Probabilistic Seismic Analysis of Slender Infinite Burried Structures

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Abstract

Using the expression for maximum stress in a buried structure due to simultaneous impact of all types of incident and reflected seismic body waves the frequency of damage f_D was calculated.

Probabilistic analysis identifies three domains for maximum stress S: safe domain where damage is impossible, potentially unsafe domain where damage is possible and unsafe domain where damage is inevitable.

A fragility curve gives the zero probability of damage in the safe domain and the unity in the unsafe domain. In the potentially unsafe domain the probability of damage varies from 0 to 1 according to the modified lognormal distribution.

Simulated distribution for the maximum stress has a power asymptotic behavior in the possible unsafe and unsafe domains.

Using the fragility curve and the asymptotic distribution for the maximum stress the frequency of a buried structure damage was calculated.

1. Introduction

The analytical relationships developed in the deterministic stress analysis of buried structures reported by Goodman and Zaslavsky [1] form the basis for this probabilistic analysis.

The expression for the maximum stress depends on soil and structure material properties, structure geometry and wave parameters. The total number of parameters is 18. Even for given soil and structure properties near the structure, the number of variable parameters is 10.

The variable parameters depend on earthquake parameters and elastic media properties. Variations of these parameters create difficulties for a deterministic analysis of buried structure. The seismic design stress level should be based on an occurrence rate that is reasonable when compared to the occurrence rate of other stress producing event.

The probabilistic approach can be very beneficial by eliminating excessive conservatism in the calculation of maximum stress.

2. Wave propagation parameters

The variable parameters are wave propagation parameters. These parameters include the angles θ and ϕ defining the direction of the incident waves, the phase shifts ϕ_1 and ϕ_2 of the SV and SH waves relatively to P-wave, the frequency ω and the peak ground velocities near the structure V_{SV}, V_{SH} and V_P due to incident SV, SH and P-waves.

The frequency ω depends on earthquake Fourier spectrum. Usually, higher frequencies are important for a buried structure because the response spectrum coincides with the earthquake spectrum.

The angle ϕ is an angle between two directions. The first direction is determined by the location of the epicenter and the point of interest on the buried structure. The second is determined by the orientation of the buried structure.

The angle θ between the direction of incident wave propagation and the vertical depends on the distance to the earthquake epicenter r and the depth of the epicenter H:

$$\theta = \text{arc tan} \frac{r}{\sqrt{r^2 + H^2}} \tag{1}$$

The phase shifts ϕ_1 and ϕ_2 depend on the initial phase shifts, the differences between the longitudinal and transverse wave propagation velocities and the variations of these velocities.

According to Campbell [2], the peak ground velocities V_{SV}, V_{SH} and V_P can be determined by formulae:

$$V_{SV} = V\chi_{SV} \tag{2}$$

$$V_{SH} = V\chi_{SH} \tag{3}$$

$$V_P = V\chi_P \qquad (4)$$

$$V = C \exp (bM) (R + C_1 \exp (C_2M))^{-d} \qquad (5)$$

Where: χ_{SV}, χ_{SH} and χ_P are direction cosines for the vector of ground velocity, M is the Richter magnitude, $R = \sqrt{r^2 + H^2}$, and C, C_1, C_2 and d are fitting parameters.

3. Parameters Distribution

The final list of parameters influencing the maximum stress include the earthquake location parameters (r, H, ϕ), the earthquake magnitude (M), the wave phase shifts (ϕ_1, ϕ_2) and ground velocity parameters (C, χ_{SV}, χ_{SH}, χ_P).

The probabilistic approach requires knowledge of the joint distribution function $f(r, H, \phi, M, \omega, \phi_1, \phi_2, C, \chi_{SV}, \chi_{SH}, \chi_P)$. Generally, this function incorporates the real data variation and some uncertainty due to incomplete state-of-knowledge. The more information available the less the spread of the distribution function and the more specific its shape.

For simplicity, we assume in this study that all five groups of parameters (earthquake location parameters, earthquake magnitude, wave frequency, phase shifts and ground velocities) are independent, so the distribution function can be expressed as follows:

$$f(r, H, \phi, M, \omega, \phi_1, \phi_2, C, \chi_{SV}, \chi_{SH}, \chi_P) = f_1(r, H, \phi)\ f_2(M)\ f_3(\omega)\ f_4(\phi_1, \phi_2) \cdot f_5(C, \chi_{SV}, \chi_{SH}, \chi_P) \quad . \qquad (6)$$

If sufficient data are available, the correlation between the Richter magnitude M and earthquake location can be included.

