A Probabilistic Concept for the Aseismic Design of Nuclear Power Plants

G. König
Technische Hochschule Darmstadt, Institut für Massivbau, Alexanderstr. 5, D-6100 Darmstadt, Germany

D. Hosser
König und Heunisch Beratende Ingenieure, Letzter Hasenpfad 21, D-6000 Frankfurt/Main 70, Germany

SUMMARY

In this paper a first-order reliability theory is used to redefine the design earthquakes and related safety factors for the aseismic design of nuclear power plant structures. A simplified probabilistic method for the derivation of safety elements for design is summarized which is based on an appropriate choice of constant design values for all scattering basic variables like loads, resistance parameters or model parameters. The design values depend on the influence upon the overall uncertainty of the design problem and on a target safety index $\beta$ as a measure for the required reliability. Partial safety factors relating those design values to appropriate nominal values can easily be determined.

The aseismic design of nuclear power plant structures presently includes a double check with two different design earthquake definitions, the extreme SSE and the more frequent OBE. Usually, "extreme" is taken to be an event with a $10^{-4}$/a probability of exceedance and "more frequent" to have a $10^{-3}$ to $2 \cdot 10^{-3}$/a probability of exceedance; the seismic input is given as 84-percetnile. It is assumed that the target safety index $\beta$ should be the same for both checks and should agree with the value required in a new German Safety Guide for usual buildings and loadings. Under this assumption, global safety factors $\gamma$ are calculated for a design equation $R = \gamma N_p(S_G + S_Q + S_E)$ with varying portions of dead loads ($S_G$), live loads ($S_Q$) and earthquake loads ($S_E$).

Given a design according to the current Reg.Guide for the design of NPP structures, the resulting safety index $\beta$ is calculated. In the case of load combinations with SSE and design with $\gamma = 1.0$ the target safety index is not fully met, especially for higher portions of earthquake loads and for brittle concrete failure. Design for combinations with OBE yields always higher reliability than design for SSE; clearly OBE presently governs the design, even if the damping effect is small. Therefore it is proposed to redefine the OBE as a 100 to 200 years event if the present safety factors are retained. In low seismic regions as the FRG, the OBE will then be only of interest if load combinations with other extreme loads have to be considered.
1. Introduction

In the Fed. Republic of Germany (FRG), the aseismic design of nuclear power plant structures is based on the Regulatory Guide KTA 2201.1 [1] dated from 1975. Design philosophy and principles are more or less taken over from 10 CFR 100 App. A [2], especially the design for two earthquake levels, the "safety earthquake" (=SSE) and the "design earthquake" (=OBE). Usually, the SSE level is defined by seismologists as the maximum earthquake potential of a larger region with an annual probability of exceedance at the site less than $10^{-4}$. Given this event, all seismic class I (safety related) systems or structural elements have to fulfill their safety function; in most cases they must not exceed an ultimate limit state. The OBE is defined as the largest historical event in the surroundings of the site, i.e. in practice an event with an annual probability of exceedance of less than $2 \cdot 10^{-3}$ and at least half the SSE amplitude. Given a multiple occurrence of such an OBE, all seismic class I systems or structural elements must be ready for further use; in practice they are not allowed to exceed the elastic or admissible limits.

Although the OBE design should primarily be an additional check of the serviceability of safety related systems, it is presently performed as a second ultimate limit state check with somewhat higher safety factors. This OBE check often governs the aseismic dimensioning of systems and structures.

In the last years, the inconsistencies of the present aseismic design philosophy became more and more obvious. It was found that two main shortcomings exist: 1) two earthquake levels and the respective load combinations and safety factors for design have been defined independently and intuitively, 2) the safety aspects of the OBE check are not clearly understood. Therefore the authors tried to formulate a uniform safety concept for the aseismic design of NPP in the FRG as basis of a new draft of [1]. It is based on the new German safety guide [3] and similar international papers, e.g. [4]. The probabilistic reliability theory is used to derive safety factors depending on appropriate nominal values and to answer the question: do we need two separate checks in aseismic design?

