Seismic Response of Multiply Supported Piping Systems

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Summary

A new floor-spectrum method for seismic analysis of multi-degree-of-freedom secondary systems with multiple attachment points, such as piping in nuclear power plant structures, is presented. The method is based on an extension of the conventional floor response spectrum concept, defined through a cross-oscillator cross-floor spectrum, which includes the effects of correlation between modal responses and between the excitations at the various support points on the primary subsystem. Through this extended concept, the most important shortcomings of the current floor-spectrum methods are resolved. Thus, the proposed method correctly accounts for tuning between modes of the two subsystems and for the cross-modal and cross-support correlations mentioned above. These effects are either neglected or improperly considered in the current methods. The new method presently does not include the effect of interaction between the primary and secondary subsystems; however, it can be extended to include this effect.

An efficient method for evaluation of the cross-oscillator, cross-floor response spectra directly in terms of the input ground response spectrum and the dynamic properties of the primary subsystem is developed. This method uses perturbation techniques to generate the modal characteristics of a generic $N+2$-degree system, which is modeled as the primary subsystem with two oscillators attached at two different support points. It is shown that the cross-oscillator, cross-floor response spectrum is proportional to the covariance of the response of the two oscillators in the $N+2$-degree system. The cross-cross spectrum is, thus, obtained as the superposition of the modal responses of the $N+2$-degree system.

In terms of the cross-oscillator cross-floor response spectra, the peak response of the secondary subsystem is obtained as a four-fold summation. Two of the summations are over the modes of the secondary subsystem and the other two are over the support points. All other quantities involved in the summations are given in terms of the secondary subsystem properties only.

Example applications for a simple system are presented which show remarkable agreement with exact results obtained by consideration of the primary-secondary system as a single structure.
Introduction

The current methods (1) of seismic analysis of secondary systems, such as piping in nuclear power plant structures, are based on the concept of floor spectrum. Briefly stated, the method involves specification of the motions at various support points of the secondary subsystem in terms of floor response spectra, and modal dynamic analysis of the secondary subsystem as subjected to these spectra. This approach has certain practical advantages: (a) it avoids the dynamic modeling and analysis of the combined primary-secondary system which can be prohibitively costly, (b) it avoids numerical difficulties that arise in the analysis of the combined system because of large differences between the properties of the two subsystems, (c) it is inexpensive relative to time-history integration methods, and (d) it allows the secondary subsystem analyst to work independently of the primary subsystem characteristics, once the floor spectra are specified.

The floor-response spectrum approach as currently applied, however, has several important shortcomings. These may include (a) neglect of interaction between the primary and secondary subsystems, which can be significant when the two subsystems have tuned or nearly tuned natural frequencies, (b) neglect or improper consideration of the correlation between closely spaced modes in the primary and secondary subsystems, (c) neglect or improper combination of the correlation between the excitations at the support points of the secondary subsystem, (d) artificial separation of the response into "pseudo static" and "dynamic" parts, because of which a proper modal combination rule can not be developed, and (e) neglect of the effect of nonclassical damping, which can be significant even when the two subsystems are individually modally damped. In addition, although response spectrum methods for generating floor spectra have recently become available (5,7), in most applications the floor spectra are generated using "spectrum-compatible" ground time histories in conjunction with time-history dynamic analysis of the primary subsystem. Besides being too costly, this approach is inappropriate since different time histories compatible with the same ground spectrum may lead to very different estimates of the peak response.

In this paper, a new floor spectrum method for seismic analysis of multi-degree-of-freedom secondary subsystems with multiple attachment points is developed. The method is based on an extension of the conventional floor spectrum concept, defined through a cross-oscillator cross-floor response spectrum, which includes the effects of the correlation between model responses of the secondary subsystem and the correlation between the excitations at the support points. The method presently neglects the interaction between the two subsystems. However all the other shortcomings listed above are resolved. The cross-oscillator, cross-floor response spectra can be evaluated directly in terms of the input ground response spectrum and an efficient method for their generation is presented. Example applications are used to demonstrate the remarkable accuracy of the method.

