

## On the Simulation of Infinite Regions in Seismic Soil-Structure Interaction Models

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### ABSTRACT

In the paper, the simulation of infinite regions in seismic soil-structure interaction is intended in the context of finite element models and is accomplished by means of "infinite element" techniques.

A two-node infinite element, to be associated with consistent finite element models, is presented in this paper based on linear displacement distributions and the possibility of alternative formulations is also discussed.

The material behaviour is assumed to be linearly elastic, hysteretic and isotropic. The shape functions assumed for the element herein presented ensure displacement continuity at the interface with the discretized inner region and represent the exponential decay of the waves propagating in the outward direction. The exponential shape functions assume wave numbers analytically derived on the element itself and, therefore, they do not allow for inter-element displacement continuity when two adjacent elements are characterized by different mechanical properties. The analytical derivation of the wave numbers leads, however, to a significant reduction in computational complexity with respect to other formulations.

This element is suitable for the analysis of general soil dynamic problems in which the energy absorption of semi-infinite media is of concern in two- or three- (axisymmetric) dimensional geometries, when spaces or half-spaces as well as soil layers underlain by a rigid bedrock are dealt with.

The accuracy of the solutions obtained for problems of the latter kind is discussed, based on some numerical examples.

## 1. INTRODUCTION

The simulation of the energy absorbing effect due to the presence of some medium extending to infinity is a significant problem in many engineering applications of numerical procedures. When localized phenomena are analyzed by means of a finite element discretization some methods are known, each one possessing advantages and disadvantages, to apply at the boundaries of the finite element mesh appropriate energy absorbing mechanisms.

Kinematic boundary conditions and dampers of various kinds are, for example, quite popular in seismic or generally in soil dynamic problems. More complicated and even more efficient or accurate procedures are also available among which one can refer, for example, to the application of the boundary element method presented by DOMINGUEZ [1], to the system identification approach due to GUPTA and PENZIEN [2], to the well known LYSMER-WAAS boundaries [3] and to their generalizations due to KAUSEL and ROESSET [4] and MULLER and WERKLE [5].

The two former approaches treat the case of isotropic, linearly elastic, viscoelastic or hysteretic half-spaces while the latter ones represent a very efficient and accurate tool for the dynamic analysis of stratified soil deposits resting on a rigid bedrock.

Another method which has also been used to solve problems of a similar nature is related to the formulation of "infinite elements". The basic idea of infinite elements is extensively discussed in the papers by BETTEES [6-7]. Applications devoted to geostatic problems were presented by BEER and MEEK [8], and detailed discussions on the development of infinite elements in fluid dynamics are given by BETTEES and ZIENKIEWICZ [9].

The concept of infinite elements is certainly the most appropriate for use in conjunction with finite element methodologies as it preserves all the basic characteristics of the method as well as the solution procedures. However, only very recently MEDINA and PENZIEN [10] presented a family of infinite elements, suitable for applications in seismic soil-structure interaction problems, whose shape functions are derived from a blend of closed form solutions to the wave propagation problem in half-spaces (P-waves, S-waves and Rayleigh waves). Nevertheless, this family of elements seems not to be usable for the analysis of soil conditions different from the half-space. Another development has been formulated by CHOW and SMITH [11].

The present paper represents an attempt to formulate another family of infinite elements which could be used for the analysis of spaces and half-spaces as well as layered soils.

## 2. ELEMENT FORMULATION

For analysis in the frequency domain, the stiffness and mass properties of an infinite element can be derived by integrating over the (infinite) volume of the element the usual matrix products

$$[B]^T [E][B] ; [N]^T \rho [N] \quad (1)$$

in which [B] is the compatibility matrix,  $\rho$  is the mass density and [E] is the elasticity matrix, eventually complex to account for material damping. The [B] matrix contains the derivatives of the shape functions [N], representing the usual displacement approximation in the directions in which the element has finite size and providing, in the directions extending to infinity, a transition from the solution in the discretized region to a far field solution, thus accounting for the energy absorption in the outer region. A typical shape function is, with reference to the element geometry of fig. 1 :

$$N(\xi, \zeta) = P(\zeta) \cdot \exp(-\alpha \xi) \tag{2}$$

being  $P(\zeta)$  a polynomial of a given order, depending from the number and the position of the nodes that match the boundary nodes of the discretized region.

