Anisotropic Creep Damage in the Framework of Continuum Damage Mechanics

J.-L. Chaboche

Office National d'Etudes et de Recherches Aérospatiales, 29 av. de la Division Leclerc, F-92320 Châtillon, France

Summary : For some years, various works have shown the possibility of applying continuum mechanics to model the evolution of the damage variable, initially introduced by Kachanov. Of interest here are the complex problems posed by the anisotropy which affects both the elastic behaviour and the viscoplastic one, and also the rupture phenomenon.

The main concepts of the Continuum Damage Mechanics are briefly reviewed together with some classical ways to introduce anisotropy of damage in the particular case of proportional loadings. Based on previous works, two generalizations are presented and discussed, which use different kinds of tensors to describe the anisotropy of creep damage :

- The first one, by Murakami and Ohno introduces a second rank damage tensor and a net stress tensor through a net area definition. The effective stress-strain behaviour is then obtained by a fourth rank tensor.

- The second theory, by the author, uses one effective stress tensor only, defined in terms of the macroscopic strain behaviour, through a fourth-order non-symmetrical damage tensor.

The two theories are compared at several levels : differences and similarities are pointed out for the damage evolution during tensile creep as well as for anisotropy effects. The possibilities are discussed and compared on the basis of some existing experimental results, which leads to a partial validation of the two approaches.

1. Introduction : Lifetime predictions in structures are the result of three main steps :

- determination of constitutive equations for the stress-strain behaviour of the material, including the time, temperature and hardening effects,

- computation of stress and strain fields in the structure, knowing the applied loads, the temperature and their evolution. This calculation is carried out in the framework of Continuum Mechanics, for example with the finite element technique, using the constitutive laws for the material,

- prediction of damage in the structure or, more precisely of the time or number of cycles before crack initiation. This prediction uses the damage laws of the material and the stresses and strains calculated in the previous step.

Classically, the above three steps are applied successively and independently, the structural analysis using the stress-strain laws to provide data for the crack initiation relationships. The present and future development of computers, but also the need for better predictability in particular practical situations, promote the research of new concepts and new procedures to treat the problem. Two aspects can be considered :

- crack initiation is the result of several physical processes, which description is necessary to better predict history effects and the cumulative damage under service loadings. The classical life parametrization are then replaced by damage rate equations, providing the rupture life as a result of their integration for a given loading.

- The presence and development of an internal damage lead to a progressive loss of strength of the material. Taking into account that effect is the main purpose of the Continuum Damage Mechanics.

Such a Damage Mechanics [1] [2] considers the volume element of the damaged material as macroscopically homogeneous and the damage state can be described by using the general framework of the thermodynamics with internal variables.

One of the main problems of the approach is the anisotropy of damage. During the past ten years, several theories were developed in order to introduce such anisotropy in an intrinsic way, independent of the considered loading conditions. Most of these works apply to the creep situation [2] [3] [4] [5] [6] [7] even if some of them were done in the context of plasticity [8], dealing with the forming processes.

In the present paper, two of these theories are briefly reviewed, discussed, and compared on the basis of several examples :

- the first one is the theory of Murakami and Ohno [2] [9], which uses a net-stress definition and a symmetric second order tensor,

- the second one was developed by the author [5], using an effective stress concept and a non symmetric fourth order tensor.

2. Definition of damage variables and effective-stresses :

2.1 Damage processes and its quantification at a macroscopic level : During the strain processes, by localization and accumulation of dislocation, irreversible defects appear that can be considered as damage : intercrystalline cavities in creep, persistent slip bands in fatigue, etc. These damages are the cause of the rupture of the volume element of material or, more precisely, of crack initiation.

The purpose of damage theory is to describe the processes involved prior to the initiation of a macroscopic crack. Defining crack initiation is one of the first difficulties. It depends on the scale on which the phenomenon is observed. The chosen definition corresponds to a main crack, which geometry is clearly approximable in the framework of Fracture Mechanics. The Damage Mechanics is then a tool used to deal with the defects in the intermediate domain where Fracture Mechanics is not easily applicable.