This paper summarizes the simplified probabilistic design method which is directly applicable to the derivation of safety factors for design. The present German design format and the special aspects of low seismic regions as the FRG are taken into account. Similar probabilistic studies with respect to the U.S. design practice have been described during the 6th SMIRT Conference, e.g. [5].

2. Simplified probabilistic design method

The first-order second-moment reliability theory in its advanced version, e.g. after the work of Haasofar/Lind [6], has been proved to be an efficient tool for calculating the resulting reliability in a given design situation with randomly varying variables and models. It has also been used to develop
more consistent future design codes \([3,4,7]\). Nevertheless, the price of a uniform reliability level is that design values of basic variables vary from one design situation to another; such design values are not applicable in the daily design practice and simplifications are essential for finding more reasonable design formats. Therefore, the authors proposed in \([3,7]\) a simplified probabilistic level II method which has the advantage that constant design values - and from those also safety factors - can be derived with which the reliability does not exceed given bounds.

First, the given design equation is formulated as a linear function of a resulting resistance \(R\) and a resulting action \(S\) (e.g. load, load effect, stress or deformation)

\[
Z = R - S \leq 0
\]  

(1)

Generally, the resulting resistance \(R\) and resulting action \(S\) are (linear or non-linear) functions of several basic random variables \(R_i\) and \(S_j\)

\[
R = g_R (R_1, R_2, \ldots, R_n)
\]  

(2a)

\[
S = g_S (S_1, S_2, \ldots, S_m)
\]  

(2b)

The distribution parameters of \(R\) and \(S\) can be calculated independently in a first-order second-moment sense using the error propagation law and linearizations of \(g_R\) and \(g_S\) at the so-called design points (Hasofer/Lind-points) \(r^* (r_i^*)\) and \(s^* (s_j^*)\); given non-Gaussian distribution functions \(F_{R_i}(r_i^*)\) and \(F_{S_j}(s_j^*)\) are fitted to standardized Gaussian distributions \(\Phi_{R_i}(\bar{\Phi}_i\cdot \beta)\) and \(\Phi_{S_j}(\bar{\Phi}_j\cdot \beta)\) with e.g. \(\bar{\Phi}_i = (r_i^* - \mu_{R_i})/\sigma_{R_i}\) (see \([8]\)). According to \([6]\), the resulting design values \(r^*\) and \(s^*\) are expressed with the help of global weighting factors \(\alpha_R\) and \(\alpha_S\) (Hasofer/Lind sensitivity factors) and the safety index \(\beta\):

\[
r^* = F_{R_i}^{-1} (\bar{\Phi}(-\alpha_R \cdot \beta))
\]  

(3a)

\[
s^* = F_{S_j}^{-1} (\bar{\Phi}(-\alpha_S \cdot \beta))
\]  

(3b)

As mentioned above, the weighting factors depend on the influences of the random variations of \(R\) and \(S\) upon the safety margin \(Z\) and change from situation to situation.

An always safe but uneconomic choice of constant \(\alpha\)-values would be \(\alpha_R = 1.0\) and \(\alpha_S = -1.0\); the target reliability then would be reached only if either \(R\) or \(S\) does not scatter and would be exceeded in all practical cases. If the relationship between the standard deviations \(\sigma_S/\sigma_R\) can be bounded by

\[
\min \frac{\sigma_S}{\sigma_R} \leq \frac{\sigma_S}{\sigma_R} \leq \max \frac{\sigma_S}{\sigma_R}
\]  

(4)

a better choice of constant values \(\bar{\alpha}_R\) and \(\bar{\alpha}_S\) can be found for which the safety index \(\beta\) is not smaller than a given limit \(\min \beta\) (more details are given in \([7]\)):

\[
\bar{\alpha}_S = -\frac{\min \beta}{\sqrt{1 + (\max \frac{\sigma_S}{\sigma_R})^2} - \sqrt{1 + (\min \frac{\sigma_S}{\sigma_R})^2}}
\]  

