About Seismic Qualification of Equipments by Multi-Spectra Method

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For many years, Shock Response Spectrum Modal Analysis (SRSMA), is used, in most cases,in Seismic Analysis of Equipments. This method has been usually employed as a simplified formulation (e.g. for a single support excitation). Lately, SRSMA was extended for taking into account residual flexibility (or missing mass correction). Now, a new extension is provided, in case of large piping systems or multi-level complex structures, taking into account the multiple support excitation effects (with residual flexibility).

Showing that a single support excitation SRSMA (umbrella response spectrum method) is conservative v. multiple support excitation method, calculation costs of those two methods should be investigated. Besides, in Designer's point of view, these different calculation costs must be compared with design costs, keeping in mind that a seismic qualification of an Equipment must take into account others live loads, such as thermal loads.

So, firstly, this paper presents the extended SRSMA method with taking into account residual modes participation. In second part, a few examples are presented, studing results and calculation costs of several load cases :

- Live loads,
- Seismic loads analysed by a single support excitation method
- Seismic loads analysed by a multiple support excitation method

Conclusions of this paper show that :

- Seismic analysis performed by single support excitation method is concervative v. multiple support excitation method.

- In most cases, supports reactions in thermal analysis, are in the same order that reactions given by seismic analysis, and generally, design costs are not so very modified.

- Computer costs are extended in multiple support excitation method, essentially due to higher modes participation calculation.

1 - INTRODUCTION

Response spectrum analysis is usually used from many years in design of equipments to resist earthquake induced motions. Comparatively with other seismic response analysis like time-history response or random vibration procedures, response spectrum technique is a simplified method.

More recently, some modifications to response spectrum method has been suggested such as :

- computation of the effect of higher modes |1| |2|
- computation of the effect of multi-support excitations |3| |4| |5| |6| |7|

The intent of this present paper is to present some industrial applications of these two modifications, keeping in mind that the others effects must be taken into account in a seismic qualification of an Equipment, such as dead loads, wind effect, thermal effect and so on.

2 - THEORY

The equation of motion of an equipment loading by time history forces is :

$$\begin{bmatrix} \bar{M}_{11} & \bar{M}_{12} \\ \bar{M}_{21} & \bar{M}_{22} \end{bmatrix} \begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{Bmatrix} + \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} \\ \bar{C}_{21} & \bar{C}_{22} \end{bmatrix} \begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{Bmatrix} + \begin{bmatrix} \bar{K}_{11} & \bar{K}_{12} \\ \bar{K}_{21} & \bar{K}_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad (1)$$

where M, C and K are the mass, damping and stiffness matrices, X and f are the time history total response displacements and applied forces, subscripts 1 and 2 correspond to the free and constrained d.o.f. Assuming that no force is applied and the total displacement vector is written as the sum of relative displacement vector {x}and the displacement vector {E} in the driving motion, Eq.(1) becomes after some manipulations and assumptions:

$$\bar{M}_{11} \{\ddot{x}_1\} + \bar{C}_{11} \{\dot{x}_1\} + \bar{K}_{11} \{x_1\} = - \bar{M}_{11} \{\ddot{E}_1\} - \bar{K}_{11} \{E_1\} - \bar{K}_{12} \{E_2\} \qquad (2)$$

The driving motion is a rigid body motion without elastic deformation - so :

$$\bar{K}_{11} \{E_1\} + \bar{K}_{12} \{E_2\} = 0$$

or

$$\{E_1\} = \bar{\gamma} \{E_2\} \quad \text{with} \quad \bar{\gamma} = - \bar{K}_{11}^{-1} \bar{K}_{12}$$

Thus, Eq.(2) may be written :

$$\bar{M}_{11} \{\ddot{x}_1\} + \bar{C}_{11} \{\dot{x}_1\} + \bar{K}_{11} \{x_1\} = - \bar{M}_{11} \bar{\gamma} \{\ddot{E}_2\} \qquad (3)$$

Using normal mode approach, Eq.(3) may be solved and generalized displacements are given by :

$$\{y\}_j^{ki} = \{\mathcal{F}\}_j^{ki} \left[- \frac{1}{\omega_j} \int_0^t \ddot{E}_2^{ki} (\tau)\, e^{-\xi_j \omega_j (t-\tau)} \sin \omega_j (t-\tau)\, d\tau \right] \qquad (4)$$

with

ω_j = j^{th} circular frequency of the equipment

k,i = subscript corresponding to the k^{th} support and i^{th} direction

ξ_j = damping ratio of the j^{th} circular frequency

$\{\Gamma\}_j^{ki}$ = modal support participation factor for the k^{th} support, i^{th} direction and j^{th} mode

The modal support participation factor may be written as :

$$\{\Gamma\}_j^{ki} = \{\phi\}_j^T \, \overline{M}_{11} \, \overline{D} \quad (5)$$

where $\{\phi\}_j^T$ is the orthonormalized j^{th} mode shape.