Formulation of Piping Response in Terms of Power Spectral Density

Consider a primary structure with \( N \) degrees of freedom subjected to base input \( u_b(t) \). Attached to this structure at \( n \) points is an \( n + n \) degrees of freedom secondary subsystem, which may represent an extended equipment item or a piping system. The attached degrees of freedom in the primary and secondary subsystems are selected to be \( i = 1, \ldots, n_p \) and \( i = n_p + 1, \ldots, n \), respectively, such that the unattached degrees of freedom are \( i = n_p + 1, \ldots, n \). This is schematically illustrated in Fig. 1.

Let \( \mathbf{U} = [u_a \ u_r]^T \) denote the vector of absolute displacement of the primary subsystem, which is partitioned for the attached \((u_a)\) and unattached \((u_r)\) degrees of freedom. Similarly, let \( \mathbf{u} = [u_a \ u_r]^T \) denote the partitioned vector of absolute displacements of the secondary subsystem. The coupled equations of motion for the primary and secondary subsystems can be written, respectively, as:

\[
\begin{bmatrix}
\mathbf{m} & 0 \\
0 & \mathbf{m}_r
\end{bmatrix}
\begin{bmatrix}
\dot{u}_a \\
\dot{u}_r
\end{bmatrix} + \begin{bmatrix}
\mathbf{c} & \mathbf{c}_d \\
\mathbf{c}_d & \mathbf{k}_d
\end{bmatrix}
\begin{bmatrix}
\dot{u}_a \\
\dot{u}_r
\end{bmatrix} + \begin{bmatrix}
\mathbf{k} & \mathbf{k}_d \\
\mathbf{k}_d & \mathbf{k}_r
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_a \\
\ddot{u}_r
\end{bmatrix} = \begin{bmatrix}
\mathbf{f} \\
0
\end{bmatrix}
\]  \( \quad (1) \)

\[
\begin{bmatrix}
\mathbf{m}_a & 0 \\
0 & \mathbf{m}_r
\end{bmatrix}
\begin{bmatrix}
\dot{u}_a \\
\dot{u}_r
\end{bmatrix} + \begin{bmatrix}
\mathbf{c}_a & \mathbf{c}_d & \mathbf{c}_a \\
\mathbf{c}_d & \mathbf{k}_d & \mathbf{c}_d \\
\mathbf{c}_a & \mathbf{c}_d & \mathbf{k}_d
\end{bmatrix}
\begin{bmatrix}
\dot{u}_a \\
\dot{u}_r
\end{bmatrix} + \begin{bmatrix}
\mathbf{k}_a & \mathbf{k}_d & \mathbf{k}_a \\
\mathbf{k}_d & \mathbf{k}_d & \mathbf{k}_d \\
\mathbf{k}_a & \mathbf{k}_d & \mathbf{k}_r
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_a \\
\ddot{u}_r
\end{bmatrix} = \begin{bmatrix}
\mathbf{f} \\
0
\end{bmatrix}
\]  \( \quad (2) \)

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In the above, \( M, C, K \) are the conventional mass, damping, and stiffness matrices of the primary subsystem, respectively; \( R \) is the influence vector that couples the ground motion to the degrees of freedom of the primary subsystem; \( m, c, k \) are the fixed base matrices of the secondary subsystem, \( c_s \) and \( k_s \) are coupling matrices which include the dampings and stiffnesses of the connecting elements of the secondary subsystem, and \( m_a, c_a, k_a \) are matrices associated with the attachment points of the secondary subsystem. Note that the full matrices in eq. (2) are the mass, damping, and stiffness matrices of the secondary subsystem considered as a free-free system. The \( n_a \)-vector \( f \) represents the interaction forces exerted by the secondary subsystem on the primary subsystem. Note that for compatibility \( U_a = U_s \).

The complete solution for the combined system involves a simultaneous solution of eqs. (1) and (2). For practical reasons mentioned before, however, a solution of the secondary subsystem independent of the primary subsystem characteristics is desired. The standard simplifying approach is to neglect the interaction between the primary and secondary subsystems, which is acceptable when the secondary subsystem is sufficiently light in comparison to the primary subsystem (5). This approach will be used in this paper.

