A Strategy to Compute Plastic Post-Buckling of Structures

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ABSTRACT

The paper gives a general framework to the different strategies used to compute the post-puckling of structures. Two particular stategies are studied in more details and it is shown how they can be applied in the plastic regime.

All the methods suppose that the loads F are proportional to a simple parameter λ ; more precisely : eq (1) $F = \lambda F_0$.

F_0 being a constant load. All the methods consist in trying to solve the equilibrium equations on the actual geometry of the body ; that is : eq (2) $B^T \sigma = \lambda F_0$

The calculations are performed by increasing the load parameter λ step by step.

All these methods break down near the limit point or the bifurcation points. To overcome this difficulty we add an axillary constraint to eq (2). eq(3) $f(\Delta U, \Delta\lambda , \Delta a) = 0$.

In eq(3) ΔU is the increment of displacement vector, $\Delta\lambda$ the increment of the load parameter, Δa an imposed value. The Rik's method is a method of this type.

The auxillary equation (3) is generally a condition of the growth of a certain "norm" of the displacement vector ("norm" is taken here in a generalized sense). The increment of the load multiplier $\Delta\lambda$ is calculated from that constraint. The paper shows how these methods can be implemented in a very cimple way.

In the elastic case we show the application of the method to the calculation of post buckling response of a clamped arch which has been previously studied by E. CARNOY /‾7_7 The method is also applied to a very simple case of two bars which can be calculated analytically.

In the plastice range, the method is applied to the post-buckling of an imperfect ring which can be calcultaed analytically.

Another example is the comparison of the comparison of the computed post-buckling of a thin cylinder under axial compression, and of the experimental behavior on the same cylinder.

The limitation of these types of strategies are also mentionned and the physical signifiance of calculations in the post-buckling regime are discussed.

1. INTRODUCTION

All the methods of computation of post-buckling behavior suppose that the loads F depend on a single parameter λ called the load factor. That hypothesis can be written :

(1) $F = \lambda F_0$

They all consist in adding to the equilibrium equations a certain type of constraint. This leads to more as less modifications in the usual computation stategy. The paper will be divided into four parts.

The first one gives the different type of existing constraints.

The second part shows how they can be implemented in a very simple way.

The third gives some examples of application in the elastic and plastic post-buckling range.

The last part is a discussion of the different strategies taking into account certain physical observations.

2. DIFFERENT TYPES OF CONSTRAINTS

The equilibrium equation written on the deformed configuration are for the load factor λ.

(2) $B^T \sigma = \lambda F_0$

B being the divergence operator written on the present configuration,

σ being the cauchy stress tensor on this configuration.

The incremental methods consist in computing the actuel configuration C and the cauchy stress tensor σ by increasing the load factor λ step by step (/¯1_7) . All these methods break down near limit points or the bifurcation points. To overcome this problem the general method is to add to eq(2) the constraint.

(3) $f(U, \lambda, \Delta a, F_0) = 0$

U being the displacement field

λ the load factor

Δa a control parameter

F_0 the load vector

Constraint associated with the well known Rik's method /¯2_7 has been written as :

(4) $f = \dot{\lambda}\ (\lambda - \lambda_1) + \dot{\vec{U}} \cdot \overrightarrow{\Delta u} - \Delta a$

where $\dot{\lambda}$ is the gradient of load factor

$\dot{\vec{U}}$ is the gradient of displacement factor

The constraint associated with the method proposed by BATOZ and DHATT /¯3_7 can be written :

(5) $f = \Delta\lambda\ U^0_q + \Delta U^{NL}_q - \Delta a = 0$

This is the well known imposed displacement method

It means that the component q of the total incremental displacement is controlled. U^0 being the elastic linear displacement. U^{NL} being the non linear part of the displacement.

STEIN /¯4_7 has imposed a constraint on the increment of external work.

(6) $f = (\Delta\lambda\ \vec{U}_0 + \overrightarrow{\Delta U}^{NL})\ \vec{F_0} - \Delta a = 0$

CRISTFIELD has proposed the same type of method which has been used in CASTEM's codes /¯5_7/¯6_7.

(7) $f = (\Delta\lambda\ \vec{U_0} + \Delta U^{NL}) \cdot \vec{U_1} - \Delta a = 0$

or ..../...

$$(8)\quad \begin{cases} f = (\Delta\lambda\ \vec{U_0} + \overrightarrow{\Delta U}^{NL})\ .\ (\Delta\lambda\ \vec{U_0} + \overrightarrow{\Delta U}^{NL}) - \Delta a = 0 \\ \text{with}\ \dfrac{(\vec{U_1}, (\Delta\lambda\ \vec{U_0} + \overrightarrow{\Delta U}^{NL}))}{\|\vec{U_1}\|\ \|\Delta\lambda\ \vec{U_0} + \overrightarrow{\Delta U}^{NL}\|} > -1 \end{cases}$$

This means that the projection of the increment of the total displacement vector $\overrightarrow{\Delta U}$ on some other vector (for instance the previous total displacemnt increment) is controlled in this form (eq(7)) the method is a linearisation of the Rik's method.

