A Comparison of Floor Response Spectra Techniques

M.J. Yan, J.E. Galford

Babcock and Wilcox, Nuclear Power Generation Division, P.O. Box 1260, Lynchburg, Virginia 24505, U.S.A.

Abstract

Floor response spectra conventionally have been generated using a time-history method. Babcock & Wilcox has developed a new technique, the Fast Floor Response Spectra (FFRS) method, in which dynamic analyses are done entirely in the frequency domain. This paper compares the two techniques and demonstrates that the FFRS method complies with the "equivalency" and "conservatism" requirements of the U.S. Nuclear Regulatory Commission's Standard Review Plan.

The FFRS method uses the specified spectral forcing function for a structure to determine the dynamic, modal response at a point on the structure. Since these calculations are performed entirely in the frequency domain, analytical costs are less than those for a time-history determination. A complete description of the FFRS method is presented in "Fast Floor Response Spectra Generation Technique" (Dynamic and Seismic Analysis of Systems and Components, ASME PVP-Vol. 65, 1982, PP-1-16).

The upper end of a once-through steam generator in the B&W 205 nuclear steam supply system (NSSS) was used to demonstrate that the FFRS method is equivalent to the time-history technique. The two techniques were compared with respect to frequency content and magnitude of response for a given point on the structure. First, the specified forcing function was described in terms of an acceleration time history and an acceleration spectra enveloping that time history. The time-history forcing function was then used in a direct transient analysis to determine the response at the specified point on the NSSS. The resultant response was subsequently converted to a floor response spectra for that point. To show that the FFRS method gave equivalent and conservative results, the FFRS technique was used to determine the modal response directly from the spectral description of the forcing function.

The FFRS- and time-history-generated data agreed to within 13 (worst case on conservative side) of each other with the former cutting analytical costs by 99%.

1. Introduction

For the purpose of seismic analyses, structural models of nuclear power plants are usually divided into several separate systems. For instance, pieces of light secondary equipment are modeled and analyzed separately from the major structure to which they are attached. Floor Response Spectra (RS), generated from a dynamic analysis of the major structure, are used as input to the analysis of the light secondary equipment.

Floor RC conventionally have been generated using the time history (TH) method. The advantage of using the TH technique is that the methodology is readily available, with almost all the structural dynamic codes in the industry (e.g., ANSYS, STARDYNE, and NASTRAN) able to generate the information. However, the TH method has three major flaws: it is overly simple, too conservative, and cost ineffective. These disadvantages arise for the following reasons. First, using a single time history may not be sufficient to model postulated earthquake motions that are more properly represented by an infinite number of different time histories. Second, an earthquake excitation is defined in terms of Input Response Spectra (Input RS). When the TH method is used, these Input RS must be transformed into a corresponding input time history (or histories). This time history is consideraly more conservative than the Input RS that is already conservative. Third, the TH method can easily consume costly hours of computing time performing the necessary analyses for each plant.

Babcock & Wilcox has developed an innovative method called the Fast Floor Response Spectral [3] (FFRS) technique that overcomes these disadvantages for generating Floor RS. This new technique utilizes the response spectral analytical method (rather than the TH method). This article compares the FFRS technique with the TH method and describes the Resonance Response Spectrum (Resonance RS) required for input to the FFRS technique.

2. Comparing the FFRS Technique With the TH Method

The postulated three-story structure shown in Figure 1 was the model for which the Floor RS were generated in this study. The input time history is also shown in Figure 1.

Table 1 lists the natural frequencies, mode shapes, participation factors, and spectral acceleration values of the structure. The corresponding time history RS for both 2% and 5% damping are shown in Figure 2. The 2% Floor RS for the top floor generated using TH method is shown in Figure 3.

The Input RS to the FFRS technique for 2% damping is taken from the smooth spectrum enveloping the 2% time history spectra as illustrated in Figure 2. This figure also shows the Resonance RS (to be discussed later) required for input to FFRS technique. The 2% Floor RS for the top floor generated using the FFRS technique together with that obtained using the TH method are compared in Figure 3; the second floor comparison is shown in Figure 4. These figures readily indicate that the Floor RS generated by FFRS technique is consistent with that obtained using the TH method.

