Development of a Reliability Analysis Method for Category I Structures

M. Shinozuka, T. Kako

Dept. of Civil Engineering and Engineering Mechanics, Columbia University, Seeley W. Mudd Building,
New York, N.Y. 10027, U.S.A.

H. Hwang, M. Reich

Dept. of Nuclear Energy, Brookhaven National Laboratory, Upton, New York 11973, U.S.A.

Summary

The present paper develops a reliability analysis method for category I nuclear structures, particularly for reinforced concrete containment structures subjected to various load combinations. The loads considered here include dead loads, accidental internal pressure and earthquake ground acceleration. For mathematical tractability, an earthquake occurrence is assumed to be governed by the Poisson arrival law, while its acceleration history is idealized as a Gaussian vector process of finite duration. A vector process consists of three component processes, each with zero mean. The second order statistics of this process are specified by a three-by-three spectral density matrix with a multiplying factor representing the overall intensity of the ground acceleration. With respect to accidental internal pressure, the following assumptions are made: (a) it occurs in accordance with the Poisson law, (b) its intensity and duration are random and (c) its temporal rise and fall behaviors are such that a quasi-static structural analysis applies. A dead load is considered to be a deterministic constant.

To accomplish the stated purpose, however, the present paper concentrates on the development of an analytical procedure which permits one to estimate the conditional limit state probability of a containment structure, given that the structure is subjected to a specific load combination. In this procedure, a particular method of frequency domain reliability analysis is applied to those cases of load combinations involving earthquake ground acceleration. This reliability analysis procedure is based on the finite element method and on the theory of random vibrations. The limit state condition recently developed for reinforced concrete containment structures is also used here. The condition represents the onset of structural failure: concrete crushing at the extreme fibre or yielding of the reinforcing steel bars. The limit state condition thus defined exhibits a closed curve in the membrane stress ~ moment stress plane. The conditional limit state probability under the unit duration of a specific load combination is then evaluated as the probability that the response, when it is plotted in the membrane stress ~ moment stress plane, will reach outside this closed curve at least once in the unit duration.
1. Introduction

The purpose of the present paper is to develop a reliability analysis method for category I nuclear structures, particularly for reinforced concrete containment structures subjected to various load combinations. The loads considered here include dead loads, accidental internal pressure and earthquake ground acceleration. For mathematical tractability, an earthquake occurrence is assumed to be governed by the Poisson arrival law, while its acceleration history is idealized as a Gaussian vector process of finite duration. The vector process consists of three component processes (e.g., WE, NS and vertical components), each with zero mean. The second order statistics of the vector process are specified by a threeby-three spectral density matrix with a multiplying factor representing the overall intensity of the ground acceleration. With respect to accidental internal pressure, the following assumptions are made: (a) it occurs in accordance with the Poisson law, (b) its intensity and duration are random and (c) its temporal rise and fall behaviors are such that a quasi-static structural analysis applies. A dead load is considered to be a deterministic constant.

To accomplish the stated purpose, however, the present paper concentrates on the development of an analytical procedure which permits one to estimate the conditional limit state probability of a containment structure, given that the structure is subjected to a specific load combination. In this procedure, a particular method of frequency domain reliability analysis is applied to those cases of load combinations involving earthquake ground acceleration. The basic analytical procedure associated with this method is presented in a companion paper by Kako, et al [1] and is based on the finite element method and on the theory of random vibrations. The limit state condition recently developed for reinforced concrete containment structures is also used in this method. The condition represents the onset of structural failure: concrete crushing at the extreme fibre or yielding of the reinforcing steel bars. The limit state condition thus defined exhibits a closed curve (limit state surface) in the membrane stress - moment stress plane. The detailed derivation of this limit state condition is presented in a second companion paper by Chang, et al [2]. The conditional limit state probability under the unit duration of a specific load combination is then evaluated as the probability that the response, when it is plotted in the membrane stress - moment stress plane, will reach outside this closed curve at least once in the unit duration. These conditional limit state probabilities, together with other probabilistic characteristics of the loads such as durations and occurrence rates, provide an analytical basis for developing a probability-based load combination methodology. Such a combinatorial procedure is presented in the third companion paper by Shinozuka, et al [3]. The numerical evaluation of the limit state probabilities is carried out with the aid of a computer program called RAS (Reliability Analysis of Structures) developed at Brookhaven National Laboratory in accordance with the above mentioned reliability analysis method.

