On the Computation of FRS

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Summary

The problem of the computation of FRS is considered and discussed; a procedure is used defining the dynamic input as its power spectral density $\phi$ rather than its time history. A numerical iterative method is used to compute $\phi$. Basically three ranges are found:

a) light equipment: i.e. the cases in which the feed back action of the equipment on the structure is negligible

b) intermediate range: i.e. the cases in which the modes of the structure are not changed, however their amplitude can be changed by the equipment action

c) heavy equipment: i.e. the cases in which the modes of the structure are changed by the equipment reactions.

An equivalent system of two 1 dof oscillators each representing a mode of the structure or of the equipment is considered for cases a),b); the transfer functions of them are computed as a function of eigenfrequencies, damping and masses both of equipment and structure are considered to calculate the average quadratic value of the accelerations of equipment and structure as a function of the power spectral density of the input signal. The ratio of the average quadratic values of the accelerations is assumed to be the amplification value for the equipment. It is shown that even in the case of quite light equipments (mass ratios 0.01) a large advantage is found in the computation of acceleration peaks, particularly for narrow band excitations. In the case of heavy equipments it is excepted that a redistribution of the modes of the structure takes place; in this case it is assumed that each new eigenmode can be represented by a series expansion in terms of the "old" eigenmodes. Ralegh-Ritz method is then used to compute the eigenmodes and eigenfrequencies of the combined equipment-structure system.

One hypothesis of the method is the stationary nature of the input signal, while it is sufficiently accurate for seismic analysis it is too conservative for impulsive signals such as SHWD, and aircraft impact. In these cases a correction (based on Cauhoy-Stumpf theory) is applied.
1. **Introduction**

The computation of FRS in a structure due to some dynamic excitations (either earthquake or safety relief valve or aircraft impact) is a necessary step for the analysis of equipments inside the building. Usually time histories methods have been used, however alternative methods based on stochastic, semistochastic or other miscellaneous methods have been suggested /1,2,3/.

In all the cases one common hypothesis is the assumption that the feed back action of the equipment on the structure is so little that it can be reasonably neglected. However in the common practice this procedure is applied even for heavy equipments leading to very conservative results (an example is discussed in ref./5/); the problem has been discussed particularly in the case of resonance conditions in ref./4/. Another method which has been used is the so called two steps analysis, whereas the signal at some points of the structure is used for the analysis of a substructure (attached at that point) in some greater detail. The method, even if apparently straightforward, has large limitations, often leading to very conservative results, due to the difficulties in a correct evaluation of the modal damped of the substructure particularly in the cases, in which modes of the substructure are coupled with rigid body motions of the building on the soil.

In this paper the problem of the computation of FRS will be discussed in some detail and some computation techniques will be presented in the following ranges:

a) very light equipments in which the feed back actions on the main structure can be reasonably neglected

b) intermediate range, in which the feed back action is important and affects the response but not modes

c) heavy equipments for which the modal frequencies and shapes are affected.

2. **Light Equipments**

In this particular case the feed back action can be neglected. The method proposed herein assumes that the input signal for the structure can be represented by means of a distribution of power spectral density $\mathcal{S}(\omega)$, $\omega$ being the frequency. Then acceleration average quadratic value $\overline{\alpha}_n^2$ can be given in terms of the power spectral density and transfer function $H_s(\omega,\omega_s,\beta_1)$ where $\omega_s$, $\beta_1$ are structural frequency and modal damping (assimilating each mode to 1 dof system):

\[
\overline{\alpha}_n^2 = \int_{-\infty}^{\infty} \mathcal{S}(\omega) H_s^2(\omega,\omega_s,\beta_1) d\omega
\]

\[
H_s(\omega_s,\omega_s,\beta_1) = 1/(\omega_s^2 - \omega^2 + 2i\beta_1 \omega \omega_s)
\]

The power spectral density $\mathcal{S}(\omega)$ can be found as a function of the required response spectrum by the use of an iterative procedure, whereas letting $\alpha^2_n$ the spectral acceleration and subscript $m$ the step index, one has:

\[
\mathcal{S}_{m+1} = \mathcal{S}_m \cdot \frac{\alpha^2_n}{\alpha^2_{n,m}}
\]
For some cases linear programming techniques have been found to give some advantages (in that case a least squares technique with the constraint $0 > 0$ has been used).