The function $f_1(r, H, \phi)$ can be derived from historical data. The record of earthquake locations for the last sixty years is available for most developed areas of the world.

Following the Richter approach [3], the exponential distribution for magnitude M is assumed:

$$f_2(M) = \alpha e^{-\alpha M} \qquad (7)$$

Usually, all authors [4] try to fit the parameter α to experimental data for a particular area. However, even for the best fit the spread of observed data is very large. Therefore, the parameter α is treated as a random parameter uniformily distributed in the range:

$$\alpha_1 \leq \alpha \leq \alpha_2 \qquad (8)$$

A typical Fourier spectrum of the strong ground motion is given by Housner [5]. The frequency ω corresponding to the peak ground velocity can be derived from data presented by Bernreuter [6]. This frequency varies from 5 rad/s to 15 rad/s. The lognormal distribution for density function $f_3(\omega)$ is a good representation for these data.

We know little about function $f_4(\phi_1, \phi_2)$. Reflecting this lack of knowledge the assumption is made that ϕ_1 is a uniformly distributed random parameter between 0 and 2π. For parameter ϕ_2 two extreme cases are examined. The first case assumes that ϕ_2 is a statistically independent uniform random parameter in the range $(0, 2\pi)$. The second case assumes that $\phi_2 = \phi_1$.

The function $f_5(C, \chi_{SV}, \chi_{SH}, \chi_P)$ actually depends on three parameters because χ_{SV}, χ_{SH} and χ_P satisfy an indentity $\chi_{SV}^2 + \chi_{SH}^2 + \chi_P^2 = 1$. The direction cosines can be expressed through two independent parameters λ_1 and λ_2 according to the following formulae:

$$\chi_{SV} = \sin\left(\frac{\pi}{2}\lambda_1\right)\cos\left(\frac{\pi}{2}\lambda_2\right) \tag{9}$$

$$\chi_{SH} = \sin\left(\frac{\pi}{2}\lambda_1\right)\sin\left(\frac{\pi}{2}\lambda_2\right) \tag{10}$$

$$\chi_P = \cos\left(\frac{\pi}{2}\lambda_2\right) \tag{11}$$

Assume that λ_1 and λ_2 are statistically independent uniformly distributed random parameters in the range [0, 1].

Because of the multivalued relationship between the energy release and the earthquake magnitude, parameter C in formula (5) is also random.

Based on the available observed data the lognormal distribution for C is adopted.

4. Fragility Curve

Let S be the maximum stress in the buried structure:

$$S = \max\{(\sigma_{ik})_{max}\} \tag{12}$$

The probability of structural fracture is determined by a fragility curve F(S). This curve satisfies two boundary conditions:

$$F(S) = 0, \text{ if } S < S_e \tag{13}$$

$$F(S) = 1, \text{ if } S > S_u \tag{14}$$

where S_e is the elastic limit and S_u is the ultimate strength.

Now, let s be the stress value corresponding to the structural fracture. This value is a random number distributed in the range:

$$S_e \leq s \leq S_u \tag{15}$$

Denoting the distribution density function of this parameter as f(s), the fragility F(S) can be defined as:

$$F(S) = \int_{S_e}^{S} f(s)\, ds \tag{16}$$

It is assumed that f(s) = 0 outside the range (12).

Consider a new parameter q related to parameter s by the formula:

$$q = \ln \frac{s - S_e}{S_u - s} \tag{17}$$

The range of parameter q is given below:

$$-\infty < q < +\infty \tag{18}$$

If only the mean μ and standard deviation σ of parameter q is known, the density function f(5) for this parameter, according to the principal of minimum information or maximum entropy (Korn [7]), is the normal distribution:

$$f(q) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(q-\mu)^2}{2\sigma^2}} \tag{19}$$

The density function f(s) can be readily found from eq. (19) by using transformation (17). It is a modified lognormal distribution:

$$f(s) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{ -\frac{1}{2\sigma^2} \left[\ln \frac{s - S_e}{S_u - s} - \mu^{\;2} \right] \right\} \cdot \frac{d}{ds} \left| \left(\frac{s - S_e}{S_u - s} \right) \right| \tag{20}$$

Putting eq. (20) into eq. (16), the fragility expression is obtained:

$$F(S) = \phi \left[\frac{1}{\sigma} \left(\ln \frac{S - S_e}{S_u - S} - \mu \right) \right] \tag{21}$$

where

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left(-\frac{x^2}{2} \right) dx \tag{22}$$

The function (21) automatically satisfies conditions (13) and (14).

