Soil-Structure Interaction for a Multi-Foundations Building

P. Labbe
Electricité de France, Direction des Etudes et Recherches, 1 av. du Général-de-Gaulle, F-92141 Clamart Cedex, France

ABSTRACT

A method to compute the soil structure interaction has been proposed assuming rigid foundations and no more than one foundation for each building. A generalization of this method for multi-foundations buildings is suggested.

An application is proposed, regarding the part of a raft of a power plant between two buildings as a pin-joint, based on the two foundations beside it.
1. Introduction

Because of the soil characteristics, it is more and more frequent that the soil structure interaction has to be taken into account for computing the seismic response of a power plant.

A method has been proposed by LUCO and WONG to carry out these complex studies. It is a modal synthesis method for which many foundations are possible, but assuming rigid foundations and no more than one of them for each building. This method is the basis of the computer program CLASSI. A generalization of this method for multi-foundations buildings is suggested here.

2. Present state of the modal synthesis method

Firstly, the relationship between the soil displacements $U_s$ under the foundations and the forces on the soil $F_s$ due to the foundations must be known. A method based on Green’s functions has been proposed by LUCO and WONG to solve this problem for rigid foundations [1] and it is possible to write:

$$F_s = K_s(\omega) U_s$$  \hspace{1cm} (1)

Where $U_s$ is a harmonic motion and $K_s(\omega)$ is the complex impedance soil matrix. Because of rigidity, the dimension of $U_s$ is $6N$, $N$ being the number of foundations.

Another relationship that has to be known is between the actual harmonic motion $U_0$ of the foundation and the forces $F_b$ on this foundation due to the structures, which can be written:

$$F_b = \omega^2 M_b(\omega) U_0$$  \hspace{1cm} (2)

Where $M_b$ is complex and called the equivalent mass matrix. In the next paragraph, we focus on this matrix $M_b(\omega)$ that, as far as we know, has not yet been exhibited for a multi-foundations building.

These two relations being known, the synthesis consists of writing the equilibrium of the foundations. This then enables us to write the expression of the motion $U_0$ [2]:

$$U_0 = (I - \omega^2 K^{-1}(\omega) (M_0 + M_b(\omega))^{-1} U_s^*$$  \hspace{1cm} (3)

Where $M_0$ is the mass matrix of the foundations, $U_s^*$ is the "foundation input motion", defined has the harmonic motion of the rigid massless foundations for a harmonic seismic wave, ($U_s^*$ is not equal to the free field motion because of rigidity).

- The equivalent mass matrix

  - The equilibrium equations

The basic equation of the problem is classically:

$$M\ddot{X} + C\dot{X} + KX = F$$  \hspace{1cm} (4)

and we set:

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \quad F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad \text{forces boundary conditions}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \quad F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad \text{displacements boundary conditions}$$
For a seismic problem, we have $F_1 = 0$ and $F_2 = -F_0$, $X_2 = U_0$ with the previous notations. We know that the fundamental problem is to find an accurate basis of the displacements field, in order to decompose $X$ on it. For this reason we set:

$$X = X^* + \overline{X}, \quad \text{where} \quad X^* = \begin{bmatrix} x_1^* \br x_2^* \end{bmatrix}, \quad \overline{X} = \begin{bmatrix} \overline{x}_1 \\ 0 \end{bmatrix} \tag{5}$$

so $X^*$ is a kinematically admissible field that we regard as a driving motion and $\overline{X}$ is a homogeneous boundary conditions field that can be decomposed into eigen modes with clamped basis. Notice that we keep a choice which is to give the definition of $X_1^*$. In our opinion, a good criterium is the following one: in the restriction to a single foundation, $X^*$ has to represent the rigid body modes.

Consequently the definition of a static mode can be introduced: "with no external loading $(F_1 = 0)$ it is the displacement field obtained if all the constrained degrees of freedom are equal to zero but one, equal to unity". So we are able to define the matrix $\mathbf{V}$ of static modes and the matrix $\mathbf{R}$ of static reactions so that:

$$\mathbf{V} = \begin{bmatrix} \psi \\ 1 \end{bmatrix} (I: \text{identity}), \quad \mathbf{K}_F = \begin{bmatrix} 0 \\ \mathbf{R} \end{bmatrix} \tag{6}$$

We notice $\mathbf{H} = [\phi \psi]$ the matrix of modes, where $\phi = \begin{bmatrix} \psi \\ 0 \end{bmatrix}$ is the matrix of dynamic modes; and we set:

$$X = H \Phi, \quad Q = \begin{bmatrix} \Phi \\ 1 \end{bmatrix} \quad \text{i.e.:} \quad \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \Phi \psi \\ \phi \end{bmatrix}$$