The engineer also often refers to the remaining lifetime to express the idea of damage. Damage exists as soon as the remaining life under a given loading is less than the nominal life under the same loading. This is the definition applied for calculating the potential life of a component. Often used to express two-level test results (linear or nonlinear cumulative rules), this definition has the disadvantage of leading to a partial indetermination of the quantitative value of damage [10]. To eliminate this indetermination, we can use either microstructural defect density measurements, more global measurements involving the variation of physical quantities (resistivity [11], density [12]) or strain resistance characteristics [13]. This last approach, based on the concept of effective stress, is the one we will develop below.

2.2 Net stress and effective stress concepts : The existence of decohesions in a volume element of material, for example intercrystalline cavities under creep conditions, leads to a reduction in the effective section, which means an increase in the effective stress. Three types of stress can be considered :

- the true stress $\sigma_V \simeq \sigma(1+\varepsilon)$, which takes into account the geometric reduction in the sectional area due to the macroscopic deformation,

- the net stress $\sigma^* = \sigma_V \, S/S^* = \sigma_V/(1-\Omega)$, where the present geometrical section is reduced to $S^* = S(1-\Omega)$, where ΩS is the mean area of decohesion ,

- the effective stress $\tilde{\sigma} = \sigma_V \, S/\tilde{S} = \sigma_V/(1-D)$, which takes into account the local stress concentrations and the interactions between defects. This effective stress is thus defined as the stress $\tilde{\sigma}$ that would have to be applied to the undamaged volume element for it to attain the same macroscopic strain ε as the damaged volume element subjected to the true stress σ_V (Fig. 1-a).

The relationship between the net and effective stresses is not simple, or at least not linear, as the comparisons in the figure 1-b show. These were obtained by calculations for perforated plates [14] and for cracks perpendicular to the direction of tensile stress [15]. In what follows σ denotes the true stress.

2.3 One-dimensional creep damage model :

Damage models cannot be detailed here and we shall only summarize Kachanov and Rabotnov's equations. We suppose that the creep damage D varies between 0, for the undamaged material, and 1 for the rupture of the volume element. Kachanov's law [16], generalized by Rabotnov [17], is written as follows for pure tensile stress :

$$dD = \left(\frac{\sigma}{A}\right)^r (1-D)^{-k} \, dt \qquad (1)$$

where r, k and A are material- and temperature-dependent coefficients, determined by the constant stress (true stress) creep tests for which the integration gives :

$$D = 1 - \left(1 - \frac{t}{t_c}\right)^{1/k+1} \qquad (2)$$

$$t_c = \frac{1}{k+1}\left(\frac{\sigma}{A}\right)^{-r} \qquad (3)$$

Rabotnov introduced the concept of effective stress in the framework of the secondary creep law (Norton's law) :

$$\dot{\varepsilon}_p = \left[\frac{\sigma}{\Lambda\,(1-D)}\right]^n \qquad (4)$$

which can describe fairly accurately the tertiary creep curves as well as the creep ductility in many materials [10] [18].

3. Damage under multiaxial loading :

The three-dimensional generalization of the damage law poses two problems :

Multiaxial Criteria : Under multiaxial proportional loading, we have to define isodomage surfaces, in the stress space for example. The multiaxial criterium is usually obtained from measurements of the isochronous surfaces, which are loci of the constant stress states that lead to the the same rupture time. If we suppose that these surfaces deduces each other homothetically, they can be described for the initially isotropic material by means of three stress invariants:

- the octahedral shear stress $J_2(\sigma)$ related to the effects of shear,
- the hydrostatic stress $J_1(\sigma) = Tr(\sigma) = \sigma : 1$, which greatly affects the growth of the cavities,
- the maximum principal stress $J_0(\sigma) = \sigma_1$, which opens the microcracks and causes them to grow.

Following Hayhurst's method [19], the equivalent stress can be defined through a linear combination : $\chi(\sigma) = \alpha\, J_0(\sigma) + \beta\, J_1(\sigma) + (1-\alpha-\beta)\, J_2(\sigma)$

where α, β are coefficients dependent on material and temperature. The time to failure under a fixed multiaxial stress is expressed as :

$$t_c = \frac{1}{k+1}\left\langle \frac{\chi(\sigma)}{A} \right\rangle^{-r} \qquad (5)$$

Anisotropy of damage : The microstructural observations show that the creep defects can be distributed and orientated in planes perpendicular to the maximum principal stress. Including the anisotropy of the damage entails major difficulties. The initial formulation in which the damage variable remains scalar but affects only the main stress component [18] cannot be universally applied to any desired loading. Tensorial variables must then be introduced. Two creep theories are thus summarized below to complement these classical approaches.