In the form of eq(8) the length of the displacement increment is controlled. The second condition avoids to return on the previous point of the load displacement curve.

3. IMPLEMENTATION

The constraints introduce a modification in the classical strategy used to compute the large displacement elastoplastic response of a structure. It can be combined to any strategy. The way Riks's proceeds is the most precise way but leads to large modification in the basic procedures. I will be shown now how a very simple modification can be added to the standard procedure to add the controll parameter.

The total displacement ΔU^T is the som of two parts :

$$(9)\quad \overrightarrow{\Delta U} = \Delta\lambda\ \vec{U_0} + \overrightarrow{\Delta U}^{NL}$$

a linear point proportional to the increment of load parameter $\Delta\lambda\ \vec{U_0}$ and a nonlinear part due to the corrections of plastic and or geometrical non linear forces ΔU^{NL}. We suppose that a certain increment of the control parameter Δa is given.

The procedure is initiated by giving an increment of forces $\Delta\lambda_0$ such that for the different cases :

$$(10)\quad \begin{cases} \Delta\lambda_0\ U_q^0 = \eta\ \Delta a & \\ \Delta\lambda\ \vec{U_0}\ .\ \vec{F_0} = \eta\ \Delta a & \text{with } \eta < 1 \\ \Delta\lambda\ \vec{U_0}\ .\ \vec{U_1} = \eta\ \Delta a & (\text{eg } \eta = 0.8) \\ \Delta\lambda^2\ \vec{U_0}\ .\ \vec{U_0} = \eta\ \Delta a & \end{cases}$$

from this load increment we deduce the non linear displacement field ΔU^{NL} by the standard procedure ; from the equations (5), (6), (7) or (8), a new value of the increment of the load factor is deduced. This new value induces a new increment of non linear displacement ΔU^{NL} and the procedure is repeated until convergence is reached on the value of increment of load factor and on the residual forces. Which such a method it is very simple to modify a standard incremental algorithm to acheive post-buckling computations. From this point of view it can be easily seen that this method can be used to compute limit loads where the non linear part of the displacement is due to plastic forces only.

4. EXAMPLES

Elastic case 1 two dimensions. The methods associated with eq (5) and (7) have been used to compute the post buckling of an arch loaded by an external dead pressure and the results have been compared with results obtained by E. CARNOY [7] and no difference has been observed (see figure 1).

Elastic case 2 three dimensions. This is the computation of an imperfect cylinder under axial compression. The radius is R = 333.33 and the length is l = 63.108. The thickness is 1. The imperfection has two possible shapes :

$$(11)\quad W_1^{imp} = 0.25\ \cos\frac{\Pi z}{2e}\ \cos\frac{\Pi R\alpha}{2e}$$

(12) $W_2^{imp} = 0.25 \cos \frac{\Pi z}{e} \cos 18\,\alpha$

The Young's modulus is 21 000 ; the Poisson's ratio is 0.3.

With this type of case the method associated with eq (5) breaks down because the structure is not controlled by a simple point.

The method associated with eq (7) works quite well and gives a rather similar response that the Rik's calculations see Figure 2.

The form of the imperfection has a great influence on the plasticity path (see Figure 2 the difference between displacements in the two cases).

Plastic case 1

The plastic post buckling of a ring under external pressure has been computed by the two methods (5) and (7) and we found that the decreasing curve is govern by the equation of the limit domain :

(13) $PR\;\;W\;=M_L$

Where P is the external pressure ; R the radius of curvature ; W the normal displacement M_L the limite momentum of the beam. (see Figure 3).

Plastic case 2

The computed post buckling path has been compared with an experimental investigation done with an imposed displacement. The correlation is quite good. (see Figure 4).

5. DISCUSSION AND CONCLUSIONS

All the methods presented here give in some cases, some interesting computed solutions. It has been already remarked [4] that the different strategies do not always give the same post buckling path. More foundamentally, it has been observed that the post buckling path, when buckling is unstable, is characterized by a dynamic movement. All inertial effects are neglected in all the approaches presented here. So that the post buckling load deflections curve is valid only if there is a very little kinetic energy associated with the post buckling. The method is also, as it is presented, limited to a load depending of a simple parameter λ. The case of more than one parameter is not very clear yet.

In conclusion, the method presented here gives a way to solve class of the post buckling behavior of a structure. If the post bukcling occurs with a small kinetic energy (displacement controlled buckling) and if the loads depend of only one parameter.

These methods should give good results even into the plastic range. If the buckling is unstable and that a large kinetic energy is involved with the post buckling these methods are not realistic.