(5a)
\[ \alpha_R = \min \frac{\sigma}{\bar{\sigma}} \cdot \sqrt{1 + \left( \min \frac{\sigma^*}{\bar{\sigma}^*} \right)^2 + \alpha^*_S \cdot \min \frac{\sigma^*}{\bar{\sigma}^*}} \]  

(5b)

For usual ultimate limit state design of structural elements with \( \beta = 4.7 \) and \( \min \beta = 4.2 \) according to [3], \( \min \sigma^* / \bar{\sigma}^* = 0.15 \) and \( \max \sigma^* / \bar{\sigma}^* = 3.5 \) the following weighting factors are introduced in [3,7]:

\[ \alpha_R = 0.8 \]
\[ \alpha^*_S = -0.7 \]  

(6a)

(6b)

They are also applicable if - like in the case of the aseismic design of NPP structures with higher seismic load portions - the upper limit \( \max \sigma^* / \bar{\sigma}^* \) should be exceeded [7].

Returning to eqs. (2) design values \( r_i^* \) and \( s_j^* \) for the basic variables are defined using the global weighting factors \( \alpha_R \) and \( \alpha^*_S \) from eqs. (6) and additional weighting factors \( \alpha_{R_i} \) and \( \alpha_{S_j} \)

\[ r_i^* = F_{R_i}^{-1} (\Phi (-\alpha_R \cdot \alpha_{R_i} \cdot \beta)) \]  

(7a)

\[ s_j^* = F_{S_j}^{-1} (\Phi (-\alpha^*_S \cdot \alpha_{S_j} \cdot \beta)) \]  

(7b)

The additional weighting factors depend on the influence of \( R_i \) or \( S_j \) on the overall variations of \( R \) or \( S \) in the surroundings of the design values \( r^* \) or \( s^* \)

\[ \alpha_{R_i} = \frac{\frac{\partial \sigma R}{\partial R_i} \cdot \sigma_{R_i}^'}{\left( \sum_{k=1}^{n} \frac{\partial \sigma R}{\partial R_k} \cdot \sigma_{R_k}^' \right)^2} \]  

(8a)

\[ \alpha_{S_j} = \frac{\frac{\partial \sigma S}{\partial S_j} \cdot \sigma_{S_j}^'}{\left( \sum_{k=1}^{n} \frac{\partial \sigma S}{\partial S_k} \cdot \sigma_{S_k}^' \right)^2} \]  

(8b)

Once more, constant values \( \alpha_{R_i} \) and \( \alpha_{S_j} \) can be found by arranging the basic variables according to their influence upon the resulting standard deviations \( \sigma R \) and \( \sigma S \) and choosing for the variable number \( i \)

\[ \alpha_{R_i} = \alpha_{S_i} = \sqrt{1} - \sqrt{1 - \gamma} \]  

(9)

For practical applications only two different values seem to be sufficient yielding always conservative results:

\[ \alpha_{R_1} = \alpha_{S_1} = 1.0 \quad \text{for } i = 1 \]

(10a)

\[ \alpha_{R_i} = \alpha_{S_i} = 0.4 \quad \text{for } i \geq 2 \]  

(10b)

Load combinations are taken into account in [3,7] following the well known model of Ditteksen [9] based on Poisson renewal pulse processes. This model has the advantage that extreme value distributions
$F_{C_j, \text{max}}$ (Type I) of time variant or intermittent actions $S_{j}$ (or action effects) can be determined for fixed numbers of independent repetitions $n_{j}$ according to Ditlevsen's scheme. The design values $r_{i}^{*}$ and $s_{j}^{*}$ of all basic variables including the extremes $s_{j}$, max $n$, then are evaluated from eqs. (7) within the given distribution functions. By relating the design values to appropriately defined nominal values, partial safety factors for the practical design can be easily determined.