The equations of motion for the unattached degrees of freedom of the secondary subsystem can be written, using the partitioning in eq. (2) and the identity \( U_s = U_s \), as

\[
m\ddot{u}_s + c_s \dot{u}_s + k_s u_s = -c_s \dot{U}_s - k_s U_s
\]

(3)

Note that with the interaction neglected, \( U_s \) can be readily obtained from eq. (1) with \( f = 0 \). For the sake of simplicity, the damping terms on the right-hand side of eqs. (1) and (3), which are generally small for structural systems, will be neglected in the following.

In order to develop a response spectrum method, it is essential to use a modal approach. Let \( \Phi = [\phi_1 \phi_2 \cdots \phi_n] \) and \( \phi = [\phi_1 \phi_2 \cdots \phi_n] \) denote the modal matrices of the primary subsystem and the fixed base secondary subsystem, respectively. Also, let \( \Omega, Z, \omega, \zeta \) denote the modal frequencies and damping ratios of modes \( l \) and \( i \) of the two subsystems, respectively. Using eq. (3), and following standard techniques in stationary random vibrations, the power spectral density of the absolute acceleration at degree of freedom \( r \), \( \dot{u}_s \), of the secondary subsystem can be shown to be (2)

\[
G_{\dot{u}_s}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \sum_{l=1}^{L} \sum_{k=1}^{K} h_k b_{ik} \omega_i^2 \omega_j^2 h_l(\omega) h_l(-\omega) G_{\dot{U}_k \dot{U}_l}(\omega)
\]

(4)

in which

\[
a_{ij} = \frac{\phi_i \phi_j}{m_i \omega_i^2} \quad \text{and} \quad b_{ik} = \sum_{m=1}^{n} \phi_m k_{im}.
\]

(5)

\( h_l(\omega) = (\omega_l^2 - \omega^2 + 2i\zeta_l \omega_l \omega) \) is the complex frequency response function of mode \( i \) of the secondary subsystem, and \( G_{\dot{U}_k \dot{U}_l}(\omega) \) is the cross-power spectral density of the absolute acceleration responses of the structure at the attachment points \( K \) and \( L \). Integrating over the frequency range, the mean square of the response is obtained

\[
E[\dot{u}_s^2] = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \sum_{l=1}^{L} \sum_{k=1}^{K} b_{ik} b_{jl} \lambda_{i,j,k,l}
\]

(6)

where

\[
\lambda_{i,j,k,l} = \int_0^\infty \omega_i \omega_j h_l(\omega) h_l(-\omega) G_{\dot{U}_k \dot{U}_l}(\omega) d\omega
\]

(7)

The above term may be interpreted as the covariance of the acceleration response of two oscillators of frequency and damping \( \omega_i, \zeta_i \) and \( \omega_j, \zeta_j \), which are subjected to base acceleration inputs \( \dot{U}_k \) and \( \dot{U}_l \), respectively. An interpretation of this term in terms of floor spectra is given in the following section.
Interpretation of Response in Terms of Floor Spectra

First consider the term $\lambda_{0,\text{K}}$. This may be regarded as the mean square of the acceleration response of a light oscillator of frequency $\omega_i$ and damping $\zeta_i$ which is attached to the $K$-th degree of freedom of the primary subsystem, where the interaction between the oscillator and the primary subsystem is ignored. Figure 2(a) illustrates this concept schematically. If $\tilde{S}_K(\omega_i, \zeta_i)$ denotes the mean "floor" response spectrum associated with degree of freedom $K$, then the following relation holds (3)

$$\lambda_{0,\text{K}} = \frac{1}{p_{\text{K}}} \tilde{S}_K^2(\omega_i, \zeta_i) = \frac{1}{p_{\text{K}}} \tilde{S}_K(\omega_i, \zeta_i, \omega_i, \zeta_i)$$

(8)

in which $p_{\text{K}}$ is a peak factor associated with the response process of the oscillator, and $\tilde{S}_K(\omega_i, \zeta_i, \omega_j, \zeta_j) = \tilde{S}_K^2(\omega_i, \zeta_i)$. The reason for defining the latter term will become clear shortly.

