Application of the Finite Element Method in the Modelling of Coil Bundles

M. Shibui
*Toshiba Corporation, Advanced Engineering Dept., 2-4 Suehiro-cho,
Tsurumi-ku, Yokohama, Japan*

I.J. Zatz, J.M. Bialek
Princeton University, Plasma Physics Laboratory, P.O. Box 451, Princeton, New Jersey 08540, U.S.A.

SUMMARY

This paper proposes a procedure for the finite element modelling of coil bundles for fusion devices. In addition to addressing the simulation of the bending, shear, and torsional capabilities of a coil appropriately in a model, the effects of interlamina bonding on the behavior of the coil structure and its modelling is also examined.

Three different finite element modelling approaches are presented and evaluated as viable interpretations of an actual coil, each limited for use within specified parameter ranges. One is based on solid elements with correctly defined properties permitting the accurate representation of the global behavior of a coil bundle. The other two are more complex and are based on the combination of various elements each accounting for a different aspect of coil behavior which are best resolved via multi-level substructuring. The choice of the best model for the job rests with the analyst who must first resolve what the goals of the analysis are and given the parameters of the problem, which models can be used. The basic idea behind these models, as with any finite element model, is the application of a systematic modelling technique requiring a close correspondence between the capability of the finite elements themselves and the true mechanical behavior of that portion of the coil being simulated.

In order to have analytical solutions for confirming the bending and torsional capabilities of these coil bundle finite element models, their behavior is studied via several basic examples. Laminated beam behavior which categorizes the structural nature of many conventional coil bundles is also examined in some depth. Also discussed is a generalized computer program that was developed to accept the description of any conventional coil section and determine an effective stiffness for it to be used in finite element modelling.

The various methodologies described in this paper should be applicable to any bundled coil design. Although only conventional coils are discussed, with the proper modifications the concepts and techniques presented herein can be applied to other configurations as well, such as superconductors.

1. Introduction

Ideally, finite element models that are used for coil engineering attempt to represent actual coil bundles with enough accuracy to permit the effective evaluation of coil displacements and stresses. Localized behavior of these coils is desired where possible. In addition, a model must usually account for the accurate transition of forces from the coil to the coil case and/or supporting structure via an appropriate load path. Any study concerning the finite element idealization of coil bundles, such as conventional toroidal field coils, faces complication from a practical applications standpoint due to the following difficulties in modelling coils with existing finite element libraries:

- A conventional coil bundle for a fusion device is usually in the form of a laminated structure composed of alternating layers of conductor and insulation.
- These layers are usually not perfectly bonded together, and hence do not necessarily behave collectively as a solid, resulting in a potential finite element model with a great number of elements and nodal points.
- Despite the small relative thickness of an insulation layer (compared with a conductor layer) it may still require an accurate finite element idealization because it could contribute significantly to certain kinds of coil deformation.

In order to alleviate these difficulties, and overcome the obstacles they present, the analyst should be knowledgeable in finite element method (FEM) theory and the capabilities of FEM programs including the specific behavior and accuracy of individual elements in a variety of applications. This approach will lead to the better modelling of a coil structure and to an accurate computation of structural displacements, stresses, and strains in those areas of the coil that are of paramount interest.

2. Modelling of a Conventional Coil Bundle

Using the element library of MSC/NASTRAN [1] , three different finite element models were developed for simulating the structural behavior of a conventional coil bundle. The first two models are based on combinations of structural elements including plates, shear panels and springs and are able to simulate the mechanical behavior of a coil bundle with little or no bonding strength (zero friction) between layers, if necessary. If the bonding strength of coil layers is unknown, a conservative engineering approach would be to assume this condition in an analysis. The other model is based on the continuum solid element with appropriately defined transversely isotropic properties. Each model is created with the capability of determining major stresses with a good degree of accuracy.

Regardless of the approach used, certain criteria must be considered throughout the modelling phase of an analysis. The model should provide for proper stress transmission and continuity across winding layers by maintaining the correspondence between the capability of the finite elements used and the true behavior of the various components of the coil bundle. Also, the simplest and most efficient modelling procedure that can get the job done should be implemented for cost effectiveness.

For example, since the MSC/NASTRAN solid (CHEXA) elements are based on an isoparametric lower order concept, sufficient detail should be used for modelling a coil cross-section when attempting to determine the warping of that cross-section. Otherwise, large longitudinal stresses will be induced resulting in a model that would be stiffer than the actual coil. This is due to the limited number of nodes available for warping resulting in the restriction of desired deformation in the analysis. Therefore, the criteria of correspondence between elements and structure is of the utmost importance in modelling.

