Limitations of Modal Analysis of Damped Structures

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Summary

Quite recently discrete spring-damper elements are increasingly used for the low-tuned supports of nuclear power-plant buildings and equipment (reactor building, turbine-fundaments etc.) to reduce the vibration response due to the dynamic load cases earthquake and airplane crash. Because of this development, it is to be investigated whether the usual modal analysis method is applicable within the design process or should be changed respectively replaced in special cases.

The presented paper contributes to this discussion by demonstrating and valuing the discrepancies in the different ways for the implementation of damping. Different methods for uncoupling (energy weighting, reduction to Rayleigh-damping) are compared with the solution of the coupled equations of motion.

In particular vertical vibrations of a spring-damper-supported building on foundation (including ground springs) are examined using a two-degree-of-freedom-system. The results of coupled and (by force) uncoupled methods are interpreted concerning free vibration by comparison of the damping of natural vibrations, natural frequencies and natural mode shapes. The effect on the forced vibrations is shown by floor response spectra to an earthquake accelerogram.

1. Introduction

The dynamic analysis of complicate structures as necessary for design of nuclear power plants against dynamic loads (earthquake, airplane crash etc.) is accomplished by the solution of a system of, at first, multiple coupled differential equations of motion. A special feature of undamped systems is the possibility to uncouple the equations by a rather simple (real) transformation (modal analysis). If damping has to be considered, uncoupling is in general not possible and the solution of the initially coupled system is necessary, combined with a considerable amount of additional efforts in computation. Exceptions are structures whose damping matrix is proportional to mass matrix and stiffness matrix with α, β as constant factors (Rayleigh damping). In this case it is also possible to uncouple the differential equations of the damped system.

Because in practice the damping matrix is not given in the mentioned form, proportional damping is forced by a proper choice of α and β or the damping matrix is, at first, suppressed. In the last case each of the natural modes is instead associated with a modal damping by application of, for example, the method of energy weighting, dependent upon material and strain level of the system. These solution methods are, of course, approximations. They are justified because the essential influence of damping (energy loss) is included and the inaccuracies in the description of damping (including the physical model of the phenomena of damping and the numerical determination of damping coefficients) are rather large so that the use of chosen modal damping values seems to be sufficient.

Quite recently, however, discrete spring-damper elements are increasingly used in reactor technology to low-tune components, platforms, parts of buildings and sometimes even buildings themselves. The physical properties of these elements can be determined by testing with rather high precision. By combining these discrete spring-damper-elements a specific kind of vibration isolation shall be aimed. The problem is, whether the simplified, uncoupled calculations are still sufficient in these cases and of what magnitude the deviations may be. Further more it is important to find out if methods exist to uncouple the equations of motion, which improve the common used energy-weighting.

2. Modal Analysis of Damped Structures

The coupled equations of motion of a vibration system with n degrees of freedom

$$\underset{\sim}{M}\ddot{\underset{\sim}{x}} + \underset{\sim}{R}\dot{\underset{\sim}{x}} + \underset{\sim}{K}\underset{\sim}{x} = \underset{\sim}{p}(t), \quad \underset{\sim}{x}^T = (x_1, \ldots, x_n) \tag{1}$$

may be uncoupled by the well known (real) transformation

$$\underset{\sim}{x}(t) = \underset{\sim}{\Phi} \cdot \underset{\sim}{q}(t) \tag{2}$$

if the system is undamped ($\underset{\sim}{R} = \underset{\sim}{0}$) or if RAYLEIGH-damping in the form $\underset{\sim}{R} = \alpha\underset{\sim}{K} + \beta\underset{\sim}{M}$ is present. $\underset{\sim}{\Phi}$ is the modal matrix, composed of the eigenvectors of the undamped system. The parameters α

and β are constants. The uncoupled system of equations with matrices of diagonal form

$$\underset{\sim}{\overset{\backslash}{M}} = \underset{\sim}{\Phi}^T \cdot \underset{\sim}{M} \cdot \underset{\sim}{\Phi}; \quad \underset{\sim}{\overset{\backslash}{R}} = \underset{\sim}{\Phi}^T \cdot \underset{\sim}{R} \cdot \underset{\sim}{\Phi}; \quad \underset{\sim}{\overset{\backslash}{K}} = \underset{\sim}{\Phi}^T \cdot \underset{\sim}{K} \cdot \underset{\sim}{\Phi} \tag{3}$$

may be written as

$$\underset{\sim}{\overset{\backslash}{M}} \cdot \underset{\sim}{\ddot{q}} + \underset{\sim}{\overset{\backslash}{R}} \cdot \underset{\sim}{\dot{q}} + \underset{\sim}{\overset{\backslash}{K}} \cdot \underset{\sim}{q} = \underset{\sim}{\Phi}^T \cdot \underset{\sim}{p}(t). \tag{4}$$