The shock response method is based on the maximum value of the integral in the bracket (see Eq.(4))

$$S_{v_{max_j}}^{ki} = MAX \left| \int_0^t \ddot{E}_z^{ki}(\tau) \, e^{-\xi_j \omega_j (t-\tau)} \sin \omega_j (t-\tau) \, d\tau \right| \quad (6)$$

The response in the geometric coordinates is given by :

$$\{x\} = \left[\sum_j \{\phi\}_j \{\gamma_{max}\}_i^{ki} \right]^{1/2} \quad (7)$$

where =

$$\{\gamma_{max}\}_j^{ki} = \{\Gamma\}_j^{ki} \frac{1}{\omega_j^2} S_{a_{max_j}}^{ki}$$

$$S_{a_{max_j}}^{ki} = \omega_j \, S_{v_{max_j}}^{ki} \quad (8)$$

Eq.(7) may be expressed with others rules of modal response combination following R.G.1.92 recommandations .

In an other hand, Eq.(7) is written, taking into account the complete modal basis. However, only first modes, less than 33 Hz, are generally calculated, although it is possible that all significant modes are not in the dynamic range of the spectrum. Thus, corrections are needed in order to obtain more accurate results : this is the subject of residual flexibilities calculations. Eq.(7) may be expressed as follows :

$$\{x\} = \left[\sum_{j=1}^{P} \{\phi\}_j \{\gamma_{max}\}_j^{ki} \right]^{1/2} + \sum_{j=P+1}^{N} \{\phi\}_j \{\Gamma\}_j^{ki} \, S_{0_{max}}^{ki} \quad (9)$$

where :

$S_{0_{max}}^{ki}$ = spectral acceleration at the cutt-off frequency for the k^{th} support and i^{th} direction

P = number of modes calculated up to the cutt-off frequency

N = number of d.o.f

Because of the orthogonality of the normal modes :

$$\overline{K}_{11}^{-1} = \sum_{j=1}^{N} \{\phi\}_j \frac{1}{\omega_j^2} \{\phi\}_j^T \quad (10)$$

Thus, taking into account the expression of the modal support participation factor(Eq.(5))

$$\{x\} = \left[\sum_{j=1}^{P}\{\phi\}_j\{\gamma_{max}\}_j^{ki}\right]^{1/2} + \left[\bar{K}_{11}^{-1} - \sum_{j=1}^{P}\{\phi\}_j\frac{1}{\omega_j^2}\{\phi\}_j^T\right]\bar{M}_{11}\,\gamma\,S_{o_{max}}^{ki} \quad (11)$$

Let $\{x_{sta}\}^{ki}$ is the solution of the static problem

$$\bar{K}_{11}\{x_{sta}\}^{ki} = \bar{M}_{11}\,\gamma\,S_{o_{max}}^{ki} \quad (12)$$

The contribution of neglected terms in the modal superposition is :

$$\{x_R\} = \{x_{sta}\}^{ki} - \sum_{j=1}^{P}\{\phi\}_j\{F\}_j^{ki}\frac{1}{\omega_j^2}S_{o_{max}}^{ki} \quad (13)$$

This contribution is expressed as the difference between the solution of the static problem (12) and the response value, calculated with the P first modes, of the structure loaded by a constant acceleration S_o^{ki} .

3 - NUMERICAL EXAMPLES

As it is indicated in the preliminaries of this paper, the seismic qualification of an equipment includes, in addition of seismic analysis, others typical analysis such as dead and live loads analysis.

Examples below show that, in many cases, these last loads are more critical than environment loads.

3.1. Piping located in petroleum refinary

Pipe size : 760 mm/10mm

Thickness of the heat-insulator : 140 mm

Loading : dead weight (M = 13400 kg)
pre-tension effect
seismic loads (ZPA = 0,1 g in the lower part and 0,15 g 25m above)
wind loads (pressure = 520 MPa)
pressure loads (pressure = 0,4 MPa)
thermal effect (ΔT = 450°C + differential displacement at each end)

Boundary conditions :clamped at each end + two support at constant lift

Result

. Modal analysis

1st mode : 4 Cps

10th mode : 28,2 Cps (ZPA = 30 Cps)

Sum of effective masses : 65% of total mass in XD (horizontal)
30% of total mass in YD (vertical)
50% of total mass in ZD (horizontal)

. Seismic response (envelope response spectrum approach)

Anchor reactions (lower part) :

R_x = 11000 N R_y = 4800 N R_z = 7000 N

M_x = 5500 mN M_y = 11600 mN M_z = 11500 mN   Maximum stress = 15 MPa

. Seismic response (multi spectra response approach)

Anchor reactions (lower part) :

R_x = 7200 N R_y = 4200 N R_z= 4000 N

M_x = 4500 N M_y = 7500 mN M_z= 9700 mN

Maximum stress (at same location as above) = 11,3 MPa

. Wind effect

Anchor reactions (lower part) :