3. The FFRS Technique

Reference 3 describes the derivation of the FFRS technique. According to Reference 3, the Floor RS acceleration can be calculated by the following simple equation:

$$a_E = \sqrt{a_{EB}^2 + a_{EE}^2} \qquad (1)$$

The first acceleration component, a_{EB}, is associated with the structural modal acceleration, $a(\omega_{Bi},\zeta_{Bi})$, and is calculated as

$$a_{EB} = \sqrt{\sum_{i=1}^{N} [\alpha(\theta_i,\zeta_E)\Gamma_i\phi_{iE}\, a(\omega_{Bi},\zeta_{Bi})]^2} \tag{2}$$

where $\alpha(\theta_i,\zeta_E)$ is called the amplification factor and is calculated as

$$\alpha(\theta_i,\zeta_E) = \sqrt{\frac{1 + (2\zeta_E\omega_{Bi}/\omega_E)^2}{[1 - (\omega_{Bi}/\omega_E)^2]^2 + (2\zeta_E\omega_{Bi}/\omega_E)^2}} \tag{3}$$

The second acceleration component in equation 1, a_{EE}, is associated with the attached equipment mode, $a(\omega_E,\zeta_E)$, and is calculated as

$$a_{EE} = \left[1 + \sum_{i=1}^{N} \pm\, \beta(\theta_i,\zeta_{Bi})\Gamma_i\phi_{iE}\right] a(\omega_E,\zeta_E) \tag{4}$$

where $\beta(\theta_i,\zeta_{Bi})$ is called impedance factor and is calculated as

$$\beta(\theta_i,\zeta_{Bi}) = \frac{(\omega_E/\omega_{Bi})^2}{\sqrt{[1 - (\omega_E/\omega_{Bi})^2]^2 + (2\zeta_{Bi}\,\omega_E/\omega_{Bi})^2}} \tag{5}$$

Equation 4 is refined from equation 20 of Reference 3. Equation 4 takes into account the phase angles among the structural modes. For low damping values and off-resonance, the mode shapes are either in-phase (plus-sign) or out-of-phase (minus-sign). That is, when the equipment frequency, ω_E, is less than the ith structural frequency, ω_{Bi}, the structural mode is in phase with the equipment vibration and a plus-sign should be used. A minus-sign is used when ω_E is greater than ω_{Bi}.

In the resonance case, Equation 1 is replaced by the following expression:

$$a_E = \sqrt{a(\omega_{Bi},\zeta_{Bi})^2 + a_\gamma(\omega_{Bi},\zeta_\gamma)^2\Gamma_i\phi_{iE}} \tag{6}$$

where $a_\gamma(\omega_{Bi},\zeta_\gamma)$ is the resonance response acceleration. The following sections explain the resonance response acceleration and its associated Resonance RS.

4. Resonance Response Spectrum

The Resonance RS is defined as:

A plot of the maximum response (acceleration, velocity, or displacement) of an idealized spring-mass-dashpot oscillator, which is in resonance and supported by another spring-mass-dashpot oscillator of much heavier mass, as a function of the natural frequency of the oscillators to a specified vibratory motion input at the support of the heavier mass.

Figure 5 shows how the Resonance RS can be generated. The specified input motions, $\ddot{d}(t)$, excite the base. The two resonant oscillators are excited simultaneously. The resonance response is obtained when the heavier mass increases indefinitely, (i.e., $m/M = \mu \to 0$), while keeping the natural frequency of the two oscillators identical. The maximum response of the small mass, m, is the Resonance RS.

5. Generation of Resonance Response Spectrum

When the two spring-mass-dashpot oscillators of identical natural frequency are coupled into a two degrees-of-freedom system, the two frequencies of the coupled system are "shifted" away from the natural frequency of the individual oscillator. The amount of the frequency shift, $\omega_2 - \omega_1$, has been derived in Equation 29 of Reference 3 as

$$\omega_2 - \omega_1 = \omega\sqrt{\mu} \tag{7}$$

The time history response of the small mass, m, in the coupled system is also derived in Equation 36 of Reference 3 as

$$\ddot{x}_m(t) = \frac{1}{2\sqrt{\mu}}\left[\ddot{q}_1(t) - \ddot{q}_2(t)\right] + \ddot{x}_M(t) \tag{8}$$

where $\ddot{x}_M(t)$ is the response of the heavier mass alone (without the small mass) to the input excitation, $\ddot{d}(t)$. Equations 7 and 8 can be combined into a single equation, yielding

$$\ddot{x}_m(t) = \frac{-\omega}{2}\,\frac{\ddot{q}_1(t) - \ddot{q}_2(t)}{\omega_1 - \omega_2} + \ddot{x}_M(t) \tag{9}$$