3.1 The anisotropic theory of Murakami and Ohno [2, 9]

3.1.1. The net stress and the damage tensor : This theory uses the intercrystalline microcavities directly to define the damage variable through the reduction in the strength section (where the geometric section is reduced by the shape of the cavities). The resulting stress is thus a direct generalization of the net stress introduced in paragraph 2.2. The variable describing the present state of damage of the element of volume is the symmetrical second-rank tensor :

$$\Omega = \frac{3}{S_g(V)} \sum_{k=1}^{N} \int_V \nu^{(k)} \otimes \nu^{(k)}\, dS_g^{(k)} \qquad (6)$$

where $dS_g^{(k)}$ is the area of the grain boundary occupied by the kth cavity, and $\nu^{(k)}$ is the vector normal to the boundary. S_g (V) is the total area taken up by the boundaries in the volume element.

Under these conditions, the force vector acting on the surface $S\nu$ is $S\mathbf{t} = \sigma.(S\nu)$, where $\mathbf{t}$ is the corresponding stress vector, σ is the Cauchy tensor acting on the considered volume element. The effective surface element A^* B^* C^* is subjected to the same force vector and we have (Fig. 2):

$$S^*\mathbf{t}^* = S\mathbf{t} \qquad \sigma^*.(S^*\nu^*) = \sigma.(S\,\nu) = \sigma.(1-\Omega)^{-1}.(S^*\nu^*) \qquad (7)$$

whence the not-necessarily-symmetrical net stress tensor $\sigma^* = \sigma.(\mathbb{1}-\Omega)^{-1}$. In practice, we use the symmetrical part of this tensor which will also be denoted by σ^*:

$$\sigma^* = \frac{1}{2}\left(\sigma . \Phi + \Phi . \sigma\right) \qquad \Phi = (\mathbb{1} - \Omega)^{-1} \tag{8}$$

3.1.2. Creep damage equation : The damage evolution law, i.e. the law for the tensor Ω, is expressed for creep in the form :

$$\dot{\Omega} = \left\langle \frac{\chi(\sigma^*)}{A} \right\rangle^r \left[\gamma \mathbb{1} + (1-\gamma)\, \nu^{(1)} \otimes \nu^{(1)} \right] \tag{9}$$

where $\chi(\sigma^*)$ is the invariant describing the isochronous surfaces (see above), with $\nu^{(1)}$ being the direction of the maximum principal stress. A, r and γ are material dependant coefficients. The latter adjusts the degree of anisotropy of the damage evolution.

3.1.3. Description of stress-strain behavior of the damaged material : The net stress tensor is thus employed to express the evolution of the damage variable. On the other hand, the effect of the present damage on the strain behavior (elastic, plastic, creep) can only be introduced through a fourth-rank tensor as we will see in the next paragraph. Murakami and Ohno use an effective stress of the form [9] :

$$\tilde{S} = \frac{1}{2}\left[\Gamma : \sigma + (\Gamma : \sigma)^T \right] \tag{10}$$

where the asymetrical fourth-rank tensor Γ is deduced from the tensor Ω, or rather from $\Phi = (\mathbb{1}-\Omega)^{-1}$, by a general expression deduced from the representation theory using a linear combination of fourth-rank tensors formed by the tensor Φ and the unit tensor. Note that this expression contains second-, third- and fourth-order terms in Φ.

According to the effective stress concept, the law of viscoplasticity of the damaged material is then obtained by substituting the effective stress $\tilde{S}$ for σ. For the special case where only the isotropic hardening is taken into account, the authors use expressions of the form :

$$\dot{p} = \left[\frac{J(\tilde{S})}{K} \right]^n p^{-n/m} \qquad \dot{\varepsilon}_p = \frac{3}{2}\, \dot{p}\, \frac{\tilde{S}'}{J(\tilde{S})} \tag{11}$$

where $\tilde{S}'$ is the deviator of $\tilde{S}$.