In the static case the parameter  $\alpha$  can be reasonably estimated by empirical rules, while in the dynamic case the shape function represents a wave component propagating toward infinity and  $\alpha$  should be determined by some solution of the wave propagation problem.

One way of doing that is, for example, to search a general solution, valid for the widest possible class of cases, for a wave of the form given by eq. (2) propagating in a continuum and reproducing the nodal state of displacement at the corresponding points.

In plain strain, one can make use of the following virtual work principle :

$$\begin{aligned} & \int_0^x \int_{-1}^1 [\hat{\varepsilon}_\xi^* \sigma_\xi + \hat{\varepsilon}_\xi^* \sigma_\zeta + \hat{\gamma}_{\xi\zeta}^* \tau - \hat{\vartheta}_\zeta^* f_\zeta^I - \hat{u}^* f_\xi^I] d\zeta d\xi = \\ & = \int_{-1}^1 [-\hat{u}^*(0, \zeta) \sigma_\xi(0, \zeta) - \hat{\vartheta}^*(0, \zeta) \tau(0, \zeta) + \hat{u}^*(x, \zeta) \sigma_\xi(x, \zeta) + \hat{\vartheta}^*(x, \zeta) \tau(x, \zeta)] d\zeta + \\ & + \int_0^x [-\hat{u}^*(\xi, -1) \tau(\xi, -1) - \hat{\vartheta}^*(\xi, -1) \sigma_\zeta(\xi, -1) + \hat{u}^*(\xi, 1) \tau(\xi, 1) + \hat{\vartheta}^*(\xi, 1) \sigma_\zeta(\xi, 1)] d\xi \end{aligned} \tag{3}$$

where  $\hat{\quad}$  and  $^*$  denote respectively virtual and complex conjugate quantities and  $f^I$  are the inertia forces, repeatedly applied for virtual states of displacement reproducing the displacement components at each node. If :

$$\begin{aligned} u &= P_1(\zeta) \exp(-ik\xi) \\ v &= P_2(\zeta) \exp(-ik\xi) \end{aligned} \tag{4}$$

this procedure generates a set of homogeneous algebraic equations and the condition of non-trivial solution allows the determination of the values of  $k$  (wave numbers) for which a wave resultant from displacements of the form given by eqs. (4) can propagate inside the volume occupied by the element.

Choosing only the values of  $k$  giving rise to waves travelling toward infinity, one can set up shape functions given by a linear combination of those waves.

Even though polynomials  $P_1(\zeta)$  and  $P_2(\zeta)$  could be so chosen that displacement continuity is obtained along the interface between finite (discretized) and infinite regions, it may be observed that continuity will not be satisfied at the interface between two adjacent infinite elements of this kind when the mechanical properties of this two elements are different. However, displacement continuity at the discretized region boundary can also be renounced in order to get polynomials in  $\zeta$  more completely satisfying equilibrium require-

ments inside the element volume.

Furthermore, one can note that isoparametric-like formulations can be derived if a coordinate transformation is introduced in formulas (1) together with an appropriate numerical integration technique [9] , [10] .

Although the above described procedure could also be performed by a fully numerical manipulation, computational efficiency is greatly improved by an analytical derivation of the element shape functions. This obviously requires the choice of simple polynomials in eqs. (4).