2. Finite Element Analysis of Containment

The concrete containment structure considered is illustrated in Fig. 1. The containment consists of a circular cylindrical wall with a hemispherical dome on the top. The dome-cylinder reinforced concrete system is fixed at the base. The thickness of the containment dome is 2'-6" whereas that of the cylindrical wall is 3'. The inside radius of the dome is equal to 62' which matches the inside radius of the cylindrical portion of the containment. The height of the cylindrical wall is 150'-6", and thus, the total height of the containment is 215'.
The containment wall is reinforced with hoop and meridional rebars which are placed in two layers, i.e., one layer closer to the inner surface of the containment and the other closer to the outer surface of the containment. For the cylindrical portion of the containment, both the hoop and meridional are reinforced with No. 18 rebars spaced twelve inches apart. The hemispherical dome is reinforced with two layers of No. 14 rebars spaced twelve inches apart. These rebars are placed in the orthogonal directions. In the lower half of the dome, two layers of No. 14 hoop rebars with twelve inch spacing are added to the cross-section, one layer each near the inner and outer surfaces. For most containment structures, diagonal rebars are used to resist the shear forces. In the present analysis, the diagonal rebars are disregarded. Also, the steel liner, which is usually located on the inner surface of the reinforced concrete containment is disregarded as a load carrying structural component in the analysis. Finally, other complications such as penetrations, personal locks and equipment hatches are not included in the study.

A three-dimensional finite shell element model described in the SAP-V code is used for the structural analysis of the containment. Each element has 4 nodes, which can have up to 6 degrees-of-freedom. The containment is divided into 19 layers. With the exception of the top-most layer of the dome, each layer has 24 elements, so that the nodal points are taken every 15° in the circumferential direction. This discretization required a total of 457 nodes and 444 elements.

When a reinforced concrete containment is subjected to static and dynamic loads, its cross-section will usually produce cracks, the extent of which depends on the load history. While a linear elastic analysis cannot take into account the temporal variations of the structural stiffness which result from such a dependence on load history, it will nevertheless, in most instances, yield correct stress resultants for the various sections of the structure. This is especially the case if the section material properties are adjusted to reflect the concrete cracking. Because of the complexity of the various load combinations, however, it is difficult to predict the crack patterns for all conceivable combinations of loadings. Therefore, in the present study, the structural stiffness associated with the uncracked section was used for all loads and their combinations, thus in fact follows the procedures recommended in SRP 3.8.1.

The dynamic characteristics of the structures are represented by the natural frequencies and associated mode shapes. With the aid of the RAS computer program, the first 20 natural frequencies are evaluated. The two most significant modes (the first and second pairs of bending modes; modes 1,2,16,17) are at 4.2 Hz and 12.5 Hz. Practically no other modes participate under the unidirectional horizontal ground acceleration.

3. Material Properties

With respect to the concrete, the weight density is 150 lb/ft³, while Young’s modulus and Poisson’s ratio are 3.6 x 10⁶ psi and 0.2, respectively, and the 91 day compressive strength f‘C = 6086 psi. As for the steel reinforcement, both No. 18 and No. 14 rebars are used in the containment structure. Young’s modulus and Poisson’s ratio are 29.0 x 10⁶ psi and 0.3, respectively, while the yield strength is 71.1 ksi for both types of rebars. Possible statistical variations and uncertainties involved in the material properties ought to be taken into account and indeed have been considered in the analysis. However, the results of such a statistical and uncertainty analysis will be presented elsewhere due to the limited space available here.
4. **Limit States for the Containment**

The state of structural response is considered to have reached the limit state if the rebars begin to yield (in tension or compression) and/or if the crushing strength of the concrete is reached at the extreme fibre of the containment wall cross-section. The limit state condition introduced above can be analytically expressed as

\[
f_s \geq f_y \quad \text{and/or} \quad f_c \geq 0.85f'_c
\]  