3.1.4. Particular case in tension [2] : For tension and a simple particularization of (10), the equations (9) (11) reduce to :

$$\dot{\Omega} = \left[\frac{\sigma}{A(1-\Omega)} \right]^r \tag{12}$$

$$\dot{\varepsilon}_p = \left[\frac{\sigma}{K(1-c\Omega)} \right]^n p^{-n/m} \tag{13}$$

If we integrate the damage equation (12) for constant stress creep, we get :

$$\Omega = 1 - \left(1 - \frac{t}{t_c}\right)^{1/r+1} \tag{14}$$

Considering only the secondary creep law, $m \to \infty$ in (13), we can integrate explicitely with (14) if n = r - 2, obtaining :

$$\varepsilon_p = \varepsilon_{p_R} \left[1 - \frac{1 - t/t_c}{\left[1 - c + c\left(1 - t/t_c\right)^{1/n-1}\right]^{n-1}} \right] \tag{15}$$

where the rupture strain is :

$$\varepsilon_{p_R} = \left(\frac{\sigma}{\Lambda}\right)^n \frac{t_c}{1-c} \tag{16}$$

3.2. The anisotropic theory of Chaboche [5, 20] :

3.2.1. Definition of effective stress and damage tensor : The damage tensor is introduced through the stress-strain behavior of the damaged material. The effective stress tensor $\tilde{\sigma}$ is the one that must be applied to the undamaged volume element to obtain the same strain tensor as the one produced by the stress σ applied to the damaged volume element (from the unloaded actual state).

The linear elastic behavior of the undamaged and damaged materials write respectively:

$$\tilde{\sigma} = \Lambda : \varepsilon_e \qquad \sigma = \tilde{\Lambda} : \varepsilon_e \tag{17}$$

By definition, the effective stress is obtained :

$$\tilde{\sigma} = \Lambda : \varepsilon_e = \Lambda : \tilde{\Lambda}^{-1} : \sigma \tag{18}$$

The fourth-rank tensor $\Lambda : \tilde{\Lambda}^{-1}$ can be set in a form that generalizes the initial theories of Kachanov and Rabotnov, such that the effective stress can be expressed :

$$\tilde{\sigma} = (\mathbb{1} - \mathbb{D})^{-1} : \sigma \tag{19}$$

where $\mathbb{D}$ is an asymmetrical fourth-rank damage tensor. It can be measured through the elastic behaviors in the present state and in the initial (undamaged) state :

$$\mathbb{D} = \mathbb{1} - \tilde{\Lambda} : \Lambda^{-1} \tag{20}$$

Such an operator $\mathbb{D}$ can represent any observed behavior for the damaged elastic material. We have :

$$\sigma = \tilde{\Lambda} : \varepsilon_e = (\mathbb{1} - \mathbb{D}) : \Lambda : \varepsilon_e \tag{21}$$

and the symmetry of Λ and $\tilde{\Lambda}$ is verified.

3.2.2. Stress-strain behavior of the damaged material : The elastic behavior is obtained directly through the chosen definition for $\mathbb{D}$. To describe the viscoplastic flow, we use a potential, which depends on the effective stress. Without going into the details, the normality assumption [21] then leads to an expression that is slightly different from (11). In the case with isotropic hardening, the viscoplastic potential is written, for example :

$$\varphi^* = \frac{K}{n+1} \left[\frac{J(\tilde{\sigma})}{K} \right]^{n+1} p^{-n/m} \tag{22}$$

and the law of viscoplastic flow is :

$$\dot{\varepsilon}_p = \frac{\partial \varphi^*}{\partial \sigma} = \frac{3}{2} \left[\frac{J(\tilde{\sigma})}{K} \right]^{n} p^{-1/n} \; (\mathbb{1} - \mathbb{D})^{-1} : \frac{\tilde{\sigma}'}{J(\tilde{\sigma})} \tag{23}$$

In simple tension, this equation reduces to :

$$\dot{\varepsilon}_p = \frac{1}{1 - D} \left[\frac{\sigma}{K(1 - D)} \right]^{n} p^{-n/m} \tag{24}$$

In comparison with (13), the damage term now has an exponent n + 1 instead of n. This is because the effective stress replaces the applied stress in the potential and not in the flow law itself. Because of the anisotropic damage, we note that the plastic flow is no longer necessarily at constant volume.