Consider a simple laminated beam composed of alternating layers of conductor (copper) and insulation (epoxy). A Cartesian coordinate system with the x-axis coincident with the beam axis is used for reference (Figure 1). First consider the uniaxial tension in the x-direction, which simulates the hoop stress field caused by radial forces. In this case, the contribution of the insulation layers to the global elongation is probably negligible because the external forces usually originate with the conductor layers, and the insulation layers, in general, have poor relative shearing and tensile stiffnesses. At best the insulation would have only a volumetric contribution.

Other important deformation patterns are bending in the x-z plane and torsion about the x-axis, caused by non-uniformly distributed radial forces and overturning (out-of-plane) forces. Modelling considerations for torsion are quite different and more complex than bending for two reasons. First, the shear stress component τ_{xy} which has only a second order effect on the bending stiffness of the beam, becomes extremely important in torsion. Secondly, the fundamental deformation pattern of a coil bundle in torsion may change as the equivalent shear modulus (the combined effects of material and bond strength) of the insulation layer varies. The indication for torsion is that the distribution of shearing stress is more important than its magnitude and therefore, an accurate finite element model for a coil bundle should be one that can account for this behavior. The three finite element models for a coil bundle based on the above mentioned criteria are outlined below. The modelling techniques will be described first. Subsequent sections go into the various details and justification for each modelling configuration.

2.1 Model 1

When the individual layers of a coil bundle are not assumed to be perfectly bonded together, it becomes necessary to consider each conductor layer as an individual plate. Models 1 and 2 account for this situation. The relevant details of Model 1, which is based on the two-dimensional element library available in the MSC/NASTRAN program, are as follows:

(a) Each conductor turn is modelled with isoparametric plate (CQUAD4 or CQUAD8) elements which are capable of representing the in-plane behavior, out-of-plane bending and transverse shear in a conductor layer. The nodes of each plate element are oriented along the centroid of the corresponding conductor layer.

(b) Two pairs (four) of shear panel (CSHEAR) elements are used to simulate the shear behavior and transfer through the inter-lamina insulation layers while providing connection between adjacent plate elements. The bending stiffness of the insulation layer is usually neglected because of its small contribution to the global behavior of the total coil bundle.

(c) The compressive stiffness of conductors and insulators are idealized using scaler spring (CELAS) elements or simple beam (CBAR) elements with pin joint options.

For the above idealization, an effective shear modulus must be used in order to account for the shear stiffness contributions of the conductor and insulator regions spanned by the elements involved. It is important to note that shear panel (CSHEAR) elements should not only represent the physical properties of the insulation layer, but in this instance must also account for portions of conductor as well because the element (e.g. ABDC on Figure 2) spans the distance between the centroids of adjacent conductors. Geometrically, the actual insulation thickness EF in Figure 2 is replaced by AC in the finite element model, which thus requires an effective modulus for the shear element. Consider the behavior of the insulation layer under the conditions that the conductor surface AB (edge view) is fixed in the x-direction while a shearing force Q is applied to the conductor surface CD. The insulation material, whose thickness is represented by EF, moves to the position EF', a lateral displacement of Δ from F to F', when subjected to the pure shear deformation due to Q. This deformation, however, should actually be realized by the shear element ABDC whose height is AC. This condition defines the necessary effective shear modulus for the shear element. From Figure 2,

$$\Delta = \gamma_o T = \gamma H \tag{1}$$

The corresponding strain energy is

$$E = \tfrac{1}{2} G \gamma_o^2 T = \tfrac{1}{2} G_{eff} \gamma^2 H \tag{2}$$

Here G is the shear modulus of the real insulation layer and G_{eff} is the effective shear modulus required for the shear element. From these equations,

$$G_{eff} = \frac{H}{T} G \tag{3}$$

where H is the nodal distance separation (conductor + insulator) in the finite element model and T is the actual thickness of the insulation layer. The spring constant for the compressive actions of the conductor and insulation layers represented by bar or spring elements is defined by the ordinary linear combination of the individual spring constants.

Model 1 has many advantages, but one of its deficiencies is that there is no direct angular relation between adjacent coil laminations. This can cause an analytical problem for the case of a coil bundle with relatively high stiffness insulation layers subjected to torsion. In this instance, each conductor plate element must behave as part of a collective solid under eccentric twist loading while the elements representing the insulation layers must be able to resist the desired angular behavior of adjacent plates. This situation can be improved by assigning necessary bending and torsional stiffnesses to additional spring elements which connect adjacent plates, the details of which are left to the reader.