It is possible to solve these n differential equations with little efforts and to find the system response by superposition of the results, following eq. (2). If the damping matrix $\underset{\sim}{R}$ is general, uncoupling is not possible any more in such an easy manner. The coupled system of equations must be solved by considerable additional expense (complex modal analysis).

3. Approximations to Uncoupling

Two methods are common to avoid the additional calculation efforts in solving the coupled equations:

- a modal damping D_L is assigned to each natural vibration of the undamped system
- RAYLEIGH-damping is forced by selection of α and β in $\underset{\sim}{R} = \alpha\underset{\sim}{K} + \beta\underset{\sim}{M}$.

The main difficulty of the first method is the determination of the appropriate modal damping values. To each mode the individual damping properties, for example owing to the material (soil, concrete, steel), contribute to a different extent. By use of "energy-weighting" according to BIGGS, the contributions of the particular, elastic elements to the strain energy of the individual eigenmodes are determined, weighted by the damping coefficient of these elements and added up to the modal damping. These damping values are finally used in solving equation (4) for forced vibrations.

By use of the second method, the condition $\underset{\sim}{R} = \alpha\underset{\sim}{K} + \beta\underset{\sim}{M}$ for modal damping is followed by the condition

$$D_L = \frac{1}{2}\left(\alpha\ \omega_L + \beta\ \frac{1}{\omega_L}\right), \quad L = I, II, \ldots, n \tag{5}$$

with eigenfrequencies ω_L (see for example [1]). In consequence of this method, it is possible to fit two given modal damping values by choice of α and β. All the remaining damping values, however, are defined by eq. (5) and must be put up with.

In both methods the unique relationship between the damping matrix which is deduced from the vibration model and the following solution is split up. This facilitates the solution on the one hand, results, however, in systematic errors on the other hand.

In the following section some basic relations are deduced at the example of a vibration model with two degrees of freedom and the effects of different solution methods on natural vibrations and forced vibrations are shown.

energy weighting and forced RAYLEIGH-damping (W- and K-method).

The parameters of the vibration model are

$$m_1 = 150\ 000\ t; \quad m_2 = 150\ 000\ t$$
$$\omega_1 = 2\pi \cdot 2s^{-1} \quad ; \quad \omega_2 = 2\pi \cdot 4.1s^{-1}$$

The results are shown dependent on damping values D_1 and D_2 of the two parts of the building with $D_1/D_2 = 1/2$ in Fig. 2a and $D_1/D_2 = 1/4$ in Fig. 2b. The parameters are chosen in that way that nearly RAYLEIGH-damping exists in the first case. By contrast nonproportional damping is present in the second case.

So far it is already clear that the different solution methods result in modal damping values which partly diverge in a distinct manner. Naturally these deviations influence the calculation of the forced vibrations and in consequence the stress analysis of the structure. The method of energy weighting results in an approach of widely different dampings D_I and D_{II}, a fact that is an expected trend of energy methods.

In Fig. 3 the eigenfrequencies of the undamped system, the coupled, damped system and the uncoupled systems are shown. For the latter the frequencies are calculated following

$$\omega_{I,II} = \omega_{uI,II} \sqrt{1 - D_{I,II}^2} \tag{11}$$

The damping values of the building parts are chosen as $D_1 = 0.2$, $D_2 = 0.4$ and $D_1 = 0.1$, $D_2 = 0.4$, respectively. The numerical differences of the distinct eigenfrequencies are within the limit of variability of the model parameters (boundary conditions, material properties etc.) and are negligible.