The simplest ones represent linear variations of the displacements, being two nodes located at (0,-1), (0,1) :

$$u = \left[ \frac{1}{2}(1-\zeta) u_1 + \frac{1}{2}(1+\zeta) u_2 \right] \exp(-ik\xi) \quad (5)$$

$$v = \left[ \frac{1}{2}(1-\zeta) v_1 + \frac{1}{2}(1+\zeta) v_2 \right] \exp(-ik\xi)$$

By applying the virtual work principle, one gets the following set of equations :

$$([\mathcal{A}]k^2 + i[\mathcal{B}]k - \rho\omega^2[\mathcal{M}])\{U\} = \{0\} \quad (6)$$

where  $[\mathcal{A}]$  ,  $[\mathcal{B}]$  are complex matrices containing geometrical and mechanical properties of the element while  $[\mathcal{M}]$  is a real matrix depending only on geometrical terms.

The characteristic equation possesses the (double) roots  $k_1 = \omega/v_p = k_p$  and  $k_2 = \omega/v_s = k_s$  , being  $v_p$  and  $v_s$  the P-wave and the S-wave velocity. Correspondingly, linear wavefronts of the type :

$$\begin{Bmatrix} u \\ v \end{Bmatrix}_1 = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} ; \quad \begin{Bmatrix} u \\ v \end{Bmatrix}_2 = \begin{Bmatrix} -(\zeta+1) \frac{Gk_p^2 - \omega^2 \rho}{ik_p(\lambda+G)} \\ 1 \end{Bmatrix} \quad (7)$$

and

$$\begin{Bmatrix} u \\ v \end{Bmatrix}_3 = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} ; \quad \begin{Bmatrix} u \\ v \end{Bmatrix}_4 = \begin{Bmatrix} 1 \\ -(\zeta+1) \frac{(2G+\lambda)k_s^2 - \omega^2 \rho}{ik_s(\lambda+G)} \end{Bmatrix} \quad (8)$$

can propagate in  $\xi$  at velocities  $v_p$  and  $v_s$  , respectively.

It should be observed that waves of the type 1 and 3 only satisfy the differential equilibrium equations everywhere in the element .

The shape functions to be used in element generation are then obtained by a linear combination of these four waves.

Similar procedures, even though more complicate from the analytical point of view, can be derived by assuming parabolic variations of the displacement components in eq. (5) either in the case of two or three nodes located on the left side of the element.

The infinite element family herein discussed should therefore adequately represent the radiation damping in a sufficiently general class of cases. Now the problem is if the element formulation is also able to simulate the radiation associated with layered media, in which the finiteness of the domain in some direction introduces further complexities in the wave propagation phenomenon.

### 3. NUMERICAL EXAMPLES AND CONCLUSIONS

The infinite element described in the previous paragraph has been tested as against the solutions obtained with the program PLAXLY [3]. The cases examined included two homogeneous strata with properties  $\rho = 1900 \text{ kg/m}^3$ ,  $G = 1.1 \cdot 10^8 \text{ N/m}^2$ ,  $\nu = 0.33$ , 5% damping and heights 40 m and 160 m respectively. For each stratum, the solutions were obtained for top vertical and horizontal harmonic forces of intensity 5000 N on two mesh types. The first mesh uses three columns of elements for each side of the loaded node while the second one uses seven columns of elements. The meshes are schematically represented in figs. 2 and 3.

Figs. 2a and 2b show respectively, for the 40 m height stratum, the horizontal displacement due to the horizontal force and the vertical displacement due to the vertical force, on two horizontal and two vertical sections, at a frequency close to the resonant frequency of the stratum. The solid line indicates the solution given by PLAXLY and the dashed line is representing the solution using the infinite elements presented herein.

Figs. 3a and 3b show the analogous comparison for the stratum of 160 m height.

Figs. 4a and 4b illustrate, for the 40 m stratum, the horizontal and vertical displacement (due respectively to the horizontal and vertical forces) at node A (see fig. 2) while figs. 4c and 4d show the same quantities at node B.

Figs. 5a,b,c,d represent the analogous results obtained for the other stratum.