2.2 Model 2

Model 2 is a modification of Model 1 which requires some additional modelling but averts the above mentioned potential for analytical difficulties. Plates (CQUAD), shear panels (CSHEAR), springs (CELAS or CBAR) and rigid links (RBAR) are the elements used to simulate the mechanical behavior of the conductors, shearing and compressive stiffnesses of the insulation layers, and the inter-winding relationship between conductors and insulators, respectively.

The importance of the rigid link (RBAR) elements is shown in Figure 3, where this time ABCD is the shear panel representing the real region of the insulation without spanning any conductor. In MSC/NASTRAN, the shearing forces are distributed among grid points A, B, C and D. Therefore, it may be realized that the shearing force in a shear panel element induces not only a force but also a moment at the point F, the centroid of a conductor layer. If this effect is neglected, the plate EF will behave independently of the rotation dependent behavior in the shear panel. The actual shear modulus of the insulation material is assigned to the shear element in Model 2, and the spring elements AB and CD represent only the compressive stiffness of the insulation layer. The compressive behavior of conductor layers is neglected because of the trivial longitudinal strain in the direction normal to the plate, enabling the use of the rigid link element (RBAR) for moment transfer.

Examination of Models 1 and 2 reveals the obvious conclusion that as the equivalent shear modulus of the insulation is reduced, the Model 1 results will approach those of Model 2.

When the equivalent shear modulus of the insulation layers is set equal to zero, i.e., zero friction between conductor and insulator layers due to an absence of bond strength, Models 1 and 2 are reduced to representing the layers by conventional double noding techniques and should produce nearly identical results. This capability, in addition to the superiority in determining localized coil stresses and strains, is one of the primary advantages of these models over Model 3.

2.3 Model 3

Model 3, which is based on three-dimensional solid finite elements with transversely isotropic properties is certainly a very attractive approach because of the simplicities associated with its modelling when compared with Models 1 and 2. The solid elements can only give reasonable results for a homogeneous (i.e., perfectly bonded) body without any discontinuity between layers. Their application, however, may be permitted far into the non-homogeneous regime if local behavioral accuracy is sacrificed while retaining acceptable global qualities. The key for the successful application of solid elements in the modelling of a coil bundle is to use an adequately defined set of material properties that are consistent with the general physical symmetry of the structure.

In this modelling technique, a number of layers of insulators and conductors are simulated by individual eight or twenty-node solid (CHEXA) elements. In this regard, Model 3 will then provide reasonable results that can be used in the engineering of a coil case, a center column, or any other major structural component external of the coil bundle. The concept of modelling solid elements as isotropic based on the conventional mixing formula is not correct and will produce increasingly erroneous results as the equivalent shear modulus of the insulator decreases. The result would be a finite element model that is far stiffer than an actual coil bundle and could possibly lead to an underdesigned coil case or supporting structure. The following section describes a proper approach for determining the material properties for Model 3.

3. Equivalent Elastic Constants for Solid Finite Elements

It has been indicated that the most crucial aspect of modelling with three-dimensional solid isoparametric finite elements is the determination of suitable material properties for an analysis. By using the fundamental stress-strain relation

$$(\sigma) = [D]\,(\varepsilon) \tag{4}$$

where [D] is the 6 x 6 elasticity matrix containing appropriate material properties, the formulation for a stratified transversely isotropic solid element can be derived.

An expedient determination of the equivalent elastic constants which populate the elasticity matrix of a solid element include the following assumptions:

- the element is homogeneous and transversely isotropic
- there is no discontinuity in the displacement field
- each layer contributing to the element is composed of linearly elastic isotropic material
- Poisson's ratios are approximated for insulation materials, if necessary.

By taking a summation of strain energies for each lamination represented in a solid element and incorporating the appropriate stress-strain relationships, equivalent elastic constants can be evaluated for an analysis as demonstrated in References [2 and 3]. The equations that determine the equivalent elastic constants of a portion of a coil bundle can then be consolidated into a computer program [3]. An efficient procedure can be developed whereby the geometric and material parameters of individual laminations composing a solid element are input and the non-zero terms of the elasticity matrix, [D], are output. These values are then incorporated into the finite element model and the analysis would proceed from there. The MAT9 card would be used with MSC/NASTRAN.

4. Evaluation of the Proposed Finite Element Modelling Procedures

The capability and reliability of the three proposed finite element models for simulating a coil bundle can be demonstrated by examining the bending and torsional behavior found in simpler laminated beam models whose results can be predicted and compared with numerical solutions [3,4,5]. In both examples, the equivalent shear modulus (ESM) of an insulation layer is the variable parameter.

