Intermediate Configurations and the Description of Viscoelastic-Plastic Mate-
Behaviors rial

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SUMMARY

The paper starts from the general constitutive equation of a simple material with memory. The concept of an intermediate configuration is based on the idea of local unloading: A given strain process is continued under the condition of vanishing stress; its asymptotic limit is introduced as the unrecoverable part of the total strain. By assumption, the present value of the unrecoverable strain depends on the history of the total strain; this functional dependence is rate-independent, if the unrecoverable strain corresponds to the plastic strain in the usual sense. A viscoelastic-plastic material can be defined by means of a relaxation functional, which relates the recoverable part of the strain history to the actual stress tensor.

1. Introduction

The analysis of finite deformations of inelastic solids and structures requires a careful consideration of physical and geometrical nonlinearities. In particular, the constitutive equations, which represent the intrinsic material behaviors, must have certain properties of invariance. This implies the possibility to decide about the particular stress and strain variables which conveniently should enter a constitutive equation [1, 2]. Inelastic material behaviors are often represented in terms of a decomposition of the total strain into parts of different qualities. For the case of finite strains, the question of decomposition has been discussed within a variety of papers (see, e. g. [3 - 8]). Essentially, this question is connected with the question to find a constitutive relation between convenient measures of the stress and the recoverable part of the strain tensor [9 - 14].

On the basis of the general constitutive equation for simple materials with memory, rational possibilities for a decomposition of strain tensors follow from the concept of a natural state [15 - 17] or, equivalently, from the idea of an intermediate configuration. The present intermediate configuration corresponds to a set of local states of strain. These (generally non-compatible) states are reached asymptotically in the sense of mechanical equilibrium, which occurs as the result of a local unloading process.

2. Local unloading and unrecoverable strains

The concept of an intermediate configuration is based on the idea of local unloading: Let

$$\tilde{T}(t) = \underset{\sigma \leq t}{\mathfrak{G}}[\,C(\sigma)\,] \qquad (1)$$

be the general constitutive equation of a simple material with memory. ($\tilde{T} := (\det F)\, F^{-1}\, T\, F^{T\,-1}$ and $C := F^{T} F$ denote the 2nd PIOLA-KIRCHHOFF stress tensor and the Right CAUCHY-GREEN tensor, respectively).

If a given strain process $C(\tau)|_{\tau \leq t}$ is continued under the condition of vanishing stress, the asymptotic limit for $\tau \to \infty$ is called unrecoverable (or irreversible) strain C_u. The present value of C_u is a functional of the past strain history:

$$C_u(t) = \underset{\sigma \leq t}{\mathfrak{P}}[\,C(\sigma)\,] \qquad (2)$$

The actual field $C_u(X,t)$ of unrecoverable strain tensors is called intermediate configuration. Equivalently, the intermediate configuration can be represented by the field of unrecoverable stretch tensors

$$U_u\ (X,t) := \sqrt{C_u(X,t)}\ . \qquad (3)$$

3. Decompositions of the strain tensor

In view of a decomposition of the total strain into recoverable and unrecoverable parts, two possibilities can be motivated:

I) A multiplicative decomposition of the deformation gradient into a recoverable part and an unrecoverable stretch:

$$F = \hat{F}_r U_u \qquad (4)$$

II) An additive decomposition of the GREEN's strain tensor $E := (1/2)(C-\underline{1})$:

$$E = E_r + E_u \qquad (5)$$

The two possibilities I) and II) imply two different measures of the recoverable strain, namely $E_r := E - E_u$ and $\hat{E}_r := \frac{1}{2}(\hat{F}_r^T \hat{F}_r - \underline{1})$. The relation between these different recoverable strain tensors is given by

$$U_u \hat{E}_r U_u = E_r\ . \qquad (6)$$

4. Constitutive equations

On the basis of the intermediate configuration the general constitutive equation (1) should be written in the form

$$\hat{\tilde{T}}(t) = \mathop{\mathcal{O}}_{\tau\leq t}[\hat{E}_r(\tau)]\ , \qquad (7)$$

where

$$\hat{\tilde{T}} := (\det \hat{F}_r)\ \hat{F}_r^{-1}\ T\ \hat{F}_r^{T-1}$$

$$= \frac{1}{\det U_u} U_u \tilde{T} U_u \qquad (8)$$

corresponds to a 2nd PIOLA-KIRCHHOFF tensor, which relates the intermediate and the actual configurations.

Equations (2) and (7) imply various possibilities to respresent inelastic material behaviors in the context of finite deformations: In particular, if the functional $\mathfrak{P}$ is rate-independent, C_u corresponds to a plastic strain in the usual sense. If, in addition to the rate-independence of $\mathfrak{P}$, $\mathfrak{O}$ has a relaxation or fading memory property, eq. (7) gives the definition of a viscoelastic-plastic material. For the particular case of an elastic-plastic material, the functional $\mathfrak{O}$ reduces to a function, which relates the present values of $\hat{E}_r$ and $\hat{\tilde{T}}$:

$$\hat{\tilde{T}}(t) = f(\hat{E}_r(t)) \quad (9)$$

$$\Longleftrightarrow \quad \tilde{T} = (\det U_u)\, U_u^{-1}\, f(U_u^{-1}\, E_r\, U_u^{-1})\, U_u^{-1} \quad (10)$$

(compare [9, 10, 12 - 14]).

Particular representations of rate-independent functionals $\mathfrak{P}$ can be achieved explicitely, using arc-length-descriptions [18 - 20], or in implicit forms by means of differential equations, which may contain evolutional laws, loading functions, yield and loading conditions and flow rules, respectively.

In view of a representation of functionals $\mathfrak{O}$ with relaxation properties the literature on viscoelasticity may be consulted (see, e. g. [21, 22]).

To summarize the main arguments of this paper, a general constitutive assumption in form of equations (2) and (7) contains the classical theory of plasticity as a special case. Conversely, particular theories of plasticity, incorporating rate-independent plastic deformations and hardening effects can be generalized in order to include creep, relaxation and some creep-plasticity interactions.

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