3. Safety factors for the aseismic design

Based on section 2 and the equations in [7] for different distributions design values of the aseismic design of structural elements will be derived. Without too much loss of generality, the following design equation is presumed:

$$\mu_{R} \cdot R = S_{G} + S_{Q} + S_{E}$$  \hspace{1cm} (11)

with

$R =$ load bearing capacity in the ultimate limit state

$\mu_{R} =$ parameter to account for model uncertainties

$S_{G} =$ action (effect) due to self weight

$S_{Q} =$ action (effect) due to live loads

$S_{E} =$ action (effect) due to earthquake

Actions due to seismic induced structural or system failures are not considered in eq. (11) because in most cases the probability of a combined occurrence is extremely low (see e.g. [10]). The random variations of the basic variables are described by the distribution parameters given in Tab. 1. The distribution functions, coefficients of variation (C.O.V.) and definitions of nominal values are chosen according to the experience with probabilistic analyses of usual design problems [7] and to the proposals in [3]. The C.O.V. of the action $S_{E}$ due to earthquake includes random variations of the source signal (~40 %), of the transfer function between source and site (~40 %) and of the soil-structure vibration model (~20 %), therefore [11]

$$v_{S_{E}} = \sqrt{0.4^2 + 0.4^2 + 0.2^2} = 0.6$$  \hspace{1cm} (12)

Percentiles of the nominal values of $S_{E}$ are calculated from the probabilities of exceedance of the SSE and GBE level mentioned in section 1 multiplied by a 16 % probability of exceeding the structural response (usual procedure is to define a 84 % response spectrum of the input motion).

Using the equations from [7] partial safety factors $y_{R_{1}}$, $y_{S_{j}}$ and combination factors $\psi_{0, S_{j}}$ as defined in [3] are evaluated and summarized in Tab. 1. Eq. (11) now takes on the following possible forms representing a partial safety factor concept (so-called level I) for the aseismic design of structural elements:

$$\frac{y_{k}}{\mu_{R}} \cdot y_{R} = \psi_{0, S_{G}} \cdot y_{S_{G}} \cdot S_{G} + \psi_{0, S_{Q}} \cdot y_{S_{Q}} \cdot S_{Q} + \psi_{S_{E}} \cdot S_{E}$$  \hspace{1cm} (13a)
\[ \gamma_{S_E} = \gamma_{S_G} \cdot \gamma_{S_Q} \cdot \gamma_{S_E} + \gamma_{S_Q} \cdot \gamma_{S_E} \]  
(13b)

\[ \gamma_{S_E} = \gamma_{S_G} \cdot \gamma_{S_Q} \cdot \gamma_{S_E} + \gamma_{S_E} \cdot \gamma_{S_E} \]  
(13c)

In eq. (13a) the variations of the seismic action \( S_E \) dominate, in eq. (13b) those of the action \( S_Q \) (live load), in eq. (13c) the portions \( S_E \) and \( S_Q \) are so small that the variations of the stationary action (self weight) dominate; the latter case is only of a theoretical nature. The design results from eqs. (13a) and (13b) in the form of a global safety factor as used in the present German design practice

\[ \gamma = \frac{1}{\gamma_{S_G} + \gamma_{S_Q} + \gamma_{S_E}} \]  
(14)

are plotted in Fig. 1 for a fixed relation \( \gamma_{S_G}/\gamma_{S_Q} = 1 \) and varying portions of seismic action \( S_E \). There are also shown for comparison the global safety factors according to the design code DIN 1045 [12] for r.c. structures and the Reg. Guide Draft KTA 2201.3 [13] for NPP structures.

