techniques and to take into account the effect of prestressing and biaxial loading conditions, further independent testing should be undertaken.

9. Acknowledgements

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TABLE 1 SPECIMEN PROPERTIES

Concrete Cover in.	Steel Ratio	Thickness in	Width in.	Long. Reinf. Bar Size	Specimen Mark	
0.5		5	12	#3	L1	Remark:
	0.0147	7	12	M10	L4	1. All specimens had transverse reinforcement M10 @ 3 in.
		10	11	#4	L7	
		7	11	#4	L5	2. The height of all specimens is 30 in.
	0.0207	10	12	M15	L8	
		5	11	#4	L3	
	0.0294	7	12	M15	L6	
		10	12	M20	L9	

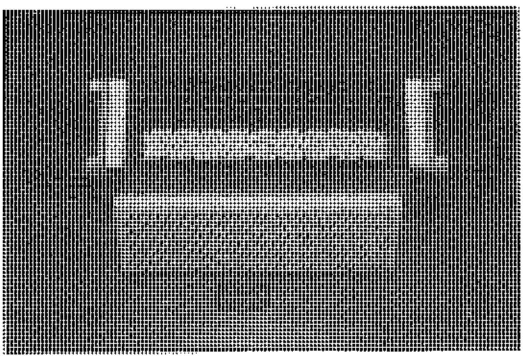

Fig. 3 Typical leakage specimen prior to casting of concrete.

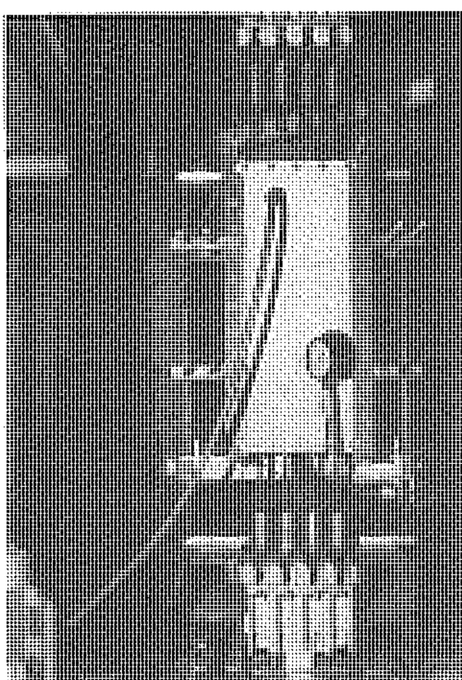

Fig. 4(a) Upstream face of testing specimen.

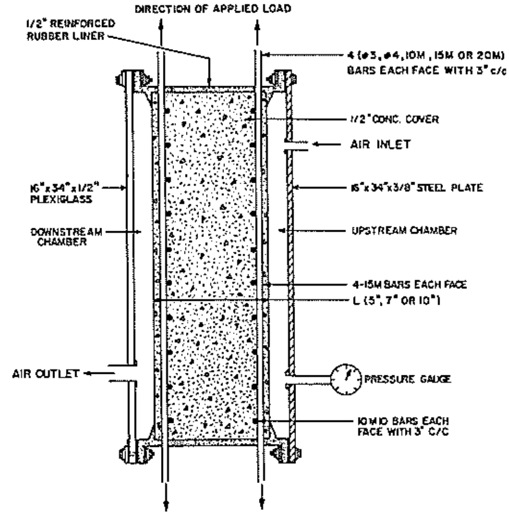

Fig. 2 Details of air chambers and rubber liners of leakage specimen.

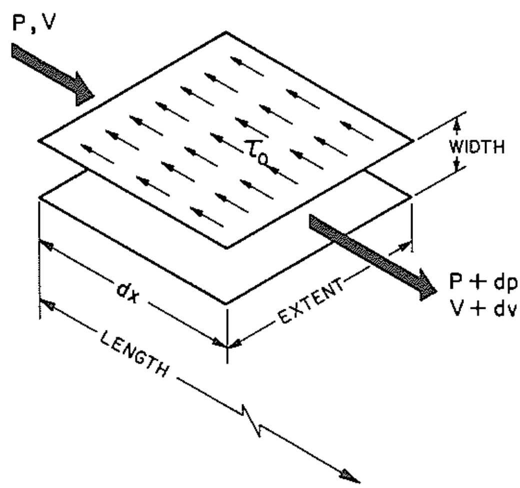

Fig. 1 Idealization of crack as a gap between parallel plates.

