Random Vibration of Cascaded Secondary Systems

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SUMMARY

The random vibration modal time history (RVMT) formulation has been extended for the analysis of cascaded, point-supported secondary systems. Fundamentally, the secondary system's modal state vector and its transition matrix have been defined. Analytical results for evolutionary modal responses to non-stationary excitation have also been determined and are given in a related research report. The effects of multiple correlated components and correlated modal responses are combined exactly to arrive at evolutionary RMS response time histories.

Two example problems are solved; one examines the accuracy of the cascading assumption for a particular system; the other illustrates the ability of treating multiple, correlated excitation components and indicates the types of studies which can be performed with the formulation.

INTRODUCTION

Equipment and piping in power plants must be qualified for the seismic extreme load condition. The qualification process often includes dynamic analysis, and so two questions arise. One is, can the equipment be treated as a secondary dynamic system cascaded from the primary dynamic system? The other is, what method of dynamic analysis should be used? A comprehensive recent review of the literature on those questions is given in Ref. [7]. Some fundamental aspects of the problems are as follows.

The ratio of the mass of the secondary system to the mass of the primary system and the ratio of the frequency of the secondary system to the frequency of the primary system are the two main variables which affect the level of coupling between two systems. If the mass ratio is small and the fundamental frequencies are far apart, the effects of the secondary system on the primary system become negligible, and the two systems can be treated as cascaded. If the frequency ratio is close to one, then even for very small mass ratios the effects of the secondary system on the primary system may be significant. Indeed, the "tuned mass damper" idea utilizes the interaction to decrease the vibration of the primary system. Studies [9] have been done to determine for which mass and frequency ratios the effects of the secondary system on the primary system become negligible. Based on such studies, cascading criteria have been proposed. The U.S. NRC uses the following [12]: If the mass ratio, \( R_m \), is < 0.01 then cascading is acceptable for any frequency ratio, \( R_f \);
if \(0.01 \leq R_m \leq 0.1\) cascading is acceptable if \(0.8 \geq R_f \geq 1.25\); if \(R_m > 0.1\) an approximate model of the subsystem should be included in the primary system model.

Assuming that two linear systems may be cascaded, several deterministic and stochastic dynamic analysis approaches are possible. For example, Biggs [1]; Scanlan and Sachs [10]; Singh and Wen [11] and others have proposed methods for deriving floor response spectra. There is continuing work on improvement of procedures for superposition of modal responses and responses from separate floor excitation components. A direct way to generate floor response spectra is from the response of the floor to an artificial time history which is consistent with a prescribed ground response spectrum. Kurokaki and Kozeki [8] have quantified the uncertainties inherent in such a procedure. They present statistics of floor response spectra ordinates from 35 spectrum-consistent artificial time histories.


Herein the random vibration modal time history (RVMTH) approach given in Ref. [5] is used for the analysis of cascaded, point-supported secondary systems. The modal coordinates of the primary system are treated as a set of filters in parallel. Only some fundamental equations of the formulation are given herein; detailed analytical results are given in Ref. [6]. Example problems are presented to show the capabilities of the formulation. Other issues studied are the accuracy of the cascaded analysis for several mass and frequency ratios, the correlation between responses of the primary system, the effects of correlation between ground excitation components and the nonstationarity of responses.

**RANDOM VIBRATION FORMULATION**

The random vibration modal time history (RVMTH) approach given in Ref. [5] was extended for the analysis of cascaded secondary systems. Briefly, the main assumptions/capabilities of the analysis are:

1) The excitation is vector-valued, nonstationary, nonwhite (defined by filters) and correlated.
2) The primary system is linear, decoupled into modes and cascaded from the filters which define the frequency content of the excitation.
3) The secondary system is linear, decoupled into modes and cascaded from the primary system.

At present the formulation considers only point supported secondary systems although the excitation at the support point can be vector-valued. The primary difference between the formulation given in Ref. [5] and that used herein is the secondary system's modal state vector and its transition matrix. The modal state vector is defined as follows: let

\[ M = \text{number of ground excitation components (also equal to the number of filters)} \]

\[ \omega_m = m^{th} \text{ excitation component} \]

\[ a_m^n = \text{filter parameter for } m^{th} \text{ excitation component} \]

\[ c_m^n = \text{filter parameter for } m^{th} \text{ excitation component} \]

\[ \omega_1 = 1^{st} \text{ modal frequency of primary system} \]
\[ \zeta_{ij} = \text{ith modal damping ratio of primary system} \]
\[ a_{ij} = \text{ith modal coordinate of primary system} \]
\[ N = \text{number of significant modes of primary system} \]
\[ N' = \text{number of excitation components for secondary system} \]
\[ \omega_{s_j} = \text{ith modal frequency of secondary system} \]
\[ \zeta_{s_j} = \text{ith modal damping ratio of secondary system} \]
\[ a_{s_j} = \text{ith modal coordinate of secondary system} \]
\[ N' = \text{number of significant modes of secondary system} \]

