Application of the Endochronic Theory to Metals with Complex Strain Histories

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Abstract

A considerable interest has been devoted to models of stable crack extending in polycrystalline solids, with particular attention focused on the crack growth processes at elevated temperatures. This work has been inspired by the service problems in pressure vessels and pressure tubing which typically operate close to one-half of the melting temperature of the material. In certain heat treatment conditions, often associated with the welding process, these materials are susceptible to creep crack growth at appreciable rates.

Here an approach is suggested to model accumulation and propagation of damage in ductile and nonlinearly visco-elastic solids. Deterioration of strength and accumulation of damage is viewed as a local process which occurs in a highly stressed volume of material and which eventually leads to initiation of cracks.

Considerations of interactions between the dominant crack and the localized damage zone, which is weakened by microvoids or microcracks, and which moves ahead of the fracture front, lead to integro-differential and differential equations describing spread of the damage front throughout material under a given crack and loading configuration.

The line-damage model recently proposed by Chrzanowski and Dusza (1980) has been extended to incorporate the effects of the finite thickness of the damage band and the stress gradient encountered at the tip of such band-damage zone, treated as a blunted crack whose front is of elliptical or hyperbolic shape. It is shown that both ductile and time-dependent fractures can be successfully modeled by such a combined CDM/LEFM approach.

1. Introduction

Variations of inner structure of polycrystalline materials leading to the failure of a structural component can be divided into two basic stages. The first one includes formation and growth of microscopic damages, such as vacancies, voids, microcracks, micropores, etc., which are randomly distributed throughout the body. This "hidden" or "incubation" phase of fracture process is terminated at the moment when the dominant crack forms. Propagation of the dominant crack constitutes the second stage of fracture process.

Rigorous treatment of a propagating crack is not available at this time, since a conventional attempt based on path-independent quantities such as J- or C*- integrals is prohibited due to the fact that a finite crack extension (excepting perhaps the very beginning of the growth process) invalidates J and C*, i.e., they cease to be path-independent integrals. It is expected that at the tip of a growing crack a new type of singular field develops which differs from the HRR (Hutchinson-Rice-Rosengren) and the HR (Hui, Riedel) singular fields established for the stationary cracks. Some argument has been given, cf. Riedel and Wagner (1981), that the results obtained for a stationary crack may still retain their validity in the growing crack situation ($\dot{a} \neq 0$), if the characteristic length, say R, over which this new type of stress field is expected to dominate, is sufficiently small against the crack length and the specimen dimensions. Riedel and Wagner (1981) give the following estimates of the characteristic length

$$R \approx (K_I^{n-1}\, EB/\dot{a})^{2/(n-3)} \tag{1.1}$$

for steady-state growth under small-scale creep conditions, while

$$R \approx (\dot{a}/E)^{\frac{n+1}{2}} / (BC^{*\frac{n-1}{2}}) \tag{1.2}$$

for a crack growth accompanied by extensive secondary creep of the whole specimen, i.e., for a large-scale creep situation. The constants B and n are the material parameters which enter in the Norton law, $\dot{\varepsilon} = B\sigma^n$, which provides a macroscopic description of the secondary creep process. If R is small compared to all other linear dimensions involved in the problem, one may speak of K_I-controlled or C*-controlled creep crack extension, in a way analogous to the J-controlled stable crack growth under fully yielded conditions.

In order to bypass these difficulties which emerge when continuum mechanics is applied to predict time-dependent fracture in physically nonlinear materials, a number of idealized models have been proposed, all of which depart to some extent from the basic concept of fracture mechanics, i.e., treatment of the growing crack as a single and continuous defect which propagates by a continuous process of separation occurring at the crack tip. Experimental evidence seems to point toward a different sequence of events leading to an extension of a stable crack. Such events involve nucleation and coalescence of microvoids ahead of the main crack. Linking up of the dominant crack front with these voids provides the mechanism by which the fracture propagates. In ductile materials the microvoids are generally centered on large inclusions present in the materials, cf. Knott

(1980). Likewise, grain boundary cracks propagate by the growth of cavities, nucleated on inclusions or precipitates, present along the grain boundaries ahead of the crack, [7,8,21]. If, indeed, creep fracture extended in this way, the crack front velocity must depend on the microstructure of the material. Parameters such as the spacing, size and cohesive strength of inclusions can be expected to play an important role. Models which treat crack growth as fully continuous and occurring within a continuum, including the zone immediately adjacent to the crack tip, cannot account for the microstructural effects mentioned here.