3.2.3. Creep damage law : The relation selected to describe the simple tensile case provides the law for the scalar variable D representing the volume density of the defects. Moreover, the directional development of the damage is taken into account by an anisotropy tensor $\mathbb{Q}$. We suppose [5], as do Murakami and Ohno [9], that the anisotropy ratios remain fixed under proportional loading, which is expressed by :

$$\dot{\mathbb{D}} = \mathbb{Q} \, \dot{D} \tag{25}$$

where $\mathbb{Q}$ depends on the material and on the principal directions of the effective stress tensor $\tilde{\sigma}$.

In the general case, $\mathbb{Q}$ is introduced by rotating a tensor $\mathbb{Q}^*$, that does not depend on the present state but is a fixed tensor for a given material. In order to simplify the identification, we chose to define $\mathbb{Q}^*$ as a linear combination of the isotropic case and of a particular case of anisotropy, e.g. the one corresponding to planar microcracks, all parallel, developing perpendicular to the direction of the maximum principal stress :

$$\mathbb{Q}^* = \gamma \mathbb{I} + (1-\gamma)\,\mathbb{G} \tag{26}$$

The anisotropy tensor $\mathbb{G}$ is obtained from elastic analysis results [15]. Two material coefficients are needed ; one of them is ν , the elastic Poisson's ratio.

We consider that the scalar damage is an invariant of $\mathbb{D}$, for example $D = T_r(\mathbb{D})/T_r(\mathbb{Q})$, and its law is given by identification with the simple tensile case, attempting to describe the isochronous surfaces in the stress space. In order to produce eq. (1) for the simple tensile case, we select :

$$\chi(\tilde{\sigma}, D) = \alpha J_0(\tilde{\sigma}) + \frac{\beta}{1+2B} J_1(\tilde{\sigma}) + \frac{1-\alpha-\beta}{1-B} J_2(\tilde{\sigma}) \tag{27}$$

where J_0, J_1, J_2 are the three invariants already defined and $B = \frac{\nu}{1-\nu}\frac{(1-\gamma)D}{1-\gamma D}$. The damage law is expressed for the principal axes of $\tilde{\sigma}$:

$$\dot{\mathbb{D}} = \left[\gamma \mathbb{I} + (1-\gamma)\,\mathbb{G}\right] \dot{D} \qquad\qquad \dot{D} = \left\langle \frac{\chi(\tilde{\sigma}, D)}{A} \right\rangle^k \left[\frac{\chi(\sigma)}{A}\right]^{r-k} \tag{28}$$

4. Comparison of the two anisotropic theories : In the two theories described above the problems posed by the multiaxial creep loadings are solved in two steps :

- definition of a stress invariant for the creep rupture under constant multiaxial stressing. It describes the surfaces of equal time to rupture.

- Definition of a creep damage law with fixed anisotropy with respect to the principal directions. This anisotropy does not depend on time, but only on the material. The nonlinearity of the variation in the damage is described by the selection of the equation used in the simple tensile case.

The essential differences between these two theories stem from the definition of the damage variable (second-rank or fourth-rank tensor), the selection of the effective stress used in the damage law (net stress σ^* or effective stress $\tilde{\sigma}$), and then from the way the stress-strain behavior is described. The table I summarizes the differences and similarities, in particular for the form of the equations used in the simple tensile case. Note in particular the idea of equivalence that is used in the damage law for the theory of Murakami and Ohno, and in the constitutive law for the theory of Chaboche.

Not many experiments can validate these theories, in particular under multiaxial loading and in creep. Experiments carried out on plates having multiple perforations give rather precise validation [14, 22, 23], for which the present distribution of the damage (simplified by the perforations) is well known. The quantitative and directional aspects can thus be studied at the same time as in [22]. The quality and precision of the tensile experiments given in [22] thus offer :

1) the justification of the net stress concept through the idea of equivalency at rupture.

2) Verification of the same concept as a function of the direction of the tensile stress with respect to the principal axes of the drilling pattern (Fig. 3)

3) The determination of the tensor $\mathbb{\Gamma}$ as a function of the tensor Ω through the plastic strain behavior (Fig. 4).

The second theory also applies to the same type of experiment. Here, the fourth-rank tensor $\mathbb{D}$ is determined from measurements of the plastic strain . The form selected for $\mathbb{D}$ particularizes equation (28), with constant values for a and γ, independent of the hole diameter. The comparisons in figures 3 and 4 show that experimental results of [22] can be described with this second theory, for the rupture as well as for the plastic behavior.