(1)

where \(f_s\) is the stress in the rebars and \(f_c\) the compressive concrete stress at the extreme fibres. Since the stresses \(f_s\) and \(f_c\) are functions of the stress vector \((\tau)\), the limit state condition in eq. (1) is in general given in the form of \(g((\tau)) \leq 0\) where \(g(\cdot)\) is an appropriate function. The equality \(g((\tau)) = 0\), representing \(f_s = f_y\) and \(f_c = 0.85f'_c\), usually indicates a closed (hyper-) surface or a limit state surface in the \((\tau)\) space. To be consistent with the SAP V finite element code used, the stress vector \((\tau)\) is given by

\[
(\tau) = [\tau_{xx} \quad \tau_{yy} \quad \tau_{xy} \quad m_{xx} \quad m_{yy} \quad m_{xy}]^T
\]  

(2)

where the first three are the membrane stress components and the last three the bending moment components of the usual definition.

Based on (a) the above definition of the limit state, (b) the assumption of a linear stress-strain relationship, and (c) the conventional theory of reinforced concrete, which asserts that concrete cannot take any tension, the limit state surface in terms of the membrane stress component \(\tau\) (e.g. \(\tau_{xx}\)) and the corresponding bending moment component \(m\) (e.g. \(m_{xx}\)) can be established for the cross-section at the finite element boundaries as shown by Chang, et al [2]. Such a limit state surface is schematically shown in Fig. 2 in which point "a" represents a limit state under pure (uniform) compression and point "g" a limit state under pure (uniform) tension. Also, straight lines I (ac and ac'), lines II (approximated by ce and c'e'), lines III (ef and e'f') and lines IV (fy and f'y) indicate those parts of the limit state surface in which the limit states are reached in concrete crushing with cross-sections remaining uncracked (lines I), in concrete crushing with partially cracked cross-sections (lines II), in yielding of rebars in tension with partially cracked cross-sections (lines III) and in yielding of rebars in tension with totally cracked cross-sections (lines IV). All other possible limit states, such as those based on shear stress or strain are not considered at this time.

5. **Containment Loads**

A containment structure will be subjected to various static and dynamic loads during its lifetime. In this study only four types of loads are taken into consideration. They are: dead load, live load, internal pressure and earthquake ground acceleration. Other loads on the containment such as the SRV load will be considered in a future study.

5.1 **Dead and Live Loads**

The dead load is the weight of the dome and the cylindrical wall. The weight density of the reinforced concrete is taken to be 150 lb/ft\(^3\). The dead load is obviously static and assumed to be deterministic.

Because several floors are connected to the containment structure, some live loads act on the containment at the locations where the floors are connected to the containment. The locations and design values of the corresponding live loads are shown as follows:

---

---

M 5/3
Elevation | 865′ | 828′ | 803′ | 770′ | 755′  
Live Load (kip/ft) | 0.707 | 3.00 | 0.940 | 1.02 | 0.930

It is noted that there are some uncertainties as to the actual magnitude of the live load. For the purpose of the present analysis, however, the live load is assumed to be deterministic and equal to the design values.

5.2 Internal Pressure

The internal pressure is considered a quasi-static load uniformly distributed on the containment wall. Moreover, it is idealized as a rectangular pulse and will occur at a prescribed mean interval during the containment life. Three parameters are used to model the internal pressure: the occurrence rate \( \lambda_p \) (per year), the mean duration \( \mu_{dp} \) (in seconds) and the intensity \( P \). The intensity \( P \) is treated as a Gaussian random variable.

The containment considered in this study was designed for an internal pressure of 15 psi. For the present reliability analysis, however, two different kinds of internal pressure are considered. One is the accidental pressure \( P_L \) due to a large LOCA, but not followed by a hydrogen burn, and the other is the accidental pressure \( P_H \) caused by a hydrogen burn (deflagration) following a large LOCA.