Then the three equilibrium equations for the filters, the primary system and the secondary system are, respectively:

\[ \ddot{y}_{fm} + 2\zeta_{fm}\omega_m \dot{y}_{fm} + \omega_m^2 y_{fm} = -w_m \]  \hspace{1cm} (1)

\[ \ddot{a}_i + 2\zeta_i \omega_i \dot{a}_i + \omega_i^2 a_i = \sum_{m=1}^{N} \gamma_{mi}(-2\zeta_{fm}\omega_m \dot{y}_{fm} - \omega_m^2 y_{fm}) \]  \hspace{1cm} (2)

\[ \ddot{a}_{s_j} + 2\zeta_{s_j} \omega_{s_j} \dot{a}_{s_j} + \omega_{s_j}^2 a_{s_j} = \sum_{i=1}^{N'} \sum_{j=1}^{N} \nu_{i'j'}(-2\zeta_{i'j'} \omega_{i'j'} \dot{y}_{i'j'} - \omega_{i'j'}^2 y_{i'j'}) \]  \hspace{1cm} (3)

\( \gamma \)'s are constants determined from the eigenvectors of the systems. The modal state vector for the secondary system is then defined as:

\[ \mathbf{z}_{s_j}^T = \begin{bmatrix} a_{s_j} & \cdots & a_{s_1} \cdots a_{s_N} & \cdots & y_{f_2} & \cdots & y_{f_{N'}} & y_{f_{N+1}} & \cdots & y_{f_{N+2M}} \end{bmatrix}_{i=1}^{N+2M} \]  \hspace{1cm} (4)

With this definition a modal equilibrium equation for a cascaded secondary system may be written as:

\[ \ddot{\mathbf{z}}_{s_j} = A_{s_j} \mathbf{z}_{s_j} + B_{s_j} \mathbf{w} \]  \hspace{1cm} (5)

The matrices \( A_{s_j} \) and \( B_{s_j} \) and the analytical expressions for the transition matrix for such a system are given in Ref. [6]. The remainder of the formulation is analogous to that described in Ref. [5]. Equations which govern the evolution of the modal mean matrix and of the cross-covariance matrix between two modal state vectors are written and solved analytically for nonstationary excitation having a piece-wise linear function. Desired response vectors are expressed as linear functions of the modal state vectors.

Modal superposition is then used to compute evolutionary mean and covariance matrices of the desired response vectors. All modal responses and responses from multiple, correlated excitation components are combined exactly. There are no approximations made.

**EXAMPLE PROBLEMS**

Two problems with different frequency and mass ratios are solved to show the capabilities of formulation. The first problem uses simple lumped mass stick models as shown in Fig. 1 with two degrees of freedom (DOF) in the primary system and three DOF in the secondary system. RMS distortion responses of the combined and the cascaded systems are compared to check the formulation and cascading criteria.

The second problem consists of a two storied frame as the main system and a stick model as a secondary system as shown in Fig. 2. It demonstrates the ability of the formulation to handle multiple excitation components to a point-supported secondary system and
to capture the effects of correlated vector-valued ground excitation. The correlation between excitation components to the secondary system and their effects on overall response are examined.

**Problem One - Systems** - Table 1 indicates the modal characteristics of the primary system. The fundamental frequency, \( f_1 = 6.57 \text{ Hz} \), lies in the range which is typical for nuclear power plants. The modal damping value for the primary system and the secondary system is 2% in all modes.

The secondary system is supported at the top mass of the primary system as shown in Fig. 1. Results from eigenvalue analyses of the cascaded and fully coupled secondary system are given in Ref. [6] for mass ratios of 0.005 and 0.1 and frequency ratios of 0.79 and 1.25. (Frequency ratios of 0.8 and 1.25 are used by the U.S. NAC in their cascading criteria). The mass ratio is defined as the total mass of the secondary system divided by the total mass of the primary system. The frequency ratio is defined as the fundamental frequency of the secondary system divided by the fundamental frequency of the primary system.

The natural frequencies of the cascaded secondary system are reflected in the modal properties of the fully coupled system, especially for the smaller mass ratios [6].

**Excitation** - Scalar evolutionary white noise is used as input to the filter. Its strength function is shown in Fig. 3; the maximum intensity, \( q_{\text{max}} = 100 \text{ in}^2/\text{sec}^3 \) corresponds approximately to an RMS ground acceleration of 0.06g. The filter parameters \( \omega_f \) and \( \zeta_f \) are 2\( \pi \) and 0.4 respectively.

**Results** - Figs. 4 a,b show evolutionary RMS distortion and absolute acceleration responses of the coupled and cascaded secondary system for \( R_m = 0.003 \) and \( R_f = 0.79 \). Peak RMS responses of the cascaded secondary system are higher by approximately 1-4%; differences are almost equal for all masses. For \( R_m = 0.005 \) and \( R_f = 1.25 \), Figs. 4 a,b show differences of approximately 7%. Although not shown, the case of \( R_m = 0.01 \) and \( R_f = 1.25 \) yielded peak cascaded secondary responses about 14% greater than coupled responses. Figs. 5 a,b, for \( R_m = 0.1 \) and \( R_f = 0.75, 1.25 \), show that differences of 100% between cascaded and coupled responses can occur at such values of \( R_m \) and \( R_f \).