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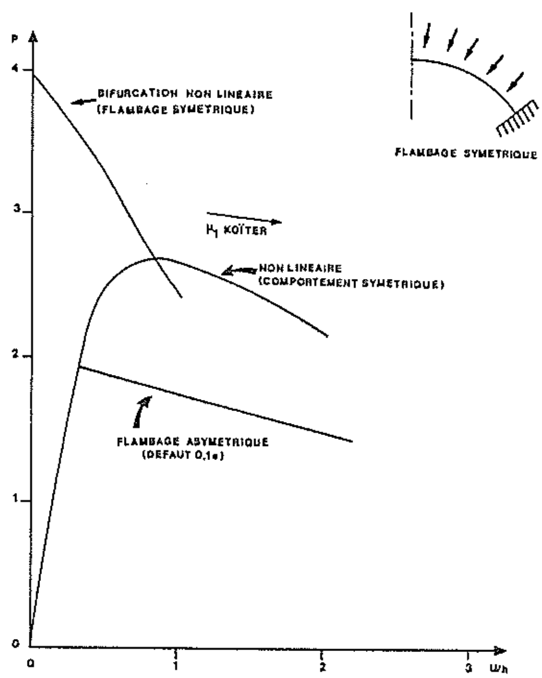

FIGURE 1 : Post buckling of an arch under external pressure

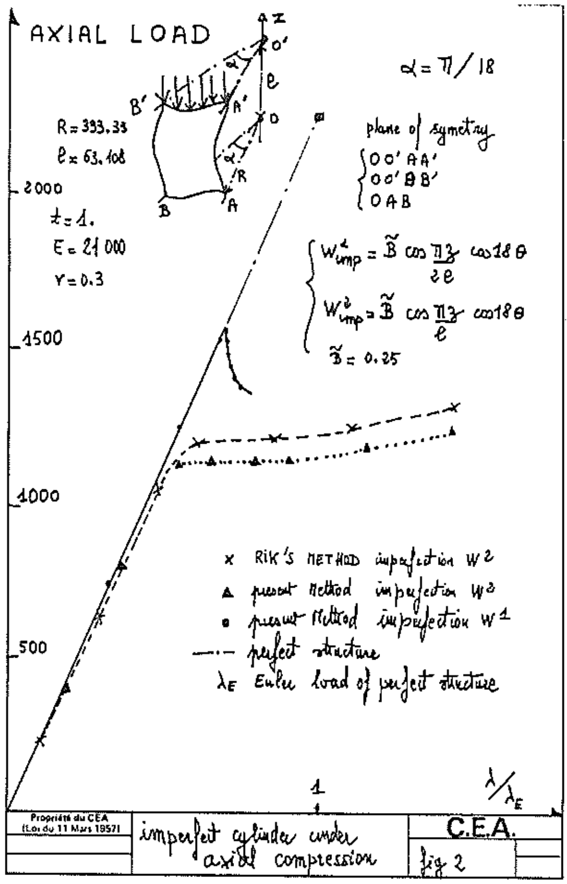

FIGURE 2 : Post buckling of an imperfect cylinder under axial compression

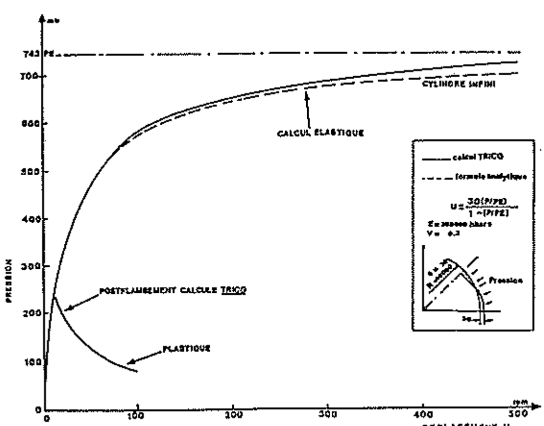

FIGURE 3 : Plastic post buckling of a ring under external pressure

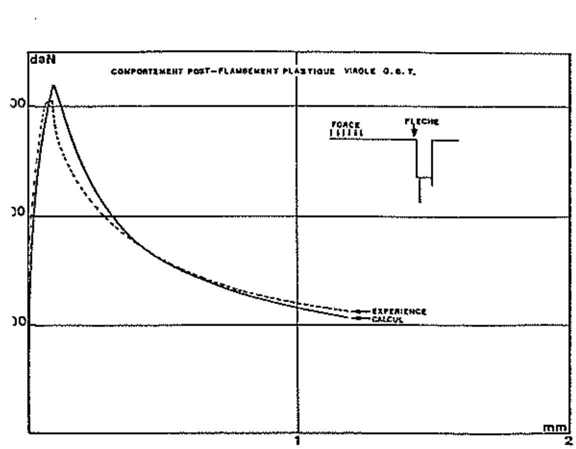

FIGURE 4 : Plastic post buckling of an axisymetric structure comparison of computed and experimental load deflection curves.