Now consider the term $\lambda_{0,\text{KL}}$. This may be regarded as the covariance of the acceleration responses of two light oscillators of frequency and damping $\omega_i, \zeta_i$ and $\omega_j, \zeta_j$ which are both attached to the $K$-th degree of freedom of the structure. An illustration of this case is shown in Fig. 2(b). This term may not be interpreted in terms of the conventional floor spectra; however, it can be used to define an extension of the floor-spectrum concept, which may then be used to evaluate it. We define the relation

$$\lambda_{0,\text{KL}} = \frac{1}{p_{\text{K}}p_{\text{L}}} \tilde{S}_{KL}(\omega_i, \zeta_i, \omega_j, \zeta_j)$$

(9)

$\tilde{S}_{KL}(\omega_i, \zeta_i, \omega_j, \zeta_j)$ may be interpreted as a cross-oscillator floor response spectrum associated with the $K$-th degree of freedom of the primary subsystem. Note that this term has the dimension of the square of the floor spectrum.

Next consider the term $\lambda_{0,\text{KL}}$. This term may be regarded as the covariance of the acceleration responses of two identical light oscillators of frequency $\omega_i$ and damping $\zeta_i$, which are attached to the degrees of freedom $K$ and $L$ of the primary subsystem, as shown in Fig. 2(c). Following the above idea, we define the relation

$$\lambda_{0,\text{KL}} = \frac{1}{p_{\text{K}}p_{\text{L}}} \tilde{S}_{KL}(\omega_i, \zeta_i, \omega_i, \zeta_i)$$

(10)

where $\tilde{S}_{KL}(\omega_i, \zeta_i, \omega_i, \zeta_i)$ may be interpreted as the cross-floor response spectrum associated with degrees of freedom $K$ and $L$.

Finally, consider the term $\lambda_{0,\text{KL}}$. It should be clear that this is the covariance of the acceleration response of two light oscillators of frequency and damping $\omega_i, \zeta_i$ and $\omega_j, \zeta_j$ which are attached to degrees of freedom $K$ and $L$ of the primary subsystem, respectively, as illustrated in Fig. 2(d). We define

$$\lambda_{0,\text{KL}} = \frac{1}{p_{\text{K}}p_{\text{L}}} \tilde{S}_{KL}(\omega_i, \zeta_i, \omega_j, \zeta_j)$$

(11)

where $\tilde{S}_{KL}(\omega_i, \zeta_i, \omega_j, \zeta_j)$ can be interpreted as cross-oscillator, cross-floor response spectrum associated with the degrees of freedom $K$ and $L$. For convenience, this most general term may be defined as cross-cross flow spectrum or, in short, CCFS.

Using the relation $E[\tilde{u}_{r,\text{max}}] = p E[\tilde{u}^2]^{1/2}$ between the mean of the peak and the mean-square response, where $p$ is the peak factor, and substituting eq. (11) in eq. (6), the mean of the peak acceleration at the degree of freedom $r$ of the secondary subsystem is obtained

$$E[\tilde{u}_{r,\text{max}}] = \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}d_{ij} \sum_{K=1}^{n} \sum_{L=1}^{n} b_{KL} b_{KL} p_{\text{K}} p_{\text{L}} \tilde{S}_{KL}(\omega_i, \zeta_i, \omega_j, \zeta_j) \right]^{1/2}$$

(12)

It can be shown (3) that the peak factors are relatively insensitive to the characteristics of the response processes and the ratios $p_{\text{K}}/p_{\text{K}}$ are near unity. Thus, the above expression can be simplified to

$$E[\tilde{u}_{r,\text{max}}] = \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}d_{ij} \sum_{K=1}^{n} \sum_{L=1}^{n} b_{KL} b_{KL} \tilde{S}_{KL}(\omega_i, \zeta_i, \omega_j, \zeta_j) \right]^{1/2}$$

(13)

It is noted that in principle there is no need to make the above approximation and it is done here for the sake of simplicity.
Equation (13) provides the mean peak response of the secondary system in terms of the CCFS's. Note that the coefficients \(a_\nu\) and \(b_k\), as given in eq. (5) are functions of the characteristics of the secondary subsystem only. It is emphasized that this formulation not only includes the effect of correlation between modal responses of the secondary subsystem, but also the correlation that exists between the excitations at the various attachment points. As indicated in the introduction, these effects are commonly neglected or improperly handled in the current practice (1).