Therefore, a substantial effort has been recently directed toward a more realistic description of both stages of fracture, i.e., (1) formation of the dominant crack, and (2) extension of such crack within a body containing micro-defects. The present work falls in this framework since it suggests a model for a time-dependent fracture based on two fundamental premises:

(a) the damage kinetics law of Kachanov governing spread of the damage zone ahead of the dominant crack, and

(b) a concept of simultaneous propagation of the dominant crack and the associated damage zone wherein both entities interact with each other.

It is obvious that such idealization of the creep rupture process derives its basic concepts from the classical mechanics of fracture, say LEFM, and from a more recent class of analyses, known under a collective name, coined by Hult in his paper co-authored with Janson (1977), namely, "continuous damage mechanics," or briefly, CDM.

Since we consider damage kinetics law in its scalar form as given by Kachanov, only a one-dimensional problem of damage enlargement is treated. However, in contrast to certain earlier line-damage models, such as the one suggested by Janson and Hult (1977) and then developed by Chrzanowski and Dusza (1980), we do incorporate (a) the effect of the finite thickness of the damage zone, and (b) the effect of interaction between the accumulation of damage and the stress concentration brought about by the existence of the dominant crack. The main crack is considered to be blunted at the very onset of its growth while the stresses ahead of the fracture front are evaluated under an assumption that the profile of the blunted crack is either elliptical or hyperbolic.

2. Damage Band and Its Interaction with the Dominant Crack

Let us consider a one-dimensional damage zone propagating ahead of the dominant crack and interacting with the state of stress induced by the approaching front of fracture. According to this model, the damage zone forms a band of width 2ρ (ρ denotes also the radius of the blunted crack tip) within which the material defects of various kinds, such as microcracks or microvoids, are localized. We assume that the process of damage localization and its subsequent propagation is influenced by the advance of a crack and, vice versa, the rate of spreading crack is affected by an increasing density of micro-defects accumulated within the damage zone. We consider a quasi-static process of crack extension so that most of the quantities involved are time-dependent, including the internal damage parameter, $\omega = \omega(t)$, which varies between the limits $\omega = 0$ and $\omega = 1$. While the lower limit of ω corresponds to an undamaged material, the upper limit is tantamount to an attainment of the critical state identified here with a local collapse occurring at a material point where $\omega \to 1$.

To follow the history of deformation at a generic point P located a critical distance Δ from the tip of a sharp crack, or placed directly on the elliptical (or hyperbolic) front of a blunted crack, i.e., at the distance $r = \rho/2$ from the focus of the ellipse as shown in Fig. 1, we shall consider the sequence of stress states induced at this point by a moving dominant crack. The stress at P due to an approaching crack whose length varies between a_0 and a can be derived from a well-known LEFM formulae (compare Creager and Paris, 1967):

$$\sigma_x = -\frac{K_I}{\sqrt{2\pi r}}\frac{\rho}{2r}\cos\frac{3\theta}{2} + \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] + \quad \ldots$$

$$\sigma_y = +\frac{K_I}{\sqrt{2\pi r}}\frac{\rho}{2r}\cos\frac{3\theta}{2} + \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] + \quad \ldots \tag{2.1}$$

$$\tau_{xy} = -\frac{K_I}{\sqrt{2\pi r}}\frac{\rho}{2r}\sin\frac{3\theta}{2} + \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} + \quad \ldots$$

In what follows we shall assume that only the component σ_y is responsible for cleavage fracture. This may not be an entirely realistic assumption, but it certainly is the simplest and most justifiable for a quasi-brittle solid. Next, it is easily shown that at the crack periphery that is at $\theta = 0$ and $r = \rho/2$, the stress σ_y reduces to

$$[\sigma_y]_{\substack{\text{crack}\\ \text{front}}} = \frac{2\sqrt{2}\,K_I}{\sqrt{2\pi\rho}} \qquad \text{(blunted crack)} \tag{2.2}$$

which differs from the corresponding expression obtained for the case of a sharp crack merely by the numerical constant, i.e.,

$$[\sigma_y]_{r=\Delta} = \frac{K_I}{\sqrt{2\pi\Delta}} \qquad \text{(sharp crack)} \tag{2.3}$$