To complete the analysis of free vibrations the eigenvectors of both parameter sets ($D_1 = 0.2$, $D_2 = 0.4$ and $D_1 = 0.1$, $D_2 = 0.4$, respectively) are shown in Fig. 4. The way of reproduction was chosen to clarify the phase shift between the maximal deflections of mass 1 and mass 2 by use of coupled calculation. For the undamped or uncoupled case the maximal deflections are always in phase or in phase opposition. This is not true anymore with the damped, coupled system (complex eigenvectors) as may be seen in Fig. 4. Thereby the deflection of mass 1 and 2 in the eigenmode corresponds to the projection of the rotating, complex vector with rotation velocity ω_I and ω_{II} on the real axis, respectively. The eigenmodes of the undamped system are opposed to the results of coupled calculation. The influence on the eigenmodes is negligible for the combination $D_1 = 0.2$, $D_2 = 0.4$. For the case $D_1 = 0.1$, $D_2 = 0.4$, however, considerable deviations between uncoupled and coupled calculations become evident.

Summarizing the results it may be stated that:

- the deviation between coupled and uncoupled calculation of modal damping is partly considerable.
- the influence of different calculation methods on the eigenfrequencies is small.
- the absolute value of the damping of building parts is yet no measure for the error in determination of the eigenmodes by use of uncoupled calculation. Rather the deviation from RAYLEIGH-damping is critical.

4. Results of Vibration Model with Two Degrees of Freedom

The vibration model with two degrees of freedom is particularly qualified to clearify basical problems in structural dynamics, because it includes all essential properties of vibrational systems with many degrees of freedom. In many cases it enables the reproduction of the global vibration behaviour of an examined structure.

Fig. 1 shows the model for vertical vibration of a building low-tuned by springs (and dampers) the outer shell of which is again supported by springs and dampers (as idealisation of the soil). By use of the parameters of the building parts (damping coefficients D_1 and D_2, frequencies ω_1 and ω_2, masses m_1 and m_2)the elements of damping- and stiffness matrix may be calculated:

$$r_{11} = 2D_1\omega_1 m_1; \quad r_{12} = -2D_1\omega_1 m_1; \quad r_{22} = 2D_1\omega_1 m_1 + 2D_2\omega_2 m_2 \tag{6}$$

and

$$c_{11} = \omega_1^2 m_1; \quad c_{12} = \omega_1^2 m_1; \quad c_{22} = \omega_1^2 m_1 + \omega_2^2 m_2 \tag{7}$$

In general the equations of motion (see equation (1)) cannot be uncoupled for given D_1 and D_2. Only if

$$D_1/D_2 = \omega_1/\omega_2 \tag{8}$$

is true, RAYLEIGH-damping is present, and the system of equations of motion can be solved uncoupled.

If the damping values D_1, D_2 are not interpreted as damping elements parallel to the springs, RAYLEIGH-damping can be forced by different assumptions. Two of these approaches which uncouple the equations of motion that way are called in this paper W- and K-method. Because of limited space they cannot be presented in detail.

In the following there are always compared the results of the solution of the coupled system of equations with the results of uncoupled calculation (energy-weighting and forced RAYLEIGH-damping).

Free Vibrations

To resolve the eigenvalue problem the complex zeros of the characteristic equation must be calculated:

$$\lambda_{1,2} = \delta_I \pm i\omega_I; \quad \lambda_{3,4} = \delta_{II} \pm i\omega_{II}. \tag{9}$$

ω_I and ω_{II} are eigenfrequencies and δ_I and δ_{II} are related to the modal damping ratio

$$D_{I,II} = \delta_{I,II}/\omega_{uI,II} \tag{10}$$

with $\omega_{uI,II}$ as eigenfrequencies of the undamped system.

In Fig. 2 the so calculated modal damping values D_I and D_{II}, are compared to the results of

Forced Vibrations, Earthquake Loading

In this section the effects of the differences between the results of coupled and uncoupled calculation, concerning free vibrations, on the forced vibrations are shown with the vibration model in Fig. 1 for the earthquake loading. A time history matched to the USNRC-spectrum (D = 0.02) is used. The acceleration responses of the interior building (mass m_1 in Fig. 1) are compared.

Fig. 5 shows the response spectra (D = 0.01) once of coupled and once of uncoupled (energy weighting) calculation with the damping values of $D_1 = 0.2$, $D_2 = 0.4$. In this case nearly RAYLEIGH-damping is present and so the methods based on forced RAYLEIGH-damping (W-, K-method) result in coincidence with the coupled solution. From Fig. 5 only little influence on the shape of the spectra can be found but there is a considerable difference between the maxima.

The results with the damping parameters $D_1 = 0.1$, $D_2 = 0.4$ are given in Fig. 6. Again there are little differences concerning the shape of the spectra. Uncoupling by energy weighting results in spectral values which are below those of the coupled calculation. The methods due to forced RAYLEIGH-damping lead to considerable deviations in the results, together, however, they are bounding the coupled solution.