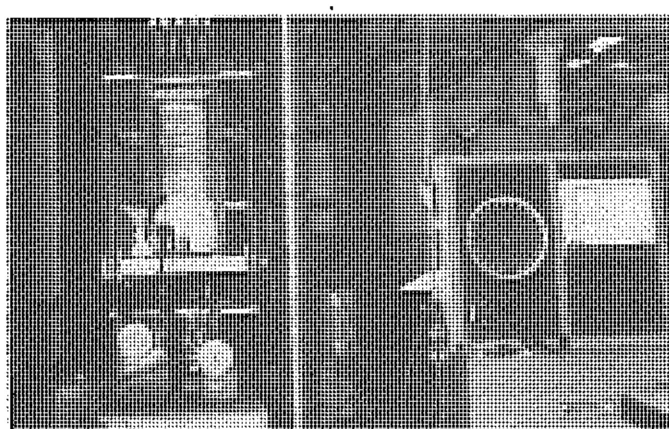

Fig. 4(b) Downstream face of testing specimen.

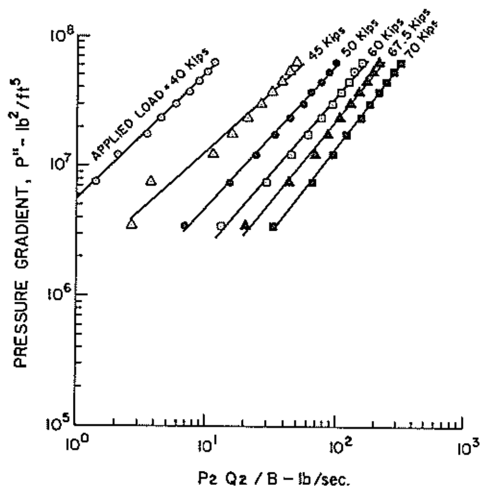

Fig. 5 Measured rate of flow per unit extent of crack for given pressure gradient - specimen L4.

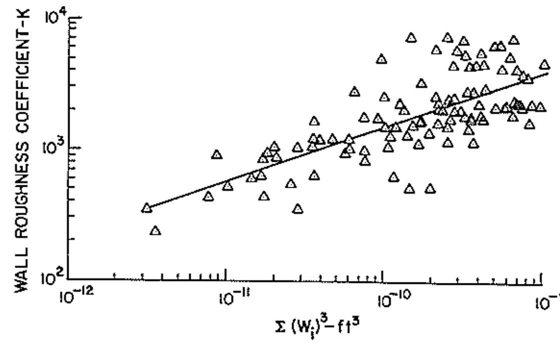

Fig. 6 Wall roughness coefficient, K, as function of summation of cube of measured crack widths.

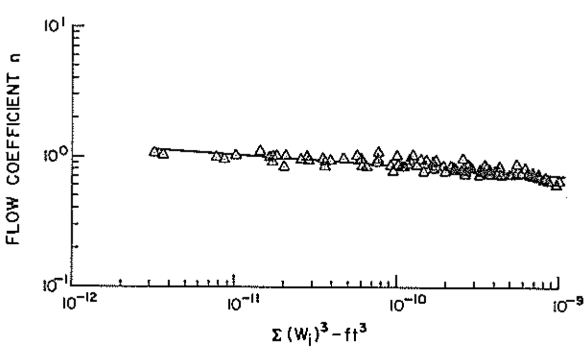

Fig. 7 Flow coefficient, n, as function of summation of cube of measured crack widths.

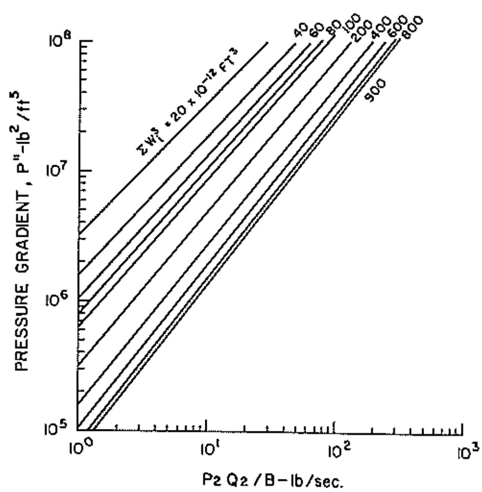

Fig. 8 Rate of flow of air for different pressure gradients and crack widths.