4. Discussion and conclusions

From Fig. 1 the following conclusions can be drawn:

1) The safety concept of [12] agrees well with eq. (13b)

2) The safety factor for SSE and concrete failure in [13] is too small whilst the safety factor for failure of the reinforcing steel agrees well with eq. (13a)

3) The safety factors of [13] for combinations with OBE are much higher than those from eq. (13a) both for concrete and steel failure.

A possible correction of 2) would be to increase the global safety factor for concrete failure in [13], e.g. to \( \gamma = 1.2 \) (shortly dotted line in Fig. 1). Point 3 could be corrected either by reducing the safety factors or by redefining the OBE as an event to be really expected to occur. The necessary \( \gamma \)-values for an OBE defined as 100 years event are shown in Fig. 1 (shortly dotted). A better feeling for the effect of these corrections of the design concept [13] is gained by calculating the (level II) safety index \( \beta \) resulting from the different design formats. The results are depicted in Fig. 2.

Obviously, the partial safety factor concept of eq. (13) yields a relatively uniform reliability for all portions of seismic actions and the three different earthquake definitions of Tab. 1. The load combinations with dominating live loads (eq. (13b)) are well approximated by the global safety factor concept of [12] (only selfweight and live load considered) with sufficient reliability up to about 40 % seismic action. A less satisfactory approximation of the safety index results from the aseismic design concept of [13]. The general conclusions drawn from Fig. 1 are verified by Fig. 2.

The abovementioned corrections of the aseismic design concept have the expected effects. Due to the increase of the safety factor for SSE and
concrete failure the target reliability is now observed for all portions of seismic action. The reduction of the OBE level to a 100 years event reduces the safety index to acceptable values; for steel failure and very high portions of seismic action the safety index is at the lower tolerance bound. The amplitudes of the redefined OBE decrease in low seismic regions as the FRG to 1/4 - 1/3 the SSE amplitudes and can no longer govern the design. The OBE check can only be of interest with respect to load combinations with actions due to more frequent external or internal events.

References
### Table 1: Distribution Assumptions and Safety Factors for the Basic Variables

<table>
<thead>
<tr>
<th>Basic Variable</th>
<th>Coef. of Variation</th>
<th>Percentile of Nominal Value</th>
<th>Distribution</th>
<th>$\sigma_0/\sigma_S$</th>
<th>$\alpha_{Gj}$</th>
<th>$\alpha_{sk}$</th>
<th>$\gamma_{sk}$</th>
<th>$\gamma_{sk}/\sigma_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>a) 0.15</td>
<td>0.05</td>
<td>lognormal</td>
<td>0.8</td>
<td>1.0</td>
<td>1.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) 0.06</td>
<td>0.05</td>
<td></td>
<td>0.8</td>
<td>1.0</td>
<td>1.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) 0.06</td>
<td>0.02</td>
<td></td>
<td>0.8</td>
<td>1.0</td>
<td>1.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.5</td>
<td></td>
<td>0.8</td>
<td>0.4</td>
<td>1.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_0$</td>
<td>0.07</td>
<td>0.5</td>
<td>normal</td>
<td>-0.7</td>
<td>1.0</td>
<td>1.23</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>$s_0$</td>
<td>0.5</td>
<td>0.99</td>
<td>Extr. 1/1 year</td>
<td>-0.7</td>
<td>1.0</td>
<td>1.66</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>$s_0$</td>
<td>0.6</td>
<td>a) 0.999984 (SSE)</td>
<td></td>
<td>-0.7</td>
<td>1.0</td>
<td>0.73</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) 0.99968 (OBE)</td>
<td></td>
<td></td>
<td>-0.7</td>
<td>1.0</td>
<td>0.95</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>c) 0.9984</td>
<td></td>
<td></td>
<td>-0.7</td>
<td>1.0</td>
<td>1.15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 1:** Global safety factors for the design of r.c. structural elements with different design formats ($s_{sk}/s_{sk} = 1$)

**Fig. 2:** Safety index of r.c. structural elements designed according to different design formats ($s_{sk}/s_{sk} = 1$)