5. Conclusions

The comparison of coupled and uncoupled solution of the equations of motion show that the different approaches may result in considerable deviations. Especially this has to be considered if the combination of spring-damper-elements is to be designed to produce a specific characteristic of vibration isolation.

Up to now the presented methods of uncoupling based on forced RAYLEIGH-damping can only improve the common used energy-weighting in special cases. Further investigations are still necessary to minimize the resulting deviations in general.

[1] Marguerre, K., Wölfel, H., "Technische Schwingungslehre", Bibliographisches Institut, Mannheim, Zürich, Wien, 1979.

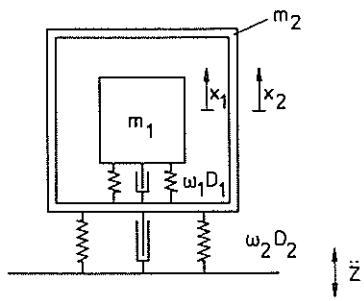

Fig. 1: Model for vertical vibrations of a building (double shell structure)

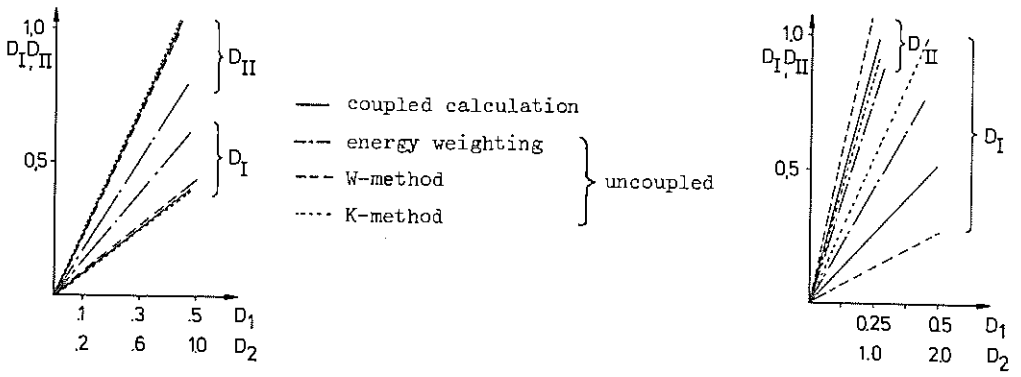

Fig. 2: Comparison of modal damping values calculated in different ways

	$D_1 = 0.2$, $D_2 = 0.4$		$D_1 = 0.1$, $D_2 = 0.4$	
natural frequencies	f_I/Hz	f_{II}/Hz	f_I/Hz	f_{II}/Hz
undamped	1.76	4.67	1.76	4.67
coupled	1.73	4.15	1.76	4.27
uncoupled energy-weighting	1.70	4.38	1.73	4.42
uncoupled W-approach	1.73	4.16	1.76	4.12
uncoupled K-approach	1.74	4.14	1.71	4.37

Fig. 3: Eigenfrequencies of the undamped, damped/coupled and uncoupled system.

eigenvectors $\phi_{I,II} = \begin{pmatrix} 1 \\ \varphi_{I,II} \end{pmatrix}, \varphi_{I,II} \in \mathbb{C}$

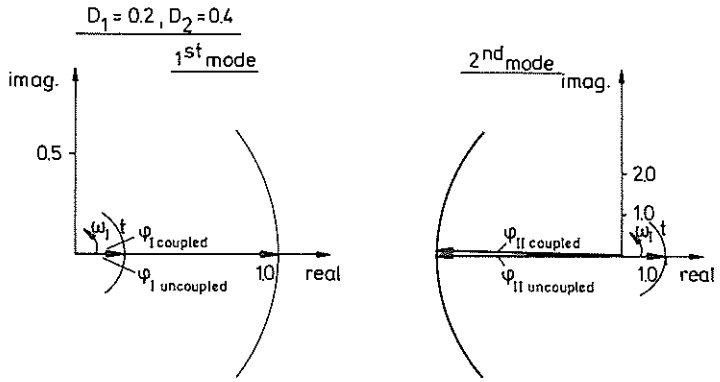

Fig. 4a: Eigenvectors of coupled and uncoupled calculation, $D_1 = 0.2$, $D_2 = 0.4$