It may be noted that the results plotted in figs. 2 and 3 are obtained with the seven element columns mesh while in figs. 4 and 5 the results obtained with both the three columns (curve 3) and seven columns (curve 2) meshes are reported for comparison with the PLAXLY solution (curve 1).

From the analysis of the results, one observes that the infinite elements need to be located at a greater distance from the source than that required by the WAAS boundary in order to get reasonable approximations. It should be observed, however, that kinematic boundaries still give erroneous solutions when the infinite elements already furnish acceptable approximations.

Secondly, one may note that infinite elements of this simple linear type closely approximate PLAXLY solution relatively far from the fundamental frequency of the stratum while they diverge significantly in the evaluation of the response at resonance. This means that, in general, this infinite element underestimates the radiation damping in the case of layers resting on a rigid bedrock.

As a final remark one can observe that better approximations could however be reached with wider meshes. In conclusion the results obtained with the simplest displacement approximation are already relatively satisfactory and therefore they encourage the development of improved infinite elements of this kind using higher order polynomials.

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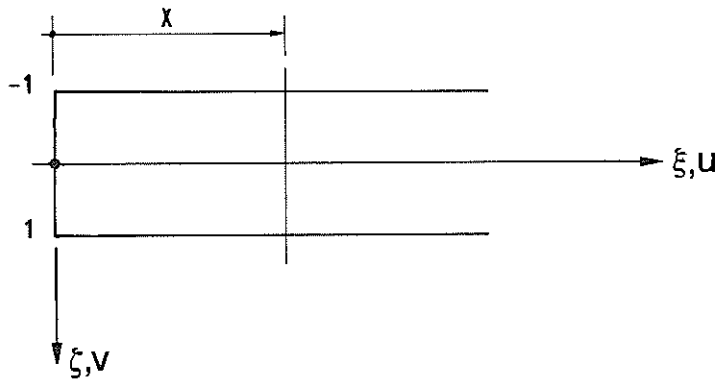


Fig. 1

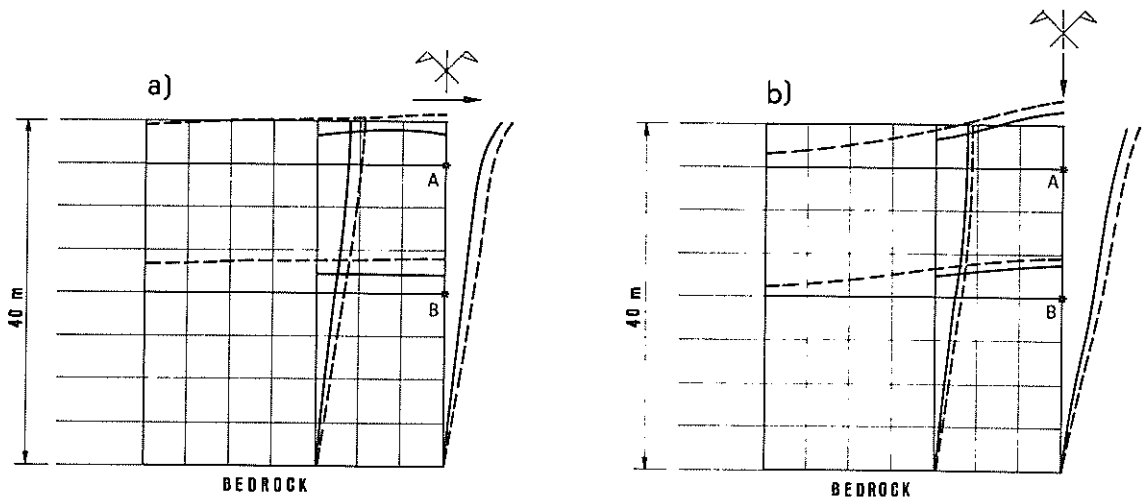


Fig. 2 - Displacement distribution - a) horizontal displacement at  $f=1.4$  Hz b) vertical displacement at  $f=2.75$  Hz

