Structural Modelling and Limit State Identification for Reliability Analysis of RC Containment Structures

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One of the most important ingredients in the safety analysis of any structures, and particularly nuclear structures, is the selection of an analytical model for the structure and identification of the limit states. An analytical model facilitates the evaluation of the significant structural response to postulated as well as actual loads, while a limit state represents a state of undesirable structural behavior which must be identified for the reliability analysis. Also, limit states must be specified in terms of response quantities obtainable from the analysis performed on the selected structural model.

For the reliability analysis of a reinforced concrete containment structure, a SAF-V finite element representation (involving shell elements) is selected as the analytical model for the structures. The limit state, which represents the onset of structural failure, is defined in such a way that it is reached when either concrete crushing occurs at the extreme fiber or the yielding of reinforcing bars occurs at any time during the structure's service life. The limit state so defined is considered at two sets of finite element boundaries, one perpendicular to the meridional direction and the other to the circumferential direction for each element. At either boundary, a symmetrical or non-symmetrical arrangement of the reinforcing bars is considered. Assuming that the concrete cracks under tensile stress, the limit state condition is derived on the membrane force vs bending moment (or membrane stress vs bending stress) plane in terms of a closed curve. The domain outside the closed curve represents the limit states in terms of the membrane force and bending moment. The limit states used in current design criteria are also examined and compared with the corresponding limit states proposed above.
1. Introduction

One of the primary considerations in evaluating the safety of nuclear structures is the definition of the limit state [1]. Essentially the term limit state refers to a state of undesirable structural behavior. The limit state must be specified in terms of the structural response which is obtainable from the selected structural analysis. Thus, for instance, if finite element methods are used for response calculations, the limit state should be defined in terms of either element forces and moments or stresses. This paper describes a method for constructing a limit state surface for shell finite elements which are used to model a reinforced concrete containment structure. Each element, consisting of many layers of rebars embedded in concrete, may be subjected to complex stress conditions, which include tension, compression, bending and shear. Ideally, a limit state encompassing all of the above stresses conditions should be defined. Sometimes this may be difficult to achieve and various alternative approaches have been investigated. One of the approaches involves treating the shear and flexural limit states separately. This paper concentrates on the flexural limit state. The shear limit state will be discussed in a separate paper to be issued shortly by Brookhaven National Laboratory (BNL).

2. Structural Modeling

In the reliability analysis study undertaken at BNL, a three-dimensional finite element model is used for evaluating the structural response. The element utilized for the analysis is the shell element described in the SAP-5 computer code. The elements are so constructed that the reinforcements are positioned parallel to the local coordinates of the element. This will facilitate the reliability calculations in a way that less mathematical manipulations will be required. This is easily accomplished for the cylindrical portion of a containment where the steel is placed in the meridional and circumferential directions. However, it may become much more involved when dealing with the modeling of the dome portion of the containment.

3. Computer-generated Limit State Surface Based on Linear Stress Distribution

If the limit state is chosen to represent the onset of the structural failure, the structural response is considered to have reached the limit state at any time of the structural service life when the rebars begin to yield (in tension or compression) and/or the crushing strength of the concrete is reached at the cross-section's extreme fibers.

Analytically, the above limit state condition can be expressed as

\[ f_s \geq f_y \text{ and/or } f_c \geq 0.85 f'_{c} \]  

where \( f_s \) is the stress in the rebars and \( f_y \) is the steel yield strength. \( f_c \) is the concrete compressive stress at the extreme fiber and \( f'_{c} \) is the concrete compressive strength.

Since under this condition, the stress distribution is at the beginning of the non-linear behavior, a linear stress distribution assumption can be utilized to simplify the computational procedures. Based on (a) the above definition of the limit state, (b) the assumption of a linear stress-strain relationship, and (c) the conventional theory of reinforced concrete, which asserts that concrete cannot take any tension, the limit state surface in terms of the membrane stress \( \tau \) and bending moment/length \( m \) can be established for a specific cross-section at the finite element boundaries. A typical limit state surface is depicted in Fig. 1.
It is to be noted that each point on the limit state surface represents a specific condition at which the limit state is reached. This condition uniquely defines a state of deformation, stress, strain and the distance from extreme fiber to the neutral axis of the cross-section. This distance is denoted as \( h \) where \( h \) is the thickness of the wall cross-section. Since the \( k \) value is unique for each point on the limit state surface, \( k \) can be used as a parameter to construct the limit state surface. This is especially convenient when the limit state surface is generated by a computer. The procedure as to how to construct a limit state surface by changing \( k \) values is discussed below.