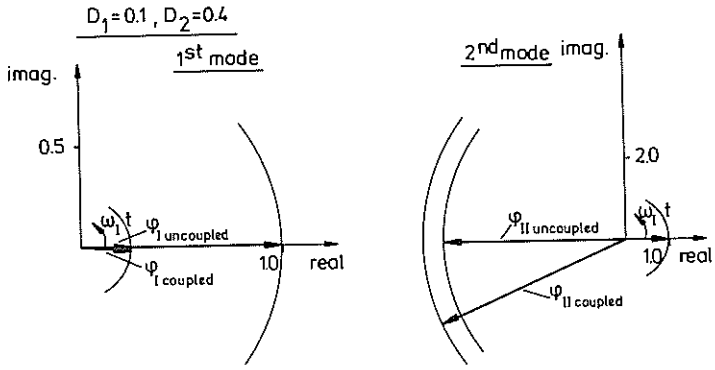

Fig. 4b: Eigenvectors of coupled and uncoupled calculation, $D_1 = 0.1$, $D_2 = 0.4$

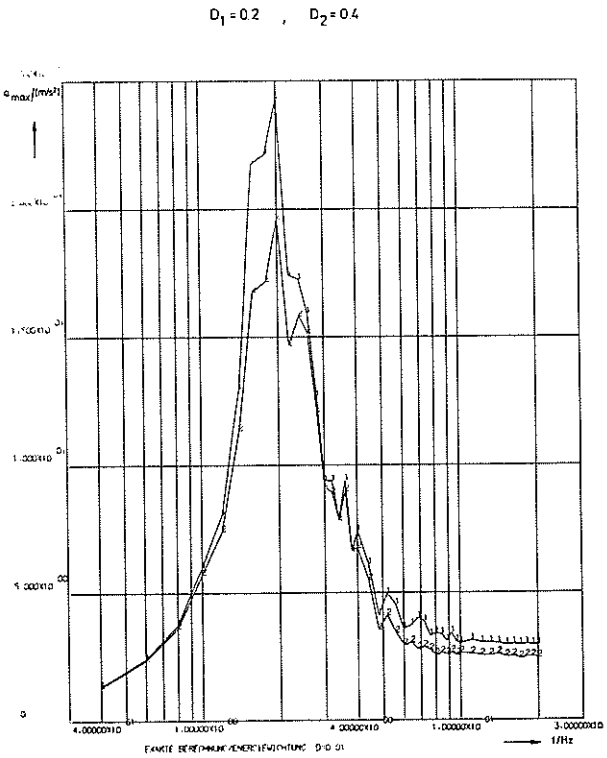

Fig. 5:

Floor response spectra (D = 0.01) of the inner building part (mass m_1) with damping values $D_1 = 0.2$, $D_2 = 0.4$

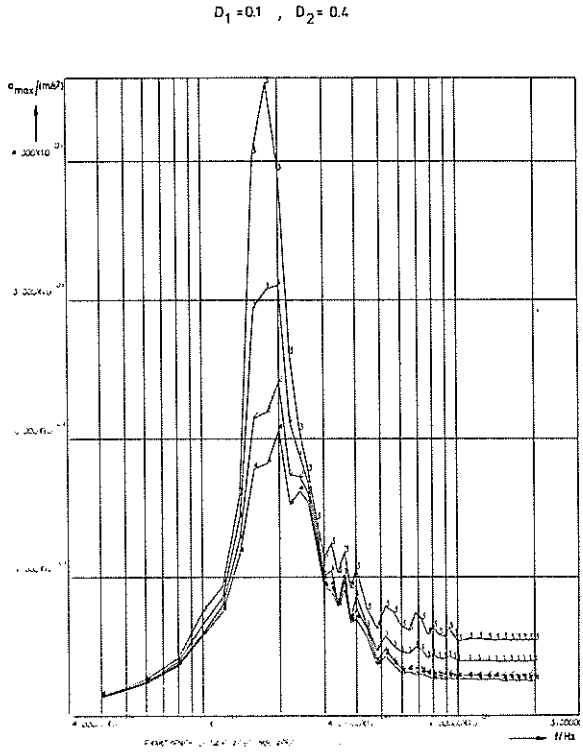

Fig. 6:

Floor response spectra (D = 0.01) of the inner building part (mass m_1) with damping values $D_1 = 0.1$, $D_2 = 0.4$