In the limiting case when the mass ratio approaches zero, $\mu \to 0$, the two system frequencies, ω_1 and ω_2, both approach each other and a common frequency, ω. Thw two modal response time histories, $\ddot{q}_1(t)$ and $\ddot{q}_2(t)$, also approach each other and a common time history response $\ddot{q}_\gamma(t)$ at the natural frequency. Thus, the first term in Equation 9 becomes a differential equation expressed

$$\lim_{\mu \to 0} \ddot{x}_m(t) = \frac{-\omega}{2}\,\frac{\partial}{\partial\omega}\,\ddot{q}_\gamma(t) + \ddot{x}_M(t) \tag{10}$$

The resonance damping values, ζ_γ, for $\ddot{q}_\gamma(t)$ can be calculated using the equations derived in Reference 4 as

$$\zeta_\gamma = \frac{\zeta_m + \zeta_M}{2} \tag{11}$$

where ζ_m and ζ_M are the damping values associated with the resonant oscillators.

Equation 10 yields quite accurate results. Figure 6 shows the resonance response time history, while Figure 7 illustrates the corresponding time history from an exact solution. The resonance frequency is 2.88 and the damping values are ζ_m = 2%, ζ_M = 5% and ζ_γ = 3.5% for both Figures 6 and 7. These two time histories are almost identical.

Figure 2 shows the 3.5% Resonance RS of the input time history shown in Figure 1, generated using Equation 10. This Resonance RS is input to the FFRS technique for the generation of the Floor RS shown in Figures 3 and 4.

6. Discussion and Conclusion

The FFRS technique is compared with the TH method by using an example case. The Floor RS generated by the FFRS method coincides with that obtained with the TH method with the maximum difference between the two less than 13%. Further, since the FFRS uses the response spectral analytic technique, the cost of computer time is negligible (less than 1%) compared with the TH method.

The Resonance RS required as input to the FFRS technique can be generated in the same manner as for the Input RS, except that two coupled resonant oscillators are used. Only a single set of resonance damping values is needed to represent the structural damping values and equipment damping values, since the resonance damping value is the average of the two.

The following conclusions can be drawn:

(1) The FFRS technique is consistent with the TH method.

(2) Tremendous savings in computer time can be realized by using the FFRS techniqué.

References

[1] R. H. Scanlan and K. Sachs, "Floor Response Spectra for Multiple-Degree-of-Freedom Systems by Fourier Transform," Transactions of the 3rd International Conference on SMiRT, K5/5.

[2] A. K. Singh and S. Singh, "A Probabilistic Model for Seismic Analysis of Nuclear-Plant Structures" Transactions of the 4TH International Conference on SMiRT, K3/3.

[3] M. J. Yan, "Fast Floor Response Spectra Generation Technique," ASME PVP-Vol. 65, pp 1-16.

[4] M. J. Yan, "Composite Modal Damping in Structure," 79-PVP-71, Presented at ASME 1979 PVP Conference, San Francisco, June 25-29, 1979.

Nomenclature

$a(\omega_{Bi},\zeta_{Bi})$	Response spectral acceleration of the ith structural mode
a_E	Floor response spectral acceleration
a_{EB}	Acceleration component of the floor response spectrum associated with the ith structural mode
a_{EE}	Acceleration component of the floor response spectrum associated with the equipment frequency
$a_\gamma(\omega_{Bi},\zeta_\gamma)$	Resonance response spectral acceleration at the ith structural mode
c,C	Damping coefficients of the small and heavy mass oscillators, respectively
$\ddot{d}(t)$	Input earthquake time history
k,K	Spring constants of the small and heavy mass oscillators, respectively
m,M	Small mass and heavy mass of the resonant oscillators, respectively

$\ddot{q}_1(t), \ddot{q}_2(t)$	The two modal response acceleration time histories of the coupled resonant oscillators
$\ddot{q}_\gamma(t)$	The resonance response time history
$\ddot{x}_m(t), \ddot{x}_M(t)$	The time history acceleration responses of the small mass and heavy mass, respectively
$\alpha(\theta_i, \zeta_E)$	Amplification factor
$\beta(\theta_i, \zeta_{Bi})$	Impedance factor
ϕ_{iE}	Magnitude of the ith structural mode shape at the equipment attachment point
Γ_i	Participation factor of the ith structural mode
$\sum_{i=1}^{N}$	Summation from i = 1 to i = N
θ_i	ith frequency ratio and is equal to $(\omega_E/\omega_{Bi})^2$
μ	Mass ratio and is equal to m/M
ω_{Bi}	ith structural frequency
ω_E	Equipment frequency or the floor response spectral frequency
$\omega, \omega_1, \omega_2$	Natural frequency and the two frequency shifts of the resonant oscillators, respectively
ζ_{Bi}	ith structural modal damping value
ζ_E	Equipment damping or the floor response spectral damping value
$\zeta_\gamma, \zeta_m, \zeta_M$	Resonance damping and damping values of the small and heavy resonant oscillators.