In the following section, an efficient method for generating the CCFS's directly in terms of the input ground response spectrum is presented.

**Evaluation of Cross-Cross Floor Spectra**

The availability of an efficient method for generating the CCFS's is clearly essential if eq. (13) is to be useful. As demonstrated above, the key to this evaluation is the \(N+2\)-degrees of freedom system shown in Fig. 2(d). In a previous study, Der Kiureghian et al. (5) used perturbation techniques to determine the response of an \(N+1\)-degree system composed of an \(N\)-degree structure and a light appendage modeled as a single-degree-of-freedom oscillator. A similar approach can be used to study the \(N+2\)-degree system here. The approach involves two basic steps: (a) Synthesis of the dynamic modal properties of the \(N+2\)-degree system using perturbation methods which exploit the relative lightness of the oscillators. This process involves the modal characteristics of the primary subsystem and the properties of the two oscillators. (b) Determination of CCFS's by superpositioning the modal responses of the \(N+2\)-degree system. Note that the approach avoids the numerical solution of the eigenvalue problem for the \(N+2\)-degree system, and for that reason is efficient and practical.

Full derivation of the dynamic properties of the combined \(N+2\)-degree system is beyond the scope of this paper. For the purpose of providing an insight to the approach, however, results for the natural frequencies and mode shapes for two extreme cases are summarized below. These include the case when the two oscillators are both detuned from the modes of the primary subsystem, and the case when the two oscillators are both tuned to the same mode of the primary subsystem. For notational purposes, capital letters superposed by an asterisk are used to denote the properties of the combined \(N+2\)-degree system. Also, the oscillator with frequency \(\omega_i\) and attached to the degree of freedom \(K\) is assigned the \(N+1\)-st degree of freedom, whereas the oscillator with frequency \(\omega_j\) and attached to the degree of freedom \(L\) is assigned the \(N+2\)-nd degree of freedom. The system will have \(N+2\) modes: \(N\) of these modes are closely linked to the primary subsystem modes and will be denoted by number 1 to \(N\), and the remaining two modes are related to the two oscillators and will be denoted by numbers \(N+1\) and \(N+2\), respectively.

The modal properties of the combined system are given in terms of the following dimensionless quantities:

\[
\gamma_{\nu K} = \frac{m_\nu}{M_j/\Phi_{ji}}, \quad \alpha_\nu = \frac{\omega_\nu^2}{\omega_i^2 - \Omega_l^2}, \quad \beta_{jK} = \sum_{i=1}^{N} \alpha_i \gamma_{iK} \frac{\Phi_{ji}}{\Phi_{Kj}}, \quad \Psi_{jK} = \begin{cases} \beta_{jK} \end{cases}
\]

(14)

In the above, \(m_\nu\) is the mass of the oscillator with frequency \(\omega_\nu\), \(M_j\) is the modal mass associated with mode \(j\) of the primary subsystem, \(\Phi_{Kj}\) is the \(K\)-th element of \(\Phi_j\), and \(\Omega_l\) is the \(l\)-th frequency of the combined system as given below.
In the case when \( \omega_i, \omega_j \approx \Omega_f \), the combined system properties to first-order approximation are:

\[
\Omega_f = \Omega_f, \quad \Omega_{N+1} = \omega_i, \quad \Omega_{N+2} = \omega_j
\]  
(15)

\[
\Phi_f = \begin{pmatrix}
\Phi_1 \\
\alpha_1 \Phi_K \\
\alpha_0 \Phi_L
\end{pmatrix}, \quad \Phi_{N+1} = \begin{pmatrix}
\omega_i - \omega_j \Psi_{IK} - \Psi_{IL} \\
\beta_{IK} \omega_i^2 - \omega_j^2 \\
\beta_{IK} \omega_i^2
\end{pmatrix}, \quad \Phi_{N+2} = \begin{pmatrix}
\beta_{IK} \omega_i^2 \Psi_{IK} - \Psi_{IL} \\
\omega_i^2 - \omega_j^2 \\
\omega_i^2 - \omega_j^2
\end{pmatrix}
\]  
(16)