If $H_e(\omega, \omega, \beta_0)$ is the transfer function for the equipment, the response of the equipment will be:

$$Q_e^2 = \int_{-\infty}^{\infty} \phi(\omega) H_e^2(\omega, \omega, \beta_0) \, d\omega$$  \(3a\)

and the amplification ratio $R_e(\omega)$ is then:

$$R_e^2 = \int_{-\infty}^{\infty} \phi(\omega) H_e^2(\omega, \omega, \beta_0) \, d\omega / \int_{-\infty}^{\infty} \phi(\omega) H_s^2(\omega, \omega, \beta_0) \, d\omega$$  \(3b\)

Note that a very simple statistics is inherently used in (3b) i.e. the amplification ratio for the equipment is equal to the ratio of the average quadratic values of the acceleration. Under this hypothesis it is important to compute the distribution of power spectral density but the multiplication of all the values for a constant is inconsequential. The FRS is then computed with the obvious correlation

$$Q_e(\beta_0, \omega_e) = Q_s(\beta_0, \omega_s); \quad R_e$$  \(4\)

the obvious advantage being the fact that no time history analysis is required as $Q_s$ can be computed with the usual modal analysis technique.

In the case of multimodal response the modal responses are combined by means of the usual technique i.e. SSRS or CCC /7/ or the method suggested in ref. /8/.

The method indicated has been devised specifically for the case of long duration signals, however in the case of short transitory behaviour, i.e. of short duration, some corrections are necessary. The transient response correction by Coughley and Stumpf /9/ has been implemented together with a change in the equivalent damping as suggested by Rosebleth and Elorduy /8/:

$$\beta = \beta_0 + 2/T \omega$$  \(5\)

where $\beta$ is the effective damping, $\beta_0$ is the material damping, $\omega$ is the pulsation and $T$ is the effective duration of the signal.

A large number of tests have been performed by means of this method implemented in the CYRANO code /6/, excellent agreement has been found in the case of transient as well as of response to earthquakes. A comparison is shown in fig.1.

3. Intermediate Range

As already mentioned, it is assumed that the influence of the equipment on the structure is not such as to modify the modes and eigenfrequencies but to alter the amplification ratio. The model which has been studied is the two single dofs system in series; in this case the transfer functions $H_s$ and $H_e$ are computed as a function of eigenfrequencies and damping of both the equipment and the structure:

$$H_s(\omega, \omega_s, \beta_e, \omega_e, \beta_s, \gamma) \quad H_e(\omega, \omega_e, \beta_e, \omega_s, \beta_s, \gamma)$$

where $\gamma$ is the mass ratio between equipment and structure; for the details see ref. /6/.

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Let then 
\[ \tilde{a}_e^2 = \int_{-\infty}^{\infty} \tilde{\omega} \tilde{\xi}_e^2 \, d\omega \]
\[ \tilde{a}_s^2 = \int_{-\infty}^{\infty} \tilde{\omega} \tilde{\xi}_s^2 \, d\omega \]
\[ \tilde{a}_{es} = \int_{-\infty}^{\infty} \tilde{\omega} \tilde{H}_e \, d\omega \]
\[ \tilde{a}_{es}^2 = \int_{-\infty}^{\infty} \tilde{\omega} \tilde{H}_s \, d\omega \]
(6a,c)
where \( \tilde{a}_{es} \) is the structural acceleration ignoring the effect of the equipment. Consequently the amplification ratio is given by:
\[ R^2 = \int_{-\infty}^{\infty} \tilde{\omega} \tilde{a}_e^2 \, d\omega / \int_{-\infty}^{\infty} \tilde{\omega} \tilde{a}_s^2 \, d\omega \]
(7)
Note that the effect of the mass ratio is always very remarkable if \( \gamma \) is not very little (even \( \gamma = 0.01 \) gives not negligible alterations) as shown in fig. 2. The effect is primarily due to local decrease of the structural acceleration \( \tilde{a}_s^2 \) as compared to \( \tilde{a}_{es}^2 \) as clearly indicated in fig. 3. The effect is particularly strong in the case of resonance between equipment and structure as shown; the method is somewhat similar to the one suggested by Sackman and Kelly /4/. Besides note that in the case of multimodal response, the structural mass is the one pertaining to the mode under consideration reduced to the attachment point (p) of the equipment. The modal mass can be computed as a result of the modal analysis of the structure by means of the correlation:
\[ m_{bi} = \sum_k \xi_{bi} \xi_k \mu_k / \xi_k^2 \]
(8)
where \( \xi_k \) is the modal shape of mode i at dof k
\( \mu_k \) is the mass at dof k
Note however that the modal acceleration is proportional to \( \xi_k \mu_k \) consequently the correction is particularly strong in the areas of strong response.