Expressions (2.3) and (2.2) become identical if the characteristic microstructural dimension Δ, identified here with the distance of the point P from the front of a sharp crack, is set equal $\rho/8$. Formula (2.3) gives the stress at the control point during the time interval associated with the final act of failure which occurs over the entire process zone ($0 \le r \le \Delta$) immediately adjacent to the crack tip. If all the preceding states of stress associated with the current crack length ($a_0 < a' < a$) are to be accounted for, then the formula (2.3) has to be replaced by (see Fig. 2):

$$[\sigma_y]_\Sigma = [\sigma_y]_{r=a-a'+\Delta} = \frac{K_I(a')}{\sqrt{2\pi(a - a' + \Delta)}} \quad , \qquad \Delta = \rho/8 \tag{2.4}$$

We employ the symbol Σ to denote the boundary of the damage zone at which the internal damage parameter attains the critical level, $\omega \to 1$.

Note that the current crack length a' is treated here as a time-like parameter which, during the propagation phase, is related uniquely to time, t', through the obvious relation

$$dt' = \frac{da'}{\dot{a}(a')} \qquad t_1 \leq t' \leq t \tag{2.5}$$

We note that the lower limit t_1 corresponds to the initiation of crack extention (or, equivalently, to the end of the incubation period during which the crack is dormant), while the upper limit t marks the instant at which the material element located at the generic point P collapses.

According to Kachonov's hypothesis, the rate of damage accumulation $d\omega/dt$ is proportional to a certain power of the stress $[\sigma_y]_\Sigma$, namely

$$\frac{d\omega}{dt} = C\left[\frac{\sigma_\Sigma}{1-\omega}\right]^\nu \tag{2.6}$$

Here ω is a certain function of time, σ_Σ is given by eq. (2.4), while C and ν are material constants. In what follows we shall consider an integral of expression (2.6), namely

$$\int_0^\omega (1-\omega')^\nu \, d\omega' = C \int_0^{t'} \sigma_\Sigma^\nu (t'')dt'' \tag{2.7}$$

When the internal damage parameter ω attains one, the upper limit in the integral on the right of eq. (2.7) approaches the instant t which marks the final collapse of a material element at P. Therefore, we have

$$C \int_0^1 (1-\omega')^\nu d\omega' = C \int_0^t \sigma_\Sigma^\nu (t')dt' \tag{2.8}$$

Alternatively, the criterion of failure (2.8) can be written in the following form

$$C \int_0^t \sigma_\Sigma^\nu(t')dt' = \Omega_c$$

$$\Omega_c = \int_0^1 (1-\omega)^\nu \, d\omega \tag{2.9}$$

If we consider the non-dimensional damage parameter Ω_c as a measure of the critical accumulation of strain at the point P, then the first of the expressions (2.9) provides the governing equation of a propagating crack. The left hand side of this equation may be subdivided into two parts: the first part corresponds to the incubation phase ($0 \leq t' \leq t_1$) during which the crack does not propagate but the internal damage parameter ahead of the crack front increases from zero to one; and the second part ($t_1 \leq t' \leq t$) which is associated with crack extension and the ensuing interaction between the damage accumulation (ω) and the stress induced by an advancing crack. The instant t denotes the termination of the second phase of fracture coinciding with collapse of the material element located at the point P. Therefore, we rewrite the first of the equations (2.9) as follows:

$$C \int_{0}^{t_1} \sigma_\Sigma^\nu(t')dt' + C \int_{t_1}^{t} \sigma_\Sigma^\nu(t')dt' = \Omega_c \tag{2.10}$$

Since the first of the integrals appearing above reduces to $\Omega_c(\Delta/\Delta+a)^{\nu/2}$, our criterion for time-dependent failure, which governs propagation of the dominant crack, assumes this form:

$$C \int_{t_1}^{t} \sigma_\Sigma^\nu(t')dt' = \Omega_c \left[1 - \left(\frac{\Delta}{\Delta + a}\right)^{\nu/2}\right] \tag{2.11}$$

If now expression (2.4) is substituted for σ_Σ and the time integral is written as an integral over the prior crack lengths, see eq. (2.5), then we have

$$C \int_{a_0}^{a} \left(\frac{K_I(a')}{\sqrt{2\pi(\Delta + a - a')}}\right)^{\nu} \frac{da'}{\dot{a}(a')} = \Omega_c \left[1 - \left(\frac{\Delta}{\Delta + a}\right)^{\nu/2}\right] \tag{2.12}$$