The basic equation is now transformed into:

$$
\begin{bmatrix}
\mathbf{H} \mathbf{K}_H \mathbf{H}^* \\
\mathbf{K}_H \mathbf{J}_H \\
\mathbf{J}_H \mathbf{K}_H \\
\mathbf{J}_H \mathbf{J}_H \\
\mathbf{K}_F \\
\mathbf{R}
\end{bmatrix} \mathbf{Q} = \begin{bmatrix}
\mathbf{0} \\
\mathbf{0} \\
\mathbf{0} \\
\mathbf{0} \\
\mathbf{F}_2
\end{bmatrix} \tag{7}
$$

Introducing the classical notations:

$$\mu = \mathbf{t}_\phi \Phi \psi, \quad \beta = \mathbf{t}_\phi \phi \psi, \quad \gamma = \mathbf{t}_\phi \phi \psi \tag{8}$$

leads easily to (we recall that $F_1 = 0$):

$$\begin{bmatrix}
\mu & \mathbf{t}_\phi \Phi \psi \\
\mathbf{t}_\phi \phi \psi & \beta & \mathbf{t}_\phi \phi \psi & \gamma & 0 & \mathbf{q} \\
\mathbf{t}_\phi \phi \psi & \mathbf{t}_\phi \phi \psi & \mathbf{t}_\phi \phi \psi & \mathbf{t}_\phi \phi \psi & 0 & \mathbf{R} & \mathbf{X}_2
\end{bmatrix} = \begin{bmatrix}
\mathbf{0} \\
\mathbf{0} \\
\mathbf{0} \\
\mathbf{0} \\
\mathbf{F}_2
\end{bmatrix} \tag{9}
$$

. Harmonic motion

The first line of the former system can be written:

$$\mu \ddot{q} + \beta q + \gamma q = -\mathbf{t}_\phi \phi \psi \dot{X}_2 - \mathbf{t}_\phi \phi \psi \dot{X}_2 \tag{10}$$

A generalized Basile assumption is introduced:

"$C$ is such that $\mathbf{t}_H \mathbf{C}_H$ is diagonal",

so $\beta$ is diagonal and $\mathbf{t}_\phi \phi \psi = 0$.

Then for a harmonic motion $X_2(t) = X_2 \ e^{i\omega t}$, we can write:

$$\mathbf{q} = D(\omega) \ \mathbf{t}_\phi \phi \psi$$

where $D(\omega)$ is the modal amplification matrix; $D(\omega)$ is diagonal, and for a choice of $\phi$ such that $\mu$ is equal to identity, we have:

$$d_{i}(\omega) = \frac{\omega_i}{1 - (\omega/\omega_i^2 + 2 \ I \ \mathbf{C}_i(\omega/\omega_i))}; \quad \gamma_i = \omega_i^2; \quad \beta_i = 2 \xi_i \omega_i \tag{12}$$
Consequently, for a harmonic motion \( x_2 \), the forces \( F_2 \) can be obtained by the relationship:

\[
F_2 = \left[-\omega^2 m + i \omega c + R - \omega^2 \gamma h D(\omega) h\right] x_2
\]  

(13)

Where \( m = m_{MN} \), \( c = c_{MN} \), \( h = h_{MN} \).

So the equivalent mass matrix is written:

\[
M_0(\omega) = m - \frac{i}{\omega} c - \frac{1}{\omega^2} R + \gamma h D(\omega) h
\]  

(14)

(c was disregarded to compute the following results).

- **The modelling of a nuclear power plant**

This power plant consists of eight buildings, some of them can be based on the same raft. Because of the stiffness of the structures each part of raft under a building can be regarded as a rigid foundation, compared to the parts between two buildings that are regarded as flexible, and that we name pin-joints.

In order to take into account this flexibility, we have regarded each pin-joint as a structure based on the two foundations beside it (figure 1) which led us to an application of the former development with very simple structures.

The complete model of the power plant (figures 2 and 3) contains eight rigid foundations on which are clamped ten stick-models and we also have twelve pin-joints.

We do not emphasize here the classical models structures ; the rigid foundations have been meshed (between 20 and 40 elements) as necessary to compute the soil impedance matrix.

The characteristics of the soil are the following ones : homogeneous, isotropic, \( E = 9500 \) MPa, \( v = 0.33 \).

The seismic wave is volumic with vertical propagation.

The synthetic Long Beach accelerogram has been used, normalized at 1 \( \text{ms}^{-2} \) for the horizontal direction and at 0.67 \( \text{ms}^{-2} \) for the vertical direction (the component North-South of the accelerogram is in the \( x \) direction).