The median S_m is defined by condition:

$$F(S_m) = 0.5 \tag{23}$$

and can be expressed in the form:

$$S_m = \frac{S_e + S_u e^{\mu}}{1 + e^{\mu}} \tag{24}$$

Using eq. (24) the expression for the fragility is rewritten as follows:

$$F(S) = \phi \left[\frac{1}{\sigma} \ell n \left(\frac{S - S_e}{S_m - S_e} \cdot \frac{S_u - S_m}{S_u - S} \right) \right] \tag{25}$$

5. Stress Domains

As mentioned above, the maximum stress S depends on some fixed parameters, which are characteristic of the soil near the structure and the buried structure and variable parameters, which are characteristic of possible earthquakes. So,

$$S = S(r, H, \phi, \alpha, M, \omega, \psi_1, \psi_2, \lambda_1, \lambda_2) \tag{26}$$

The 10-dimensional space of parameters $r, H, \phi, \alpha, M, \omega, \psi_1, \psi_2, \lambda_1, \lambda_2$ can be divided into three domains; 1) Safe, 2) Potentially Unsafe and 3) Unsafe. The maximum stress for these domains is as follows:

1. Safe: $S(r, H, \phi, \alpha, M, \omega, \phi_1, \phi_2, \lambda_1, \lambda_2) < S_e$ (27)
2. Potentially Unsafe: $S_e \leq S\ (r, H, \phi, \alpha, M, \omega, \phi_1, \phi_2, \lambda_1, \lambda_2) < S_u$ (28)
3. Unsafe: $S(r, H, \phi, \alpha, M, \omega, \phi_1, \phi_2, \lambda_1, \lambda_2) \geq S_u$ (29)

In the first domain the maximum stress is less than the elastic limit, so no failure occur. In the second domain the maximum stress is between the elastic limit and the ultimate strength, so structural fracture can occur. In the third domain, the maximum stress is greater than the ultimate strength, so structural fracture always occurs.

The joint distribution function (6) of 10 independent parameters induce the distribution f(S) for parameter S determined by formula (26). Using this function, the conditional probability of buried structure damage P_D, given the earthquake occurrence in some area of radius r_o is obtained by:

$$P_D = \int_{S_e}^{\infty} F(S)\ f(S)\ dS \tag{30}$$

Let n be the number of earthquakes per year in the area of radius r_o around the point of interest of a buried structure. Then the total frequency of damage per year f_D is determined by the formula:

$$f_D = n\, P_D \tag{31}$$

6. Algorithm and Numerical Example

Two types of algorithms could be developed for the calculation of the frequency of damage f_D.

First algorithm is straight forward. Using Monte Carlo method we simulate the distribution given by eq. (6), then calculate the probability of damage P_j for the j^{th} simulated earthquake according to eq. (25). The frequency of damage can be found by formula:

$$f_D = \frac{n}{N} \sum_{j=1}^{N} P_j \tag{32}$$

where N is a number of simulations.

This algorithm is not economical because the fraction of simulations with nonzero P_j is about 10^{-3} - 10^{-6}.

Second algorithm using Monte Carlo method simulates the density function f(S). In the Potentially Unsafe and Unsafe domains this function has a power asymptotic behavior:

$$f(S) = \frac{A}{S^k} \tag{33}$$

where A and k are parameters fitted by the code.

Putting eq. (33) into eq. (30) and eq. (30) into eq. (31), the frequency of damage f_D can be calculated. An example calculation for a buried steel pipe in an area with moderate and uniform seismicity is shown in Table I.

A special reduction technique developed for calculating the asymptotic expression given by eq. (33) directly, practically avoiding the simulation of function f(S) in the safe domain.

References

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Table I

Frequency of Damage f_D per Year as a Function of the Elastic Limit S_e

S_e (MPa)	f_D (yr^{-1})
10	1.50×10^{-3}
20	2.65×10^{-4}
30	9.62×10^{-5}
40	4.69×10^{-5}
50	2.68×10^{-5}
60	1.70×10^{-5}
70	1.11×10^{-5}
80	8.29×10^{-6}
90	6.12×10^{-6}
100	4.74×10^{-6}