This constitutes an integral equation for the unknown growth rate, $\dot{a}$, as a function of the crack length a. It is a linear Volterra type equation in which the unknown function is the reciprocal of the crack growth rate, namely $1/\dot{a}$. Although it is possible to construct a closed form analytical solution for this equation by Laplace transformation*, we shall employ a numerical technique which leads to a relatively uncomplicated and an efficient algorithm. This approach permits quick and straightforward generation of the output data pertaining to creep rupture by using a computer. The numerical procedures involved are omitted in this brief text, and only the final results and conclusions are described in the following section.

3. Discussion of Results and Conclusions

First, let us note that the integral equation (2.12) governs extension of a quasi-static crack preceded by a propagating damage band of width $2\rho = 16\Delta$, (while Δ denotes the process zone size) in which the processes of localization and accumulation of micro-defects take place. This equation has been derived based on the Kachanov hypothesis which relates the rate of damage accumulation $d\omega/dt$ to a certain power of the stress induced at the control point for which the time of rupture we wish to predict. Moreover, it has been assumed that the time-dependent internal damage parameter ω is also a history-dependent quantity due to the interaction of the damage accumulation process with the state of stress induced by the advancing crack. We have considered the damage to accumulate during all prior time increments when the dominant crack has been approaching the considered material point located on the unbroken ligament, beginning with the instant of

*H. Riedel (1981), private communication. The exact solution of eq. (2.12) may be expressed in terms of incomplete gamma functions (unpublished results obtained by Hui and Riedel).

load application, followed by the incubation and propagation phases of creep rupture, up to the point of final collapse of the chosen material element.

The equation of a moving crack, eq. (2.12), which was obtained by combining the stress distribution in a brittle solid induced by a blunted crack and the Kachanov law of damage accumulation, may be subsequently reduced to an expression for the rate of the growing crack, i.e.,

$$\frac{da}{dt} = \frac{C\,[K_I(a)/\sqrt{2\pi\Delta}]^{\nu}}{C\,\frac{\nu}{2}\int_{a_0}^{a} \frac{(K_I/\sqrt{2\pi})^{\nu}}{(\Delta + a - a')^{\nu/2+1}}\,\frac{da'}{\dot{a}(a')} + \frac{\nu}{2}\,\Omega_c\,\frac{\Delta^{\nu/2}}{(\Delta + a)^{\nu/2+1}}} \tag{3.1}$$

This form is obtained from eq. (2.12) by the differentiation of both sides of eq. (2.12) with respect to a. Equation (3.1) predicts the rate of crack growth as a certain function of the crack length, a, treated here as a time-like variable. In particular, the initial rate of crack extension, evaluated at the instant of termination of the incubation phase of creep rupture, i.e., when $t = t_1$ and $a = a_0$, is predicted from (3.1) as follows:

$$\left(\frac{da}{dt}\right)_{ini} = \frac{2C}{\Omega_c}\,\frac{K_I^{\nu}(a_0)(\Delta + a_0)^{\nu/2+1}}{\nu\left(\sqrt{2\pi}\,\Delta\right)^{\nu}} \qquad \text{at } t = t_1 \tag{3.2}$$

It turns out that the parameters Ω_c and t_1 are related by a simple formula

$$t_1 = \frac{\Omega_c}{C\,\sigma_0^{\nu}} \tag{3.3}$$

in which

$$\sigma_0 = \frac{K_I(a_0)}{\sqrt{2\pi(\Delta + a_0)}} \tag{3.4}$$

Numerical integration of eq. (3.1) allows for determination of the function $\dot{a} = f(a)$. This has been accomplished for four different pre-cracked specimen configurations, namely: (a) single-edge notch, (b) double-edge notch, (c) three-point bend, and (d) four-point bend. In all instances, it has been shown that in a constant load test the growth rate decreases initially to a minimum which is only slightly lower than the starting rate given by eq. (3.2), and then it increases rapidly to a certain final (sometimes unbounded) level. This rapid increase in the rate of creeping crack is interpreted as the global failure of the specimen. If we denote the duration of the incubation phase (i.e., the dormant crack) by t_1 and the time of final rupture by t_*, then it is of ultimate interest to predict the numerical values of times t_1 and t_* for any specific geometry of the structural element, the type of loading (tension or bending) and the crack configuration involved (single- or double-edge notch). This has been done by direct integration of the governing equation (3.1). Finally, it is interesting to compare the ratios t_*/t_1 predicted from the present model with those obtained from the standard CDM (continuous damage mechanics) approach in which no dominant crack is considered and no interaction between the macro-defect and the damage zone is allowed for. For tensile rupture, we obtain