For the accidental pressure \( P_L \) caused by a large LOCA, the occurrence rate \( \lambda_{PL} \) and the mean duration \( \mu_{dp_L} \) are taken to be \( 1.0 \times 10^{-4} \) year and \( 1.0 \times 10^6 \) seconds, respectively, while the intensity \( P_L \) is Gaussian with a mean value of 15 psi and standard deviation of 3 psi. If the probability is assumed to be 0.1 for a LOCA to be followed by a hydrogen burn, the occurrence rate of the hydrogen burn \( \lambda_{PH} \) is \( 1.0 \times 10^{-5} \). It is further assumed that the mean duration \( \mu_{dp_H} \) of the pressure resulting from the hydrogen burn is 600 seconds and that its intensity \( P_H \) is a Gaussian variable with a mean value of 45 psi and standard deviation of 9 psi.

For mathematical simplicity, moreover, the hydrogen accident is assumed to occur independently of the LOCA without, however, allowing their simultaneous occurrence. Although this scenario is somewhat different from the actual situation, the limit state probability based thereupon is expected to be close to that which would follow from the actual sequence of events.

5.3 Earthquake Ground Acceleration

The earthquake ground acceleration is assumed to act only along the global x (horizontal) direction. It is further assumed that the ground acceleration can be idealized as a segment of finite duration of a stationary Gaussian process with mean zero and Kanai-Tajimi spectrum;

\[
S_{ggxx}(\omega) = S_0 (1 + 4\zeta_g^2 (\omega/\omega_g)^2) / [(1 - (\omega/\omega_g)^2)^2 + 4\zeta_g^2 (\omega/\omega_g)^2]
\]

where the parameter \( S_0 \) represents the intensity of the earthquake and \( \omega_g \) and \( \zeta_g \) are the dominant ground frequency and the ground damping ratio, respectively. The values of \( \omega_g \) and \( \zeta_g \) depend on the soil conditions of the chosen site. For the present study, \( \omega_g = 9\pi \) rad/sec and \( \zeta_g = 0.6 \) are used. Also, the mean duration \( \mu_{de} \) of the earthquake acceleration is assumed to be 10 seconds. The peak ground acceleration \( A_p \) given an earthquake, is assumed to be \( A_p = p_g \sigma_g \) where \( p_g \) is the peak factor which is assumed to be 3.0 and \( \sigma_g \) is the standard deviation of the ground acceleration such that

\[
\sigma_g = \sqrt{\omega_g^2 (2\zeta_g + 1/(2\zeta_g))} \cdot S_0
\]

--- 231 ---
and therefore
\[ A_1 = a_g \sqrt{\sigma_0} \quad \text{with} \quad a_g = \rho_g \sqrt{\sigma_0} \left( 2 \sigma_g + 1/(2 \sigma_g) \right) \]  \hfill (5)

If the earthquake occurs in accordance with the Poisson law at a rate $\lambda_E$ per year, it is easy
to show that the probability distribution $F_A(a)$ of the annual peak ground acceleration $A$ is
related to the probability distribution $F_{A_1}(a_1)$ of $A_1$ in the following fashion.

\[ F_A(a) = \exp(-\lambda_E [1 - F_{A_1}(a)]) \quad \text{or} \quad F_{A_1}(a) = 1 + \frac{1}{\lambda_E} \ln F_A(a) \] \hfill (6)

Therefore, if $a_0$ indicates the minimum peak ground acceleration for any ground shaking to be
considered an earthquake, $F_A(a_0) = 0$ and hence $\lambda_E = -\ln F_A(a_0)$. Assuming that $F_A(a)$ is of
the extreme distribution of Type II, $F_A(a) = \exp[-(a/a_0)^{-\alpha} - u]$ with $\alpha = 2.61$ and $u = 0.01$, one
finally obtains

\[ F_{A_1}(a) = 1 - (a/a_0)^{-\alpha} \quad a \geq a_0 \] \hfill (7)

Under these conditions, one finds that $\lambda_E = 1.50 \times 10^{-2}$/year provided that $a_0 = 0.05g$.
Combining eqs. (5) and (7) and writing $Z$ for $\sqrt{\sigma_0}$, one further obtains the probability
distribution and density functions of $Z$ in the forms, respectively,

\[ f_Z(z) = 1 - (a_g z/a_0)^{-\alpha} \quad f_Z(z) = a(a_g z/a_0)(a_g z/a_0)^{-\alpha+1} \] \hfill (8)