In the case when \( \omega_i = \omega_j = \Omega_f \), i.e., when both oscillators are perfectly tuned to the \( T \)-th frequency of the primary subsystem, the modal properties of the combined system to first order are:

\[
\Omega_f = \Omega_f, \quad \Omega_f = \left(1 + \frac{1}{2 \sqrt{\gamma_{123}}} \right) \omega_i, \quad \Omega_{N+1} = \omega_i, \quad \Omega_{N+2} = \left(1 - \frac{1}{2 \sqrt{\gamma_{123}}} \right) \omega_i
\]  
(17)

\[
\Phi_f = \begin{pmatrix}
\Phi_1 \\
\alpha_1 \Phi_K \\
\alpha_0 \Phi_L
\end{pmatrix}, \quad \Phi_f = \begin{pmatrix}
\Phi_T \\
\frac{\Phi_{IK}}{\sqrt{\gamma_{123}}} \\
\frac{\Phi_{IL}}{\sqrt{\gamma_{123}}}
\end{pmatrix}, \quad \Phi_{N+1} = \begin{pmatrix}
\Phi_{LT} \Psi_{IK} - \Psi_{IL} \\
\Phi_{LT} - \Phi_{KT} \\
\Phi_{LT}
\end{pmatrix}, \quad \Phi_{N+2} = \begin{pmatrix}
\Phi_{KT} \Psi_{IK} - \Psi_{IL} \\
\Phi_{KT} - \Phi_{LT} \\
\Phi_{KT}
\end{pmatrix}
\]  
(18)

where \( \gamma_{123} = \gamma_{12} + \gamma_{13} \). Results similar to the above are also obtained for cases when only one oscillator is tuned to a mode of the primary subsystem and when the two oscillators are tuned to two different modes of the primary subsystem (2).

With the modal properties of the \( N+2 \)-degree system determined, the covariance term \( \lambda_{0,IKL} \) can be evaluated in terms of the ground response spectrum using the modal superposition procedure described in Ref. 3. The result is

\[
\lambda_{0,IKL} = \sum_{I=1}^{N+2} \sum_{J=1}^{N+2} \Phi_{N+1,I} \Phi_{N+2,J} \Gamma_I \beta_{h_0} \frac{1}{p_I p_J} \tilde{S}_e(\Omega_f, Z_I) \tilde{S}_e(\Omega_f, Z_J)
\]  
(19)

where \( \Gamma_i \) is the participation factor for mode \( I \) of the combined system, \( \beta_{h_0} \) is the modal correlation coefficient for the combined system and is given in terms of the derived modal frequencies and damping ratios, \( p_I \) is the peak factor for mode \( I \) of the combined system, and \( \tilde{S}_e(\Omega_f, Z_I) \) is the ground acceleration response spectrum ordinate associated with mode \( I \) of the combined system with frequency \( \Omega_f \) and damping \( Z_f \). Using the definition in eq. (11), and approximating the ratios \( \beta h_0 (p_I p_J) / (p_I p_J) \) by unity, the cross-coupled floor spectrum ordinate is given by

\[
\tilde{S}_{IKL}(\omega_i, \xi_i; \omega_j, \xi_j) = \sum_{I=1}^{N+2} \sum_{J=1}^{N+2} \Phi_{N+1,I} \Phi_{N+2,J} \Gamma_I \beta_{h_0} \frac{1}{p_I p_J} \tilde{S}_e(\Omega_f, Z_I) \tilde{S}_e(\Omega_f, Z_J)
\]  
(20)

The above formulation uses the masses \( m_i \) and \( m_j \) of the two oscillators. These quantities, however, are not known. To be consistent with the method developed in the previous section, which ignored interaction, it is clear that \( m_i = m_j = 0 \) must be assumed. On the other hand, it is evident from the eqs. (16) and (18) that the modal properties will diverge when the oscillator masses are set equal to zero. It can be shown that the proper limits for the responses of the two oscillators are derived by first superpositioning the modal responses, as in eq. (20), and then letting \( m_i \) and \( m_j \) go to zero. This procedure was demonstrated for a single oscillator in Ref. 5. Details of the analysis for two oscillators are described in Ref. 2.