R_x = 3000 N R_y = 130 N R_z= 30 N

M_x = 420 mN M_y = 2200 mN M_z= 7900 mN

Maximum stress (at same location as above) = 5 MPa

. Pressure load + thermal load + pre-tension effect

Anchor reactions (lower part) :

R_x = 470 N R_x = 4500 N R_x= 1100

M_x = 6100 mN M_y = 5560 mN M_z= 11400 mN

Maximum stress (at same location as above) = 8 MPa

. Computer costs (CDC 7600 computer, CA.ST.OR code)

Modal analysis : 20 UC

Seismic response, effect of neglected modes included (envelope response spectrum approach) : 46 UC

Seismic response, effect of neglected modes included (multi spectra response approach) : 85 UC

Other static calculations : 60 UC.

NOTE : UC = Unit of count of computer

4 - CONCLUSION

As it is seen in the industrial example above, there are no significant differences between stresses and reactions calculated by envelope spectrum approach and multi-spectra technique. However, computer costs increase significantly with the second method. Besides, stresses and reactions issued from others static calculations (needed for seismic qualification) are in the same order that results of seismic response calculations. Thus, before a choice between these two spectrum response methods, we must keep in mind that an increase of costs calculations must be balanced by a decrease of piping and support design costs.

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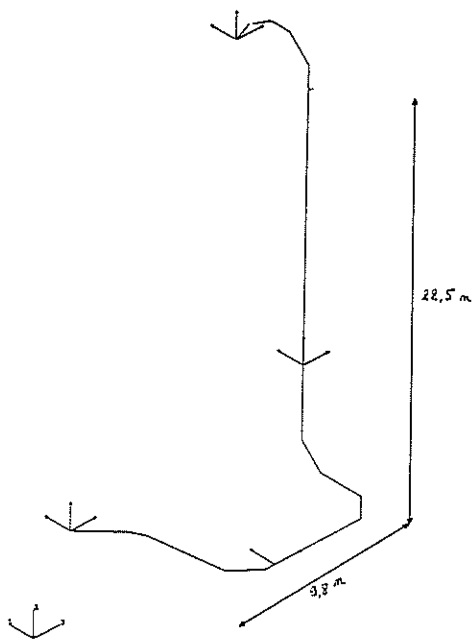

figure 1 - mesh of piping

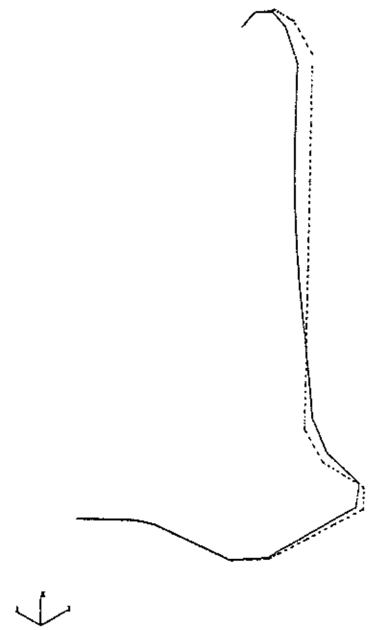

figure 2 - modal shape 1 - F = 4 cps

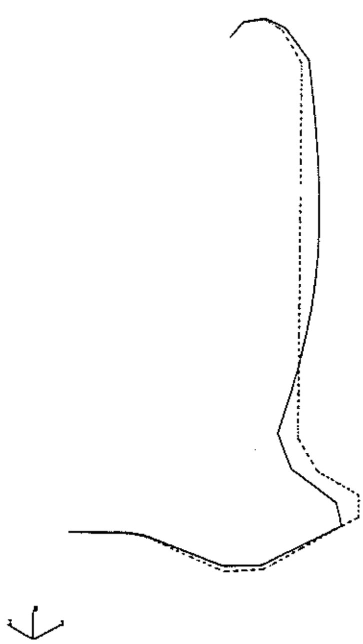

figure 3 - modal shape 2 - F = 4,9 cps

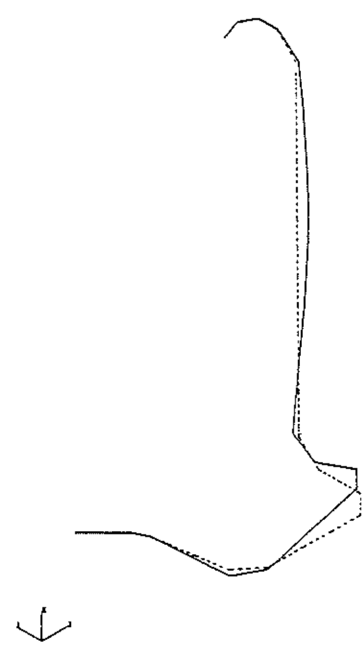

figure 4 - modal shape 3 - F = 6,2 cps