6. **Conditional Limit State Probabilities**

The limit state surface is expressed in terms of the segments of the following eight straight lines which define the octagonal area shown in Fig. 2.

\[ R_j = (A_j)^T (\tau_i(e)) = 0 \quad (j=1,2,\ldots,8) \] \hfill (9)

where $(\tau_i(e))$ is the element stress vector, and $R_j$ and $(A_j)$ are constants and constant vectors,
respectively. The vector $(\tau_i(e))$ is at most the sum of three vectors, $(\tau_i(e))_0$, $(\tau_i(e))_p$ and
$(\tau_i(e))_d$ respectively representing the stresses due to the dead and live (D/L) loads, due to the
accidental internal pressure $(P)$ and due to the earthquake acceleration $(E)$. The vector $(\tau_i(e))_0$ is time-invariant and deterministic since so are the D/L loads, while $(\tau_i(e))_p$ can be
written as $P + (\tau_i(e))_p$ where $(\tau_i(e))_p$ is the stress due to the unit internal pressure $P = 1$
psi and $P$ is a Gaussian random variable with mean $\bar{P}$ and standard deviation $\sigma_P$. On the other
hand, as shown by Kako, et al [1], the vector $(\tau_i(e))_d$ has been shown to be $(\tau_i(e))_d = 2[\sigma(e)]$
$\cdot[\sigma(e)]_d$ respectively representing the stresses due to the dead and live (D/L) loads, while
$(\tau_i(e))_p$ can be written as $P + (\tau_i(e))_p$ where $(\tau_i(e))_p$ is the stress due to the unit internal pressure $P = 1$
psi and $P$ is a Gaussian random variable with mean $\bar{P}$ and standard deviation $\sigma_P$. On the other
hand, as shown by Kako, et al [1], the vector $(\tau_i(e))_d$ has been shown to be $(\tau_i(e))_d = 2[\sigma(e)]$
$\cdot[\sigma(e)]_d$ respectively representing the stresses due to the dead and live (D/L) loads, while
$(\tau_i(e))_p$ can be written as $P + (\tau_i(e))_p$ where $(\tau_i(e))_p$ is the stress due to the unit internal pressure $P = 1$
psi and $P$ is a Gaussian random variable with mean $\bar{P}$ and standard deviation $\sigma_P$. On the other
hand, as shown by Kako, et al [1], the vector $(\tau_i(e))_d$ has been shown to be $(\tau_i(e))_d = 2[\sigma(e)]$
$\cdot[\sigma(e)]_d$ respectively representing the stresses due to the dead and live (D/L) loads, while

\[ I_m = \text{max identity matrix} (m = \text{number of modes considered}). \]  \hfill (10)

The vector $(\tau_i(e))_d$ is the generalized coordinate vector when $Z = \sigma_0 = 1/\text{in}^2/\text{sec}^2$. Thus, $(\tau_i(e))_d = 2[\sigma(e)](\nu_0)$ where $[\sigma(e)] = [\sigma(e)]_d [L_d]_d$ and

\[ (\tau_i(e))_d = (\tau_i(e))_0 + P \cdot (\tau_i(e))_p + 2[\sigma(e)](\nu_0) \] \hfill (11)

Substituting eq. (10) into eq. (9) and writing $R_j' = R_j - (A_j)^T (\tau_i(e))_0$ one obtains

\[ R_j' - D(e)P - Z(A_j)^T (\nu_0) = 0 \quad \text{or} \quad r_j(e) = d_j(e)P - Z(\nu_j(e))^T (\nu_0) = 0 \] \hfill (11)

where $D(e) = (A_j)^T (\tau_i(e))_p$, $(A_j)^T (\tau_i(e)) = (A_j)^T [\sigma(e)]$, $r_j(e) = R_j/A_j$, $d_j(e) = D_j/A_j$ and
\( n_{j}^{(e)} = n_{j}^{(e)} / A_{j}^{(e)} \). It can be shown (Kako, et al [1]) that the probability distribution of \( Y_{j}^{(e)} \) is approximated by \( F_{j}^{(e)}(x) = 1 - \sum_{0}^{\infty} \sum_{a,b}^{\infty} n_{a}^{(e)}\exp(-i\lambda_{a}x) \) with \( n_{j}^{(e)} = \sum_{a=1}^{\infty} \sum_{b=1}^{\infty} a_{a,b}^{(e)}d_{b}E[I_{0}V_{0}V_{0}]/(2\pi) \) \( (12) \)