3.1 Calculation of Stresses

On the limit state surface, there exists a point which corresponds to a stress state such that the extreme fiber concrete stress \( f'\) reaches a value of 0.85 \( f_c \) while at the same time the largest tensile stress in the rebar reaches \( f_y \). This state is often referred to as the balanced point. At this stress state the \( k \) value is denoted as \( k_b \). Its value can be calculated from the expression

\[
k_b = \frac{f_c k_m}{f_c + f_y/n}
\]

where \( k_m \) is the distance ratio for the \( m \)th rebar layer located at the furthest distance from the edge and \( n = E_s/E_c \) is the ratio of Young's modulus of the steel to that of concrete. The stress expressions in four ranges of \( k \) encompassing the limit state surface as depicted in Fig. 1 is described below.

3.1.1 \( -\infty < k < 0 \)

For this range of \( k \) values, the cross-section becomes fully cracked. Since the concrete fiber is assumed not to be able to sustain tensile stress, the limit state will be reached when the largest tensile stress in the steel rebar reaches \( f_y \). Using the stress relation shown in Fig. 2, the stress for the \( i \)th layer of steel rebars can be obtained

\[
f_{si} = f_y \left(\frac{k-k_m}{k-m}\right)
\]

Note that the stresses are taken as positive when in compression and negative when in tension. This notation applies throughout the entire discussion.

3.1.2 \( 0 < k < k_b \)

For this range of \( k \) values, the stress in outer steel rebar will reach \( f_y \) before the stress in extreme fiber of concrete reaches 0.85 \( f_c \). Thus the limit state is essentially given by

\[
f_m = f_y
\]

where \( f_m \) is the stress in the \( m \)th layer of rebars located in this manner as depicted in Fig. 2. Under this condition, the concrete stress of extreme fiber of the cross section is

\[
f_c = f_y \frac{k}{k_m-k} < 0.85 f_c'
\]

and the stress in the \( i \)th layer or rebar is

\[
f_{si} = f_y \frac{k-k_m}{k_m-k}
\]

It is also to be noted that for this range of \( k \) the element cross-section will be partially cracked.
3.1.3 \( k_h < k < 1 \)

For this range of \( k \) values, the stress of the extreme concrete fiber will reach 0.85 \( f'_c \) before the stress of outer steel rebar reaches \( f_y \). The stresses in the \( i \)th layer of rebar are

\[
f'_{si} = 0.85 f'_c \frac{k - k_i}{k_i} \quad (7)
\]

As in the previous case, the element cross-section for this range of \( k \) will be partially cracked.

3.1.4 \( 1 \leq k < \infty \)

For this range of \( k \) the cross-section is under compression with no cracking taking place. Furthermore, since the ratio of \( E_s/E_c \) is usually smaller than the ratio of \( f_y/(0.85 f'_c) \), it can be assumed that the concrete will reach 0.85 \( f'_c \) prior to the yielding of the rebars in compression. Under these conditions, the extreme concrete fiber stress is

\[
f'_c = 0.85 f'_c \quad (8)
\]

while the stress in the \( i \)th layer of steel rebar are

\[
f'_{si} = nf'_c \frac{k - k_i}{k_i} \quad (9)
\]

3.2 Calculation of Force Components and Limit State Capacities

The force component for the \( i \)th steel layer is

\[
F_i = A_{si} f'_{si} \left( 1 - H(k - k_i)/n \right) \quad (10)
\]

where \( H(\cdot) \) is the Heaviside function. The term \( H(k - k_i)/n \) represents the effect of the concrete area under compression occupied by the steel rebars. The force component for the concrete, when the section is fully uncracked or partially cracked, are respectively given by

\[
c = \frac{2k - 1}{2k} f'_c bh, \quad C = bh f'c/2 \quad (11)
\]

If the bending moment is taken about the center line of the section, then the bending moment contributed from \( i \)th layer of rebars is

\[
M_i = F_i \left( \frac{h}{2} - k_i h \right) \quad (12)
\]

Bending moment contributed from the concrete stress block under compression when the section is partially cracked or fully uncracked are respectively given by

\[
M_c = C \left( \frac{h}{2} - \frac{kh}{3} \right) \quad (13)
\]

\[
M_c = C \left( \frac{h}{2} - \frac{h}{2} - \frac{3k - 2}{3} - \frac{3k - 2}{2k - 1} \right)
\]

The limit state capacity for the membrane force, \( F \), and bending moment, \( M \), are then obtained from the expressions:

\[
P = \sum_{i=1}^{m} F_i + C
\]

\[
M = \sum_{i=1}^{m} M_i + M_c
\]

3.3 Construction of Limit State Surface

Using the pair \( P/bh \) and \( M/b \), a point on the limit state surface can be constructed. This is, of course, related to a particular \( k \) value. By using different \( k \) values the entire limit state surface can be generated.

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It is to be noted that since the membrane force and bending moment in each cross-section of a shell element can be both positive and negative, the limit state surface will consequent-
ly result in a closed surface as shown in Fig. 1. Any point falling inside of the surface will never exceed the limit state.