$$\frac{t_*}{t_1} \begin{cases} =1 & , \quad \text{standard CDM} \\ >1 & , \quad \text{present model} \end{cases} \tag{3.5}$$

while for creep rupture under predominant bending we have

$$\frac{t_*}{t_1} \begin{cases} =3 & , \quad \text{standard CDM} \\ >3 & , \quad \text{present model} \end{cases} \tag{3.6}$$

Such a range of values finds strong support in the experimental evidence.

We may conclude, therefore, that the description of creep crack propagation based on consideration of interacting macro- and micro-defects (or, a "cross" between LEFM and CDM) leads to a more realistic mathematical model. The approach followed here provides a greater flexibility in data reduction and interpretation as compared to the standard CDM model. This is due to the incorporation of two distinct sets of parameters governing the rate of crack growth and time to failure for any specified load/crack configuration. The first of the two sets corresponds to the quantities used in the macroscopic or field theory of fracture, such as the stress intensity factor K_I and the material toughness K_{IC}, while the second results from the microstructural considerations underlying the laws which govern formation and spread of micro-defects in a stressed solid.

We realize that further refinements of the model are necessary. In particular, it is desirable that certain novel constitutive relations and damage growth laws are developed to better represent the behavior of various types of materials. One such possibility is offered by the recent work of Krajcinovic (1981, 1982).

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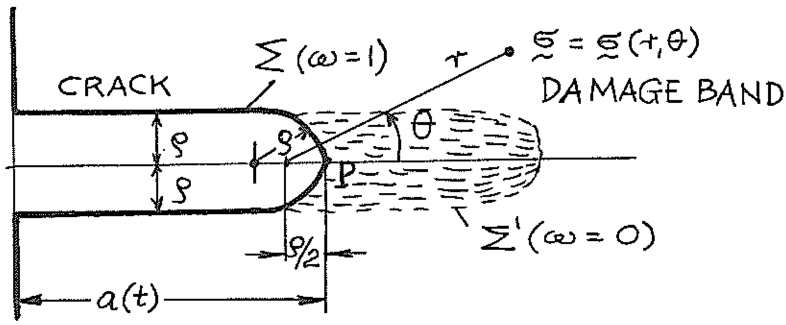

Fig. 1. Crack of finite width and the associated damage band. Note that the crack front (Σ) is defined as a locus of points at which $\omega = 1$.

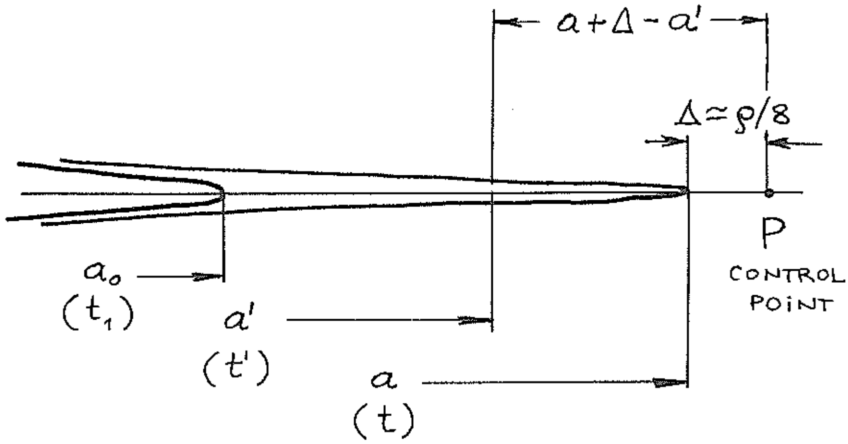

Fig. 2. Stages of crack advancement toward the control point: (a) initiation at t_1, (b) location of the crack front at an intermediate instant t' and (c) collapse occurs at the control point, time = t.