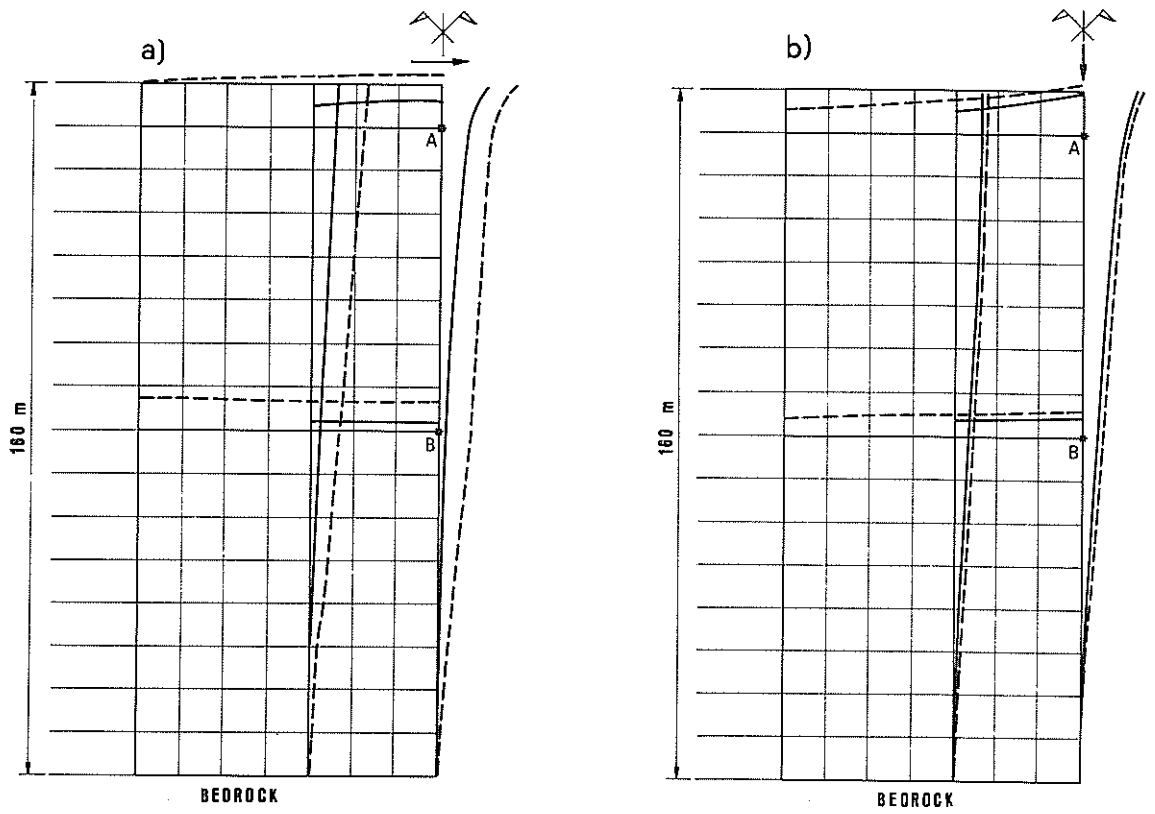


Fig. 3 - Displacement distribution - a) horizontal displacement at  $f=0.28$  Hz b) vertical displacement at  $f=0.6$  Hz