**Problem Two - Systems** - Problem two illustrates the case of correlated, vector-valued excitation for both the primary system and the point-supported, cascaded secondary system. The structure model is shown in Fig. 2; it is a plate frame with a secondary system at one point. Only one mass ratio, \( R_m = 0.005 \), and only one frequency ratio, \( R_f = 1.29 \), is considered. Ref. [6] gives properties of the first six modes of the primary system and of the first four modes of the cascaded secondary system. Lumped masses were assumed to have masses in the horizontal and vertical directions only. The rotational mass moments of inertia were neglected. Damping of 2% was assumed for all modes of both systems.

**Excitation** - Two-dimensional, filtered, non-stationary excitation is used as input to the primary system. The strength function of the horizontal component is shown in Fig. 3. The vertical component is assumed to have the same strength variation but with an intensity \( 2/3 \) that of the horizontal component. Filter parameters were \( \omega_f = 2\pi \) and \( \zeta_f = 0.4 \) for both components. Only responses to perfectly correlated and uncorrelated components are presented. Although such an excitation model is simplified, it must be emphasized that the formulation can accept components having different frequency contents, strength functions and time-varying correlation with other components.
Results – The time variation of cross correlation coefficient between the absolute acceleration responses of the primary system at points a and b (refer to Fig. 2) for cross correlation coefficients between ground excitation components equal to 0.0 and 1.0 are plotted in Figs. 6 and 7, respectively. Fig. 6 shows the correlation between response 4a, i.e., the horizontal acceleration at point a, and the other five responses. Clearly cross-correlation coefficients attain stationarity much faster than individual RMS responses. Responses 4a, 6a, 10 and 12 are highly correlated because they are dominated by the same mode(s) of the dynamic system. Conversely, response 4a is not correlated with 5a and 11 because the modes which contribute to 4a do not contribute significantly to the vertical acceleration responses. Fig. 7 which shows the correlation between acceleration response 5 and the other responses, can also be explained by the primary system's modal characteristics. If the ground excitation components are perfectly correlated, then all responses, including 5a and 11 become correlated, as shown in Figs. 6a and 7a.

The RMS relative displacement and absolute acceleration responses of the secondary system with $R_m = 0.005$ and $R_f = 1.29$ are shown in Fig. 8. Fig. 8(a) shows responses to uncorrelated (base) excitation components; Fig. 8(b) shows responses to perfectly correlated (base) excitation components. RMS horizontal relative displacements were dominant, indeed the vertical displacements could not be plotted on the same scale. However, RMS vertical accelerations were of the same order of magnitude as horizontal accelerations. Comparing Figs. 8a) and b), it can be seen that the effects of correlation between base excitation components are very small. This is so because one group of responses, i.e., 1, 4, 7 are largely determined by the horizontal excitation component and one set of modes while another group of response; i.e., 2, 5 and 8, are largely determined by the vertical excitation component and different modes.

CONCLUSIONS

The random vibration modal time history (RVMTH) formulation of Ref. [5] has been extended for the analysis of cascaded, point-supported secondary systems. Fundamentally, the secondary system's modal state vector and its transition matrix have been defined. Analytical results for evolutionary modal responses to non-stationary excitation have also been determined and are given in a related research report [6]. The effects of multiple correlated components and correlated modal responses are combined exactly to arrive at evolutionary RMS response time histories.

Two example problems were solved; the objectives of the problems were not to derive general statements on the behavior of secondary systems; rather, one simply examined the accuracy of the cascading assumption for a particular system; the other illustrated the ability of treating multiple, correlated excitation components and indicated the types of studies which can be performed with the formulation.

The random vibration analysis can be part of the seismic qualification process. First the eigenvalue problem for the secondary system is solved. A RVMTH analysis can be performed if the ground excitation components and the properties of the first few modes of the primary system are specified. No floor excitation needs to be specified. The way evolutionary RMS responses are used for design depends on the reliability criteria for the specified load intensity. Or, design responses are those which have the specified probability of non-exceedance. In effect they will be multiples of peak RMS responses.
REFERENCES


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**TABLE 1**

<table>
<thead>
<tr>
<th>Mode ( j )</th>
<th>Period (sec)</th>
<th>Frequency (Hz/sec)</th>
<th>Normalized Node Shapes ( \phi_j )</th>
<th>Participation Factor ( Y_j )</th>
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<td>1</td>
<td>0.1523</td>
<td>41.2674</td>
<td>0.3731, 0.6787</td>
<td>1.0824</td>
</tr>
<tr>
<td>2</td>
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<td>95.42</td>
<td>-0.6057, 0.5272</td>
<td>-0.6442</td>
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