The two definitions of damage used in these two theories can be related together, for example for the tensile creep case. Comparison of (2) and (14) shows that :

$$D = 1 - (1 - \Omega)^{r+1/k+1} \tag{29}$$

The difference between the two concepts is illustrated in fig. 1-b where the continuous lines are calculated by means of (29) with $(r+1)/(k+1) = 0.6$ in the circular hole case, 0.33 in the case of parallel cracks.

Another comparison can be made in terms of the description of tertiary creep and of the tensile creep ductility. By neglecting primary creep stage, the theory of Murakami and Ohno gives (15) and (16). In the second theory, the same asumption ($m \to \infty$) and an explicit integration of (1) (24) leads to :

$$\varepsilon_p = \varepsilon_{p_R} \left[1 - \left(1 - \frac{t}{t_c}\right)^{\frac{k-n}{k+1}} \right] \qquad \varepsilon_{p_R} = \left(\frac{\sigma}{\Lambda}\right)^n t_c \frac{k+1}{k-n} \tag{30}$$

We then clearly see the link existing between the two approaches and the relation between the phenomenological coefficients : $c \simeq (n+1)/(k+1)$.

5. Conclusion : The prediction of lifetime of structures requires in many situations a better description of the progressive deterioration of the material before initiation of a macroscopic crack. The Continum Damage Mechanics is a good tool to introduce such deterioration processes, especially for their influence on the stress-strain behavior.

The particular case of creep damage and elastic-viscoplastic behaviour was considered in the present paper, with an emphasis on the development of damage theories valid for any multiaxial loading, especially non proportional loading. This needs the use of anisotropic damage tensors.

The two theories considered yield similar possibilities but should be evaluated more thoroughly in comparison with experimental data. The definition of the fourth-rank tensor $\mathbb{D}$ fits easily into a thermodynamic framework and describes the elastic behavior of the damaged (anisotropic) medium. The theory of Murakami is more complex in use and in identification for the description of the behavior. Furthermore, the measurement of the tensor $\mathbb{D}$, and thus the identification of the corresponding evolution equation, is easier using the effective stress concept, whereas the second-rank tensor introduced by Murakami et al. requires measurements of defects on the microstructural level. Thus, the difficulty associated with the large number of variables to be used in the general case (36 instead of 6) is partially compensated by more ease in identification.

Among the phenomena still poorly reflected in the current models, we may mention :

- the evolution of the creep damage under compression (for lack of significant tests) and the possibilities of damage decrease ($\dot{D} < 0$) that may occur in these phases of compression by the diminution of the cavities.;

- the separation, during tertiary creep, of the effects related to a recovery of the hardening from the damage effects. These two phenomena increase the creep rate and they must be separated to apply the concept of effective stress correctly.

In summary, the equations describing the behavior and damage of metals and alloys offer large possibilities. Additional validations and improvements are to be expected from research undertaken in the fields of multiaxial loading, interactions with the microstructural modifications and with the environment.

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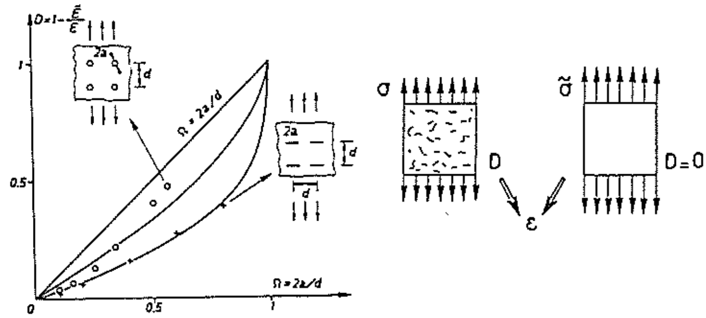

Fig. 1 — (a) effective stress concept.
(b) damages defined by the net area reduction and by the effective cross-section. o+ : results of calculations in elasticity.

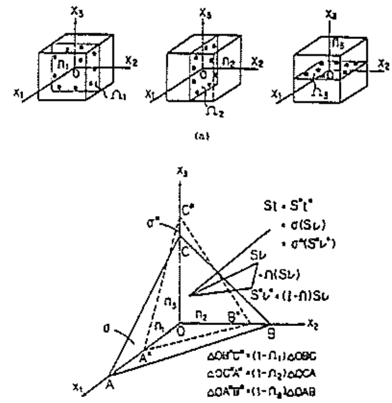

Fig. 2 — The net stress concept :
(a) reduction of the cross-section on the principal planes of the damage tensor
(b) reduction of the cross-section on any given plane.