4. Heavy Equipments
In this range it is assumed that the equipment response is such as to modify appreciably the modal shapes and frequencies of the structure. A typical example is the interaction between the containment and the polar crane; as discussed in ref. /5/.

The method proposed herein assumes that the structure modal analysis has been made and that \( [\xi_k] \) and \( [\omega_k] \) are the modal shapes and eigenfrequencies. Assume now that such modal shapes are slightly changed by the adding of the mass of some heavy equipment and that \( \gamma_{nm} \) and \( \omega_n \) are the modified modal shapes and eigenfrequencies, then let us assume that:
\[ \xi_{sn} = \sum_k \lambda_{nk} \xi_k \]
(9)
where \( \lambda_{nk} \) are the unknown multipliers of the old modes. The potential and kinematic energy of the structure can be computed as a function of the \( \lambda_{nk} \) coefficients as:
\[ Z_{sn} = 0.5 \sum_k m_{kp} \Omega_k^2 \omega^2_{pk} \omega_n^2 \]
kinetic energy
(10a)
\[ \zeta_{sn} = 0.5 \sum_k m_{kp} \Omega_k^2 \omega^2_{pk} \omega_k^2 \]
potential energy
(10b)
where \( m_{kp} \) is the effective mass of the old Kth mode reduce to the attachment point p
\( \omega_{nk} \) eigenpulsation of new n mode
\( \omega_{kn} \) eigenpulsation of the old Kth mode

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In the same way the kinetic and potential energy of the equipment are computed as:

\[ \mathcal{E}_m = 0.5 \, m_e \, \lambda_{m e} \, \omega_m^2 \]
\[ \mathcal{E}_e = 0.5 \, k_e \left( \lambda_{m e} - \sum_k \mathcal{O}_{p k} \lambda_{p k} \right)^2 \]

(10c) 
(10d)

where \( m_e \), \( k_e \) are mass and stiffness of the equipment and \( \lambda_{m e} \) is the motion modal amplitude of the equipment. If then the difference between kinetic and potential energy is minimized with respect to the \( \lambda \) coefficients, i.e.

\[ \sum \lambda \left( \mathcal{E}_m + \mathcal{E}_e - \mathcal{E}_{m e} - \mathcal{E}_{r e} \right) = 0 \]

(11)

one finds a square matrix \( C \) function of \( \omega_m \) whose determinant must be zero in order to avoid the trivial solution of zero amplitudes.

\[ \det C(\omega) = 0 \]

(12)

Note that the method (which can be slightly modified in considering similar problems e.g. the case in which the equipment mass was originally rigidly connected to the structure) is the better, the better is the approximation in the old modes.

The method is incorporated in LISE code \( /10/ \).

The two steps analysis method already described in paragraph one is often used in these cases; however to avoid over conservative responses much care damping, which should be equal to the various modal dampings in the original analysis of the structure.

5. Conclusions

The computation of Floor Response Spectra is generally made assuming no "feed back" action from the equipment to the structure, either by means of TH procedures or stochastic or semistochastic procedures. This procedure is appropriate only for very light equipments (say with a mass less than 1/100 of the effective mass of the structure); in this case procedure is given, which very favourably compares with the usual TH techniques (particularly in the case of the assumption of spectral power density made to match the actual FRS) at much lower costs.

In the case of heavier equipments this procedure is too conservative particularly in the case of resonance conditions; in this case (for mass ratios between 1/100 to say 1/10) an original procedure is presented, which assumes no modification of the structure eigenmodes and eigenfrequencies, but takes care of the reduction of modal amplitudes only in the structure.

In the case of extremely heavy equipments, a procedure is suggested that takes into account the modes of the structure without equipments (or with rigid mass representation) as a set of independent orthogonal eigenvectors and assumes that the final modes will be a combination of all the original eigenvectors. The coefficients of the combination as well as the amplitude of the equipment displacement are established on the basis of Rayleigh-Ritz method.

Some comments are offered in the case of "two steps analysis"; it is pointed out that
if the analysis of the subsystem is performed assuming the damping is equal to the subsystem damping, the results are generally overly conservative. The analysis should be performed computing separately the contributions of the structure modes, in each case the subsystem mode should be considered with a damping equal to the mode damping rather than to the subsystem damping.

References