Table 1. Building or Support Structure Information

	First mode	Second mode	Third mode
Frequency (ω_{Bi}), Hz	2.88	20.5	52.8
Participation factor (Γ_i)	1.296	-0.516	-0.335
Mode shapes $\{\phi_i\}$	$\begin{Bmatrix} 1.0 \\ 0.56 \\ 0.18 \end{Bmatrix}$	$\begin{Bmatrix} 0.71 \\ -1.0 \\ -0.84 \end{Bmatrix}$	$\begin{Bmatrix} 0.21 \\ 0.70 \\ -1.0 \end{Bmatrix}$

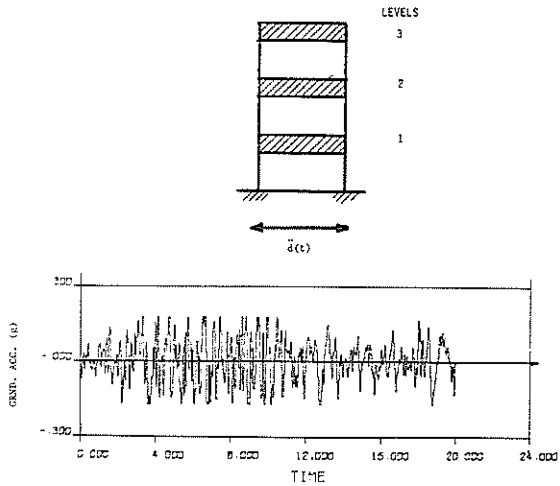


Figure 1. An Example of Strncture and Input Time History Used to Generate a Floor Response Spectrum

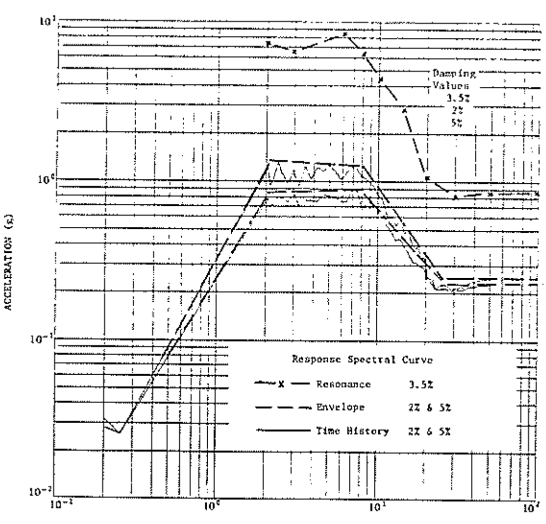
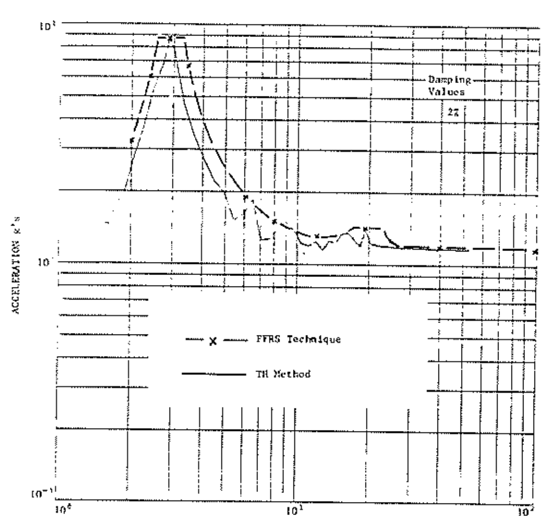


Figure 2. Resonance Response Spectrum and Corresponding Response Spectra for Time History Shown in Figure 1

Figure 3. Floor Response Spectra for Top Floor Generated by FFRS Techniqne and TH Method