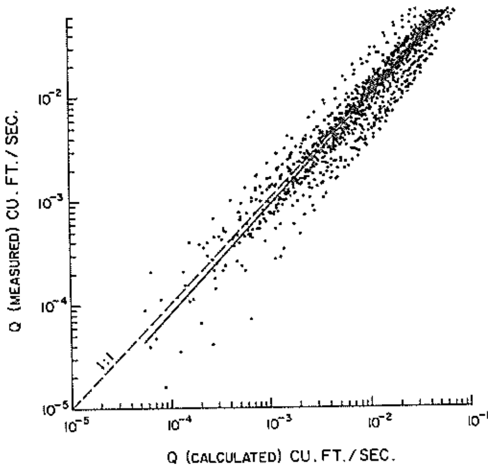

Fig. 9 Comparison of measured flow rate to predicted flow rate based on measured crack widths.

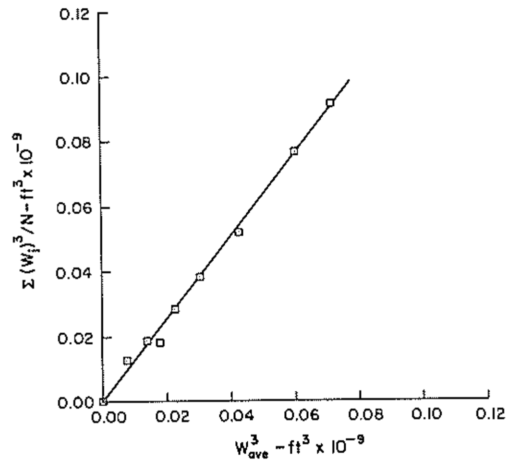

Fig. 10 Relationship between summation of cube of measured crack widths divided by number of cracks of cube of average crack width for specimen L4.

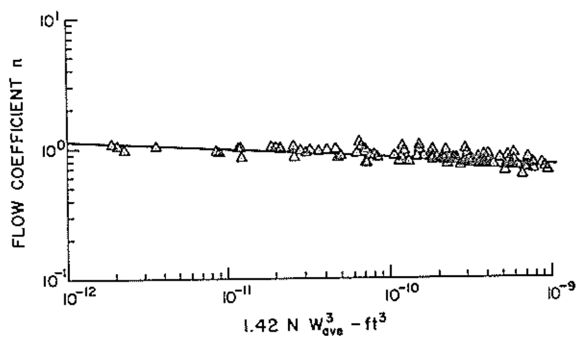

Fig. 11 Flow coefficient as a function of average crack width parameter - all tests.

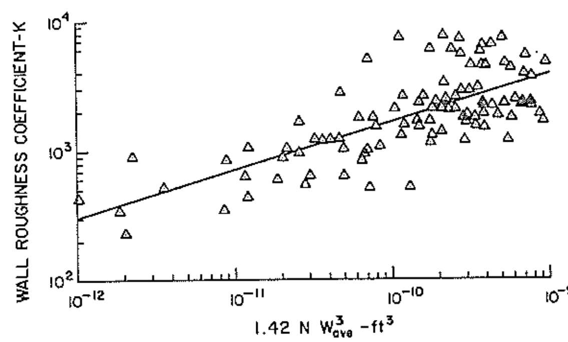

Fig. 12 Wall roughness coefficient, k, as function of average crack width parameter - all tests.

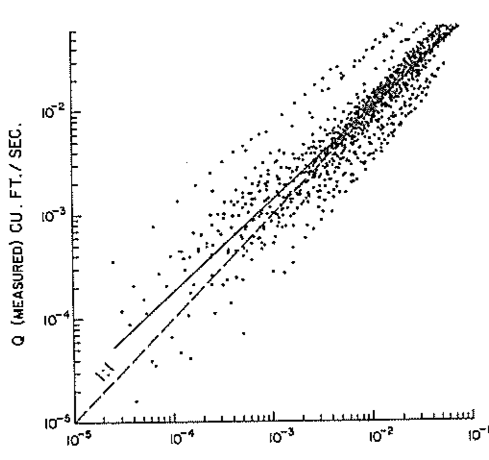

Fig. 13 Comparison of measured flow rate to predicted flow rate based on average crack width.