Another problem that must be addressed is the effect of nonclassical damping in the combined system. It has been pointed out previously that combined primary-secondary systems in general do not possess classical damping (6). The significance of nonclassical damping has been shown to be most critical in conditions of near tuning and when the difference between the modal dampings of the two subsystems is large. In the present study, exact
results for the extreme cases of perfect tuning and gross detuning are obtained which include the effect of nonclassical damping. For the cases of near tuning, approximate results are obtained by a matching process using the results for the two extreme cases. Details of this analysis are also described in Ref. 2.

Example Applications

In order to illustrate application of the method and examine its accuracy, the simple primary-secondary system shown in Fig. 3 is considered. This system is composed of a 5-degree-of-freedom primary subsystem, modeled as a shear-building, and a 3+5-degree-of-freedom secondary subsystem with the attached degrees of freedom 1 to 3 and the unattached degrees of freedom 4 to 8. The modal frequencies for the fixed base subsystems are given in Table 1. Note that modes 1 and 4 of the secondary subsystem are nearly tuned to modes 2 and 4 of the primary subsystem, respectively. Two cases of modal damping are considered for the two subsystems: (a) 5 percent of critical damping for each mode of both subsystems, and (b) 5 percent of critical damping for each mode of the primary subsystem and 2 percent for each mode of the secondary subsystem. As noted before, in the second case the effect of nonclassical damping can be important. This effect is included in the analysis through the approximate procedure mentioned in the preceding section.

To examine the accuracy of the proposed method, two parallel analyses of the system were performed using the same input ground response spectrum. In the first analysis, the modal superposition rule in eq. (13) and eq. (20) for the CCFS’s were used to determine the response of the secondary subsystem. In the second analysis, the combined primary-secondary system was considered as a single structure. Solving the 10x10 eigenvalue problem for the combined system, the modal properties were derived which were then used in conjunction with the CQC method (3,8) to obtain the "exact" response of the secondary subsystem. Results for the two cases of equal and unequal damping mentioned above are summarized in Table 2. The agreement between the two sets of results is clearly remarkable.

As an item of interest, Fig. 4 shows plots of CCFS’s for several selected cases. Unlike the conventional floor spectra which are always positive, the CCFS’s for unequal oscillators (i.e., \( \omega_i \neq \omega_j \)) or unequal floors (i.e., \( K \neq L \)) may assume negative values.

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References


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Table 1. Modal Frequencies of Fixed Base Subsystems

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency, rad/s</th>
<th>Primary Subsystem</th>
<th>Secondary Subsystem</th>
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<tbody>
<tr>
<td>1</td>
<td>1.273</td>
<td></td>
<td>3.221</td>
</tr>
<tr>
<td>2</td>
<td>3.716</td>
<td></td>
<td>4.472</td>
</tr>
<tr>
<td>3</td>
<td>5.857</td>
<td></td>
<td>6.799</td>
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<tr>
<td>4</td>
<td>7.524</td>
<td></td>
<td>7.746</td>
</tr>
<tr>
<td>5</td>
<td>8.582</td>
<td></td>
<td>9.132</td>
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Table 2. Secondary Subsystem Response:
Nodal Accelerations in g’s

<table>
<thead>
<tr>
<th>Degree of Freedom</th>
<th>Case (a): Equal Damping</th>
<th>Case (b): Unequal Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed Method</td>
<td>&quot;Exact&quot;</td>
</tr>
<tr>
<td>4</td>
<td>0.3869</td>
<td>0.3835</td>
</tr>
<tr>
<td>5</td>
<td>0.3317</td>
<td>0.3323</td>
</tr>
<tr>
<td>6</td>
<td>0.2517</td>
<td>0.2532</td>
</tr>
<tr>
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</tr>
<tr>
<td>8</td>
<td>0.2454</td>
<td>0.2438</td>
</tr>
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</table>

Figure 1. Schematic Illustration of Primary-Secondary System

Figure 2. N+2-Degree Systems Used in Defining CCFS

Figure 3. Example Primary-Secondary System

Figure 4. Selected Cross-Cross Floor Spectra for Example Structure