where \( x = z_{2n}^{(e)}/V_{0}V_{0} \) is the \( a \)-component of \( n_{j}^{(e)} \) and \( E[I_{0}V_{0}V_{0}] \) is the \( a \)-\( b \) component of the covariance matrix \( [V_{0}V_{0}V_{0}V_{0}] \) of \( (V_{0})^{(e)} \). The conditional limit state probabilities are then obtained as the probabilities that the left-hand side of eq. (11) becomes negative. In eq. (11), it is assumed that \( R_{j}^{(e)} > 0 \). This implies that the D/L loads alone will not produce the stresses in the limit state. When the structure is subjected to D/L and P but not to E (hence \( (V_{0})^{(e)} = 0 \)), the conditional limit state probability given this load combination becomes:

\( p_{j}^{(D/L + P)}(e) = \sigma^{(e)}(d_{j}^{(e)}/d_{j}^{(e)}/\sigma_{j}) \) if \( d_{j}^{(e)} > 0 \); \( = 0 \) if \( d_{j}^{(e)} < 0 \) \( (13) \)

where \( \sigma^{(e)} \) is the standardized Gaussian distribution function. Eq. (13) can be used for both cases in which the internal pressure \( P = P_{L} \) or \( P = P_{H} \); provided that the corresponding values of \( \bar{P} \) and \( \sigma_{j} \) are used. When the structure is subjected to D/L and E but not to P, the conditional probability for this load combination is given by, using eq. (12),

\( p_{j}^{(D/L + E)}(e) = \int_{z_{\min}}^{z_{\max}} \sum_{\nu\sigma} \left( \nu_{\nu}^{(e)}(D/L + E) \right) \exp(-i\bar{P}z)/(2\pi) \) \( \int_{z_{\min}}^{z_{\max}} \nu_{\nu}^{(e)}(D/L + E) \exp(-i\bar{P}z)/(2\pi) \) \( (14) \)

where \( \nu_{\nu}^{(e)} = \nu_{\nu}^{(e)} \). Finally, when the structure is under the simultaneous action of D/L, P and E, the conditional probability is

\( p_{j}^{(D/L + P + E)}(e) = \int_{z_{\min}}^{z_{\max}} g_{j}^{(e)}(z) f_{j}^{(e)}(z) \) \( (15) \)

where

\( g_{j}^{(e)}(z) = \sum_{1}^{\infty} \sum_{1}^{\infty} \left( z^{(e)}(\nu_{\nu}^{(e)}(D/L + E) \right) \exp(-i\bar{P}z)/(2\pi) \) \( (16) \)

with \( \nu_{\nu}^{(e)} = \nu_{\nu}^{(e)} \). The maximum value of eq. (13) with respect to \( j \) and \( e \) is evaluated as the conditional limit state probability for the entire structure. However, for the load combinations D/L + E and D/L + P + E, the maximum values of \( p^{(e)}(e) = \sum_{j=1}^{8} p^{(e)}(e) \) with respect to \( e \) are used as the upper bound of the conditional limit state probabilities. The computations carried out using the parameter values indicated in the preceding sections resulted in \( P^{(D/L + P)} = 0.192 \), \( P^{(D/L + E)} = 1.21 \times 10^{-3} \), \( P^{(D/L + P + E)} = 1.15 \times 10^{-3} \) and \( P^{(D/L + P + E)} = 0.424 \). The finite element location at which these maximum values are obtained usually depends on the load combination.

References

[1] KAKO, T., SHINOZUKA, M., HWANG, H. and REICH, M., "FEM-Based Random Vibration Analysis"


Acknowledgement

This work was supported by the Office of Nuclear Regulatory Research, US Nuclear Regulatory Commission under Contract No. A3226. The authors are grateful to Mr. H. Ashar and Drs. J. Burns and W. Anderson of the Office of Nuclear Regulatory Research, US Nuclear Regulatory Commission, for their support on various phases of this work.