- **Results and conclusions**

In order to test the efficiency of joints, computations have been made for the model with joints and without joints. Comparisons of results are shown on tables I, II and III.

On table I the maxima of differential displacements between different buildings are shown, and of course these maxima decrease with the joints that solidarize the different buildings.

Table II shows accelerations ; the strongest acceleration, at the top of stick 1 decreases significantly with the joints. On the other hand, the acceleration increases a little at the top of stick 2, which seems to be due to an acceleration increase at the base of the structure. For sticks 3 and 4 the consequence of joints is a marked decrease in acceleration at the top.

At the base of sticks 1, 2, 3 and 4 a uniform increase (10 %) in acceleration is found in the \( x \) direction and a decrease in the \( y \) direction.
For stick 5, a marked increase is recorded at the top, despite there not being a joint for the raft of this building. Naturally we could not expect the results to be exactly the same as when there is no joint in the model of the power plant; nevertheless, in our opinion, such a difference at the top of stick 5 was not to be expected. For the base of 5 and for stick 6 the results are unchanged.

Vertically the results are quite stable for all the buildings, with or without joints, as could also be expected.

The results concerning forces at the bases of sticks are shown in table III. Of course a strong correlation appears with the acceleration results. We just focus here on the decrease of moments at the base of stick 1; furthermore these moments become more or less the same in the two horizontal directions. This result was in fact the main result we hoped to obtain from the joints.

In conclusion we can say that the pin-joints can be a good solution for obtaining an as low as possible level of accelerations and forces in certain structures. Nevertheless care has to be taken not to neglect the consequences that can be summed up in saying that the lighter buildings may be "driven" by the heavier ones. What is more, the consequences have to be studied even for the buildings nearby, the rafts of which do not have pin-joints.

BIBLIOGRAPHY


The structure:

The model:

The pin-joint model:

Figure 1

Figure 2

Map of the rafts and position of pin-joints.

Figure 3

Numbers of the stick-models.
### TABLE I

**Differential displacements between buildings (mm)**

<table>
<thead>
<tr>
<th>Number stick</th>
<th>1 - 3</th>
<th>1 - 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta x )</td>
<td>( \Delta y )</td>
</tr>
<tr>
<td>Without joints</td>
<td>6.5</td>
<td>8.0</td>
</tr>
<tr>
<td>With joints</td>
<td>4.2</td>
<td>3.9</td>
</tr>
</tbody>
</table>

### TABLE II

**Accélération (m/s²) in buildings**

<table>
<thead>
<tr>
<th>Number stick</th>
<th>Without joints</th>
<th>With joints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \gamma_x )</td>
<td>( \gamma_y )</td>
</tr>
<tr>
<td>1</td>
<td>4.62</td>
<td>4.30</td>
</tr>
<tr>
<td></td>
<td>1.10</td>
<td>1.15</td>
</tr>
<tr>
<td>2</td>
<td>3.65</td>
<td>3.62</td>
</tr>
<tr>
<td></td>
<td>1.10</td>
<td>1.15</td>
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<tr>
<td>3</td>
<td>3.91</td>
<td>3.74</td>
</tr>
<tr>
<td></td>
<td>1.10</td>
<td>1.22</td>
</tr>
<tr>
<td>4</td>
<td>2.37</td>
<td>2.66</td>
</tr>
<tr>
<td></td>
<td>1.06</td>
<td>1.10</td>
</tr>
<tr>
<td>5</td>
<td>2.81</td>
<td>2.08</td>
</tr>
<tr>
<td></td>
<td>1.11</td>
<td>1.16</td>
</tr>
<tr>
<td>6</td>
<td>2.15</td>
<td>2.46</td>
</tr>
<tr>
<td></td>
<td>1.21</td>
<td>1.11</td>
</tr>
</tbody>
</table>

### TABLE III

**Forces at the base of structures (10⁶ N, 10⁷ Nm)**

<table>
<thead>
<tr>
<th>Number building</th>
<th>Without joints</th>
<th>With joints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( F_z )</td>
<td>( F_x )</td>
</tr>
<tr>
<td>1</td>
<td>2 040</td>
<td>6 280</td>
</tr>
<tr>
<td>2</td>
<td>1 900</td>
<td>4 930</td>
</tr>
<tr>
<td>3</td>
<td>2 040</td>
<td>4 190</td>
</tr>
<tr>
<td>4</td>
<td>2 190</td>
<td>4 190</td>
</tr>
<tr>
<td>5</td>
<td>2 700</td>
<td>5 670</td>
</tr>
<tr>
<td>6</td>
<td>3 400</td>
<td>7 020</td>
</tr>
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