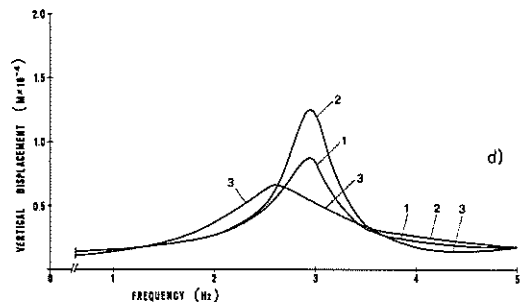
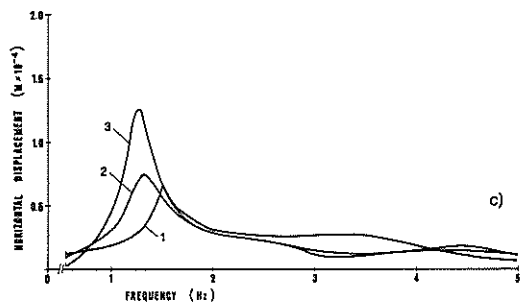
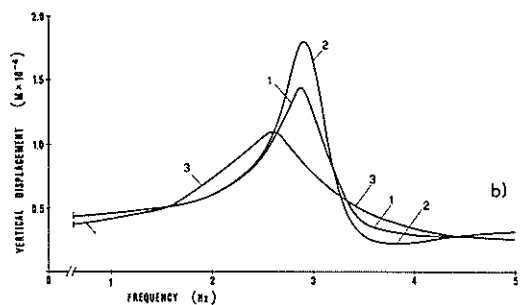
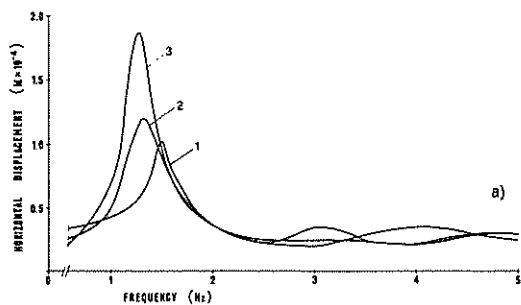


Fig. 4 - Frequency Response - stratum H= 40 m

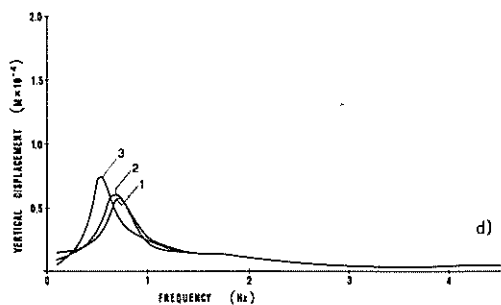
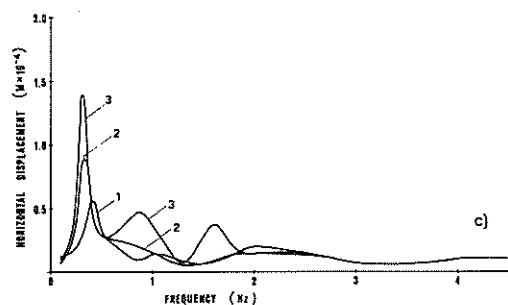
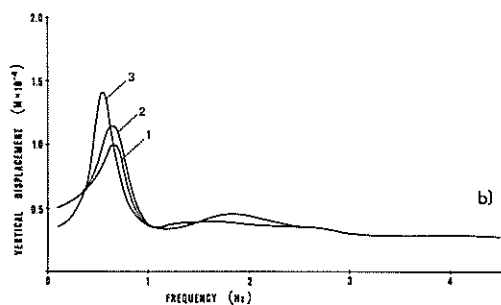
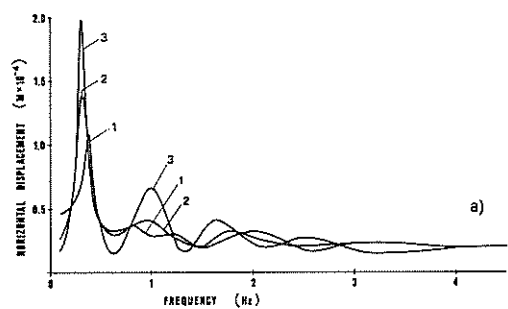


Fig. 5 - Frequency Response - stratum H= 160 m