4. Computer-generated Limit State Surface Based on Non-linear Stress Distribution

For structures involving more substantial failure, it is possible to define a limit state based on the Ultimate Strength Theory [2]. Even in this case, the parameter \( k \) can still be utilized to construct the limit state surface, since its value is unique for each point on the limit state regardless of if the stress is linear or non-linear. The procedure to construct the limit state surface with non-linear stress distribution is discussed below.

4.1 Calculation of Stress and Force Components

As discussed previously in Section 3.1, the stress can be expressed as a function of the parameter \( k \). This is also valid for the non-linear stress distribution under current limit state.

4.1.1 \( 0 < k \leq 1 \)

The limit state in this range of \( k \) is defined by a maximum compressive strain \( \varepsilon_u \) equal to 0.003 at the extreme concrete fiber. It is noted that rebar yielding does not constitute a limit state, i.e., the rebars are permitted to yield. Even though the stresses are in the non-linear range, the strain in the reinforcing steel and concrete is still directly proportional to the distance from the neutral axis, thus producing a linear relationship.

Using this linear relationship as shown in Fig. 3, the strain for the \( i \)th layer of steel rebar can be expressed as:

\[
\varepsilon_i = \varepsilon_u - \frac{k-k_i}{k_i}
\]

The force component of the \( i \)th layer of the rebars can also be shown as follows:

\[
\begin{align*}
F_i &= E_s A_s \frac{\varepsilon_i}{f_y} (1 - H(k-k_i)/n) \\
&\text{if } |\varepsilon_i| < f_y/E_s = \varepsilon_y \\
F_i &= A_s f_y \left[ \frac{\varepsilon_i}{f_y} \right] (1 - H(k-k_i)/n) \\
&\text{if } |\varepsilon_i| > f_y/E_s = \varepsilon_y
\end{align*}
\]

where \( H(\cdot) \) is the Heaviside function. As mentioned previously in Section 3.2, the term \( H(k-k_i)/n \) represents the effect of the concrete area under compression occupied by steel rebars.

Following the ultimate strength theory of reinforced concrete, the non-linear stress relationship is approximated by a rectangular stress block depicted in Fig. 3. The force component of rectangular concrete block is readily shown to be

\[
C = \alpha f_c b h
\]

where \( \alpha \) is as shown in Fig. 3.

4.1.2 \( k > 1 \)

When the neutral axis falls outside of the cross-section, i.e., \( k > 1 \), the rectangular block approach may not be proper. Several approximations for such a case have been suggested. One of these, namely the straight line approximation has been adopted for this study. In this approach \( P \) and \( M \) values corresponding to \( k = \infty \) are calculated respectively by
\[ P = 0.85 \frac{A}{C} f_c + \sum_{i=1}^{m} A_{s_i} f_y \]

\[ M = h \sum_{i=1}^{m} (1/2 - k_i) A_{s_i} f_y \] (18)

and the limit state surface is then established by connecting the two points k=1 and k=∞, by a straight line.

4.2 Calculation of Limit State Capacities

The limit state capacity of membrane force can be obtained from all the contributions including all steel layers and the entire concrete block. It is equal to

\[ P = \sum_{i=1}^{m} P_i + C \] (19)

Similarly, the total bending capacity is equal to

\[ M = \sum_{i=1}^{m} P_i (1/2 - k_i) h + C (1/2 - 3k)h \] (23)

where \( \beta \) is as shown in Fig. 3.

4.3 Construction of the Limit State Surface

As discussed in Section 3.3, the pair \( P/b \) and \( M/b \) constitute an unique point on the limit state surface. The surface can readily be constructed using different values for the pair.

5. Numerical Example

In order to demonstrate how the limit state surface is constructed, a typical wall element of a reinforced concrete containment is chosen as shown in Fig. 1 and Fig. 4. The thickness of the wall is 54". Across the thickness of the wall, four layers of #18 rebar with 12" spacing are arranged in an unsymmetrical manner as shown in the figures. Based on the limit state defined in Section 3, where the linear stress distribution is considered, the constructed limit state surface is shown in Fig. 1. On the other hand, the limit state surface based on the methods discussed in Section 4 is shown in Fig. 4. In these figures some of the limiting cases are identified by the strain distributions and their associated k values.

6. Conclusions

Two approaches for constructing a limit state surface for reinforced concrete containment structures have been presented. Both methods are simple to implement into computer methods, in particular, the finite element method. These limit state surfaces have been very useful for reliability analysis of category I structures [3, 4, 5].

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References


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**Fig. 1**  A Limit State Surface Under Linear Stress Distribution.

**Fig. 2**  Geometries of a Concrete Cross-section Under Linear Stress Distribution.
Fig. 3) Geometries of a Concrete Cross-section Under Non-linear Stress Distribution.

Fig. 4a) Limit State Surface Under Non-linear Stress Distribution.