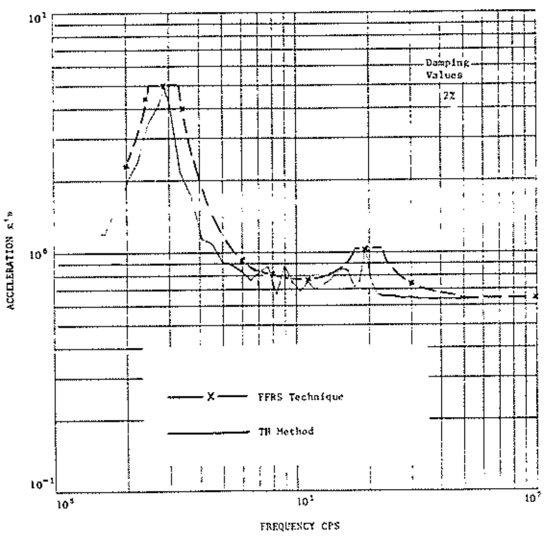


Figure 4. Floor Response Spectra for Second Floor Generated by FFRS Technique and TH Method

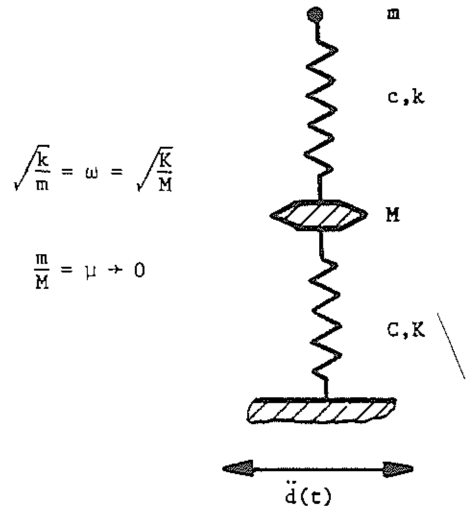


Figure 5. Resonance Response Spectrum Generation

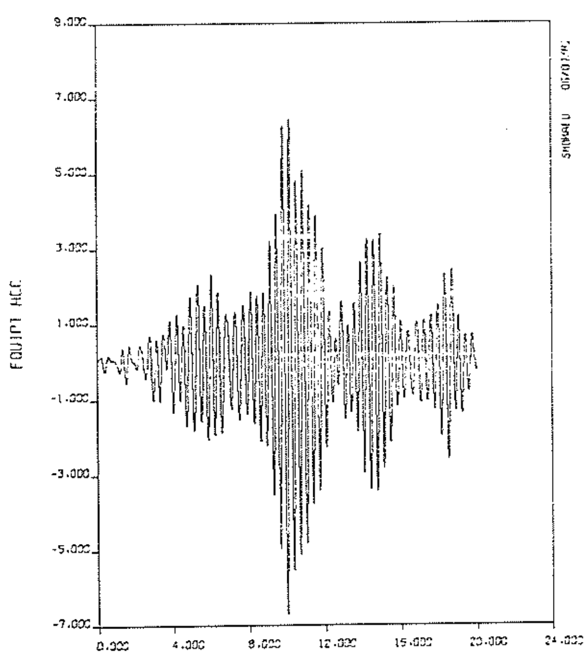


Figure 6. Resonance Response Time History Calculated by Equation 10

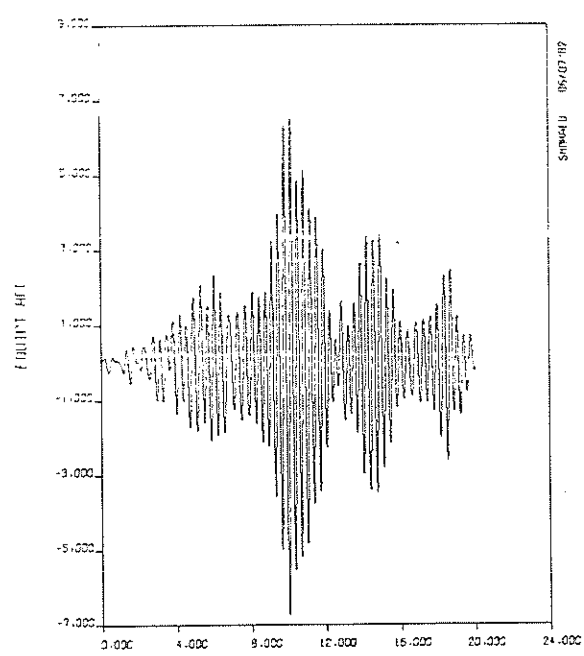


Figure 7. Resonance Response Time History From Exact Solution