Table I -

Comparison of anisotropy theories of

Murakami-Ohno and Chaboche.

	TABLE I	Theory of Murakami-Ohno	Theory of Chaboche
Definitions	Damage Definition	through the net section : $\Omega = 1 - S^*/S$	through the effective tive behavior : $D = 1 - \tilde{E}/E$
	Effective Stress	net stress : $\sigma^* = \frac{\sigma}{1-\Omega}$	effective stress : $\tilde{\sigma} = \frac{\sigma}{1-D}$
	Equivalence on the damage law ?	yes: $\dot{\Omega}(\sigma,\Omega) = \dot{\Omega}(\sigma^*,0)$	no: $\dot{D}(\sigma,D) \neq \dot{D}(\tilde{\sigma},0)$
	Equivalence on the viscoplastic law	no : $\dot{\varepsilon}_p(\sigma,\Omega) \neq \dot{\varepsilon}_p(\sigma^*,0)$	yes : $\dot{\varepsilon}_p(\sigma,\Omega) = \dot{\varepsilon}_p(\tilde{\sigma},0)$
Simplified equations	Damage law	$\dot{\Omega} = \left[\frac{\sigma}{A(1-\Omega)}\right]^r$	$\dot{D} = \frac{(\sigma/A)^r}{(1-D)^k}$
	Damage evolution during creep (σ = cte)	$\Omega = 1-\left(1-\frac{t}{t_i}\right)^{1/r+1}$	$D = 1-\left(1-\frac{t}{t_i}\right)^{1/k+1}$
	Constitutive equation (secondary and tertiary creep)	$\dot{\varepsilon}_p = B\left(\frac{\sigma}{1-i\Omega}\right)^n$	$\dot{\varepsilon}_p = \frac{B}{1-D}\left(\frac{\sigma}{1-D}\right)^n$
Generalization	Damage tensors	Second order tensor : Ω	Fourth order asymmetrical tensor : $\mathbb{D}$
	Effective stress tensors	$\sigma^* = \frac{1}{2}\left[\sigma\cdot\Phi + \Phi\cdot\sigma\right]$ $\Phi = (1-\Omega)^{-1}$ $\tilde{S} = \frac{1}{2}\left[\Gamma:\sigma + (\Gamma:\sigma)^T\right]$ $\Gamma = \Gamma(\Omega)$	$\tilde{\sigma} = (1-\mathbb{D})^{-1} : \sigma$

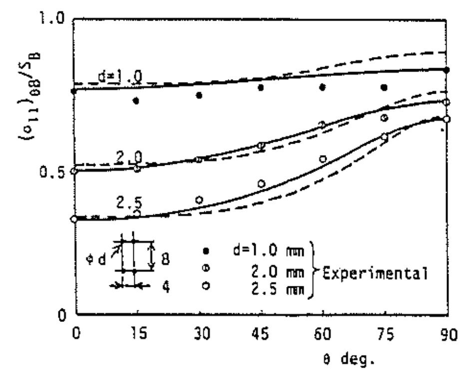

Fig. 3 — Relation between the measured and calculated rupture stress and the direction of uniaxial tension. —— theory of Murakami and Ohno. ——— theory of Chaboche. S_B is the rupture stress of the unperfored plate.

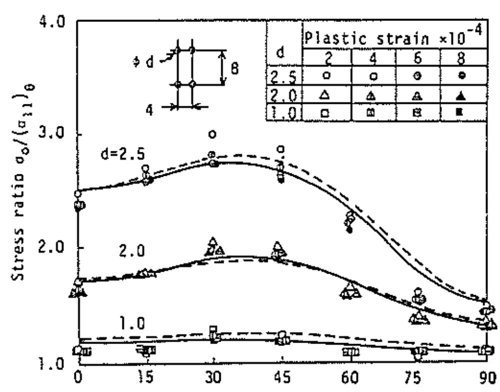

Fig. 4 — Effect of the orientation of the perforations on the uniaxial plastic strain. —— theory of Murakami and Ohno. — theory of Chaboche. σ_0 is the stress corresponding to the same plastic strain on the nonperforated plate.