4.1 Bending Test

The three coil modelling techniques were applied in a bending study of straight laminated beams carrying uniform transverse loadings over their spans. The beam cross-sections used are geometrically similar to a coil bundle with length-to-depth ratios of ten-to-one (a relatively short beam comparable to an actual coil), where ten percent of the beam volume is epoxy insulation. When displacements and strain energies from the analyses are compared with those predicted by bending theory, the results were found to compare favorably [3]. The conclusion drawn from this study is that each model is capable of simulating the limits of coil bending behavior ranging from the upper bound where the bundle behaves collectively as a solid to the lower bound where each conductor layer acts as an independent plate. It should be noted that although the maximum beam deformations due to bending are related to the insulator equivalent shear modulus, the global deformation pattern of the beam is not.

4.2 Torsion Test

The torsional stress field of a beam with a rectangular cross-section is determined by the warping deformation induced in the beam. If the cross-section of a finite element model is incapable of and thereby prevented from true warping, the result would be abnormally large normal stresses in the beam. Therefore, a model of a coil bundle must be able to account for the accurate warping of the cross-section. The dependency of the insulation layer ESM on the warping of a beam should also be determined.

When one considers the shear flow responsible for torsion in a coil bundle, if the equivalent shear modulus of an insulation layer is small when compared to a conductor layer, the shear flow may only circulate within each conductor layer. This can lead to the situation where each conductor layer behaves independently and result in the induction of large normal stress components, the magnitude of which will affect the warping of the cross-section. Thus, the torsional deformation pattern of a coil bundle will depend largely on the value of the insulator ESM. In this regard, torsional behavior is different than bending whose global deformation pattern is not directly affected by the ESM.

The same beam configuration that was used in bending is also used to study torsion, this time, however, the beam models are cantilevered and the loads are applied to the free end. The first torsion test was performed on a 'laminated' beam in which the constituent material properties are all assumed to be equal, i.e. a homogeneous solid beam. This represents the extreme upper bound of coil bundle behavior. The results indicate that Model 3 is very accurate and Model 2 is quite satisfactory. Model 1, however, is not correct because the 'insulation' layers in the model are incapable of resisting the desired angular behavior of the adjacent 'conductor' layers. This was pointed out in Section 2.1.

The effects of the insulator equivalent shear modulus on the warping of Model 3 was studied next. The results are shown on Figure 4. As expected the warping pattern changes significantly with the change in the ESM.

The torsional capabilities of each model are compared on Figure 5 which shows the amount of moment required to twist the free end of the beam 5 x 10^{-3} radians as a function of the insulation equivalent shear modulus. Models 1 and 3 are quite sensitive to the ESM; the former being rather accurate for values of the ESM less than 5 x 10^4 psi (where the conductor layers are assumed to behave independently) while the latter behaves quite well for values of the ESM greater than 10^5 psi. But, as was indicated earlier, Model 3 can be used with values of the ESM that are less than 10^5 psi in the conservative analysis of the structural behavior of a coil case or intercoil support system. Model 2 behaves quite well over a wide range of ESM values.

As a final note, it should be pointed out that the beam used in these torsional analyses is not long enough to permit the total dissipation of St. Venant forces. Therefore, only the twisting moment values are compared because of the variation in support conditions between plate and solid finite elements at the fixed end of the beam.

5. Conclusion

Three finite element modelling techniques for simulating a laminated coil bundle have been presented and evaluated. The first two configurations have the capability of representing sufficient detailing of a coil bundle to permit their use in the engineering of the coil. Multi-level substructuring would probably be required for such analyses. The other configuration is capable of describing the global behavior and stiffness of a coil bundle which can then be used in the engineering of other machine components. The single most important factor in the selection of a coil bundle modelling approach is to realize the applicability and limits of the elements employed when compared to the mechanical behavior of the actual structure that is being studied.

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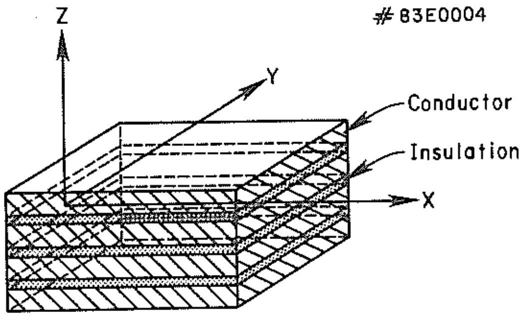

Figure 1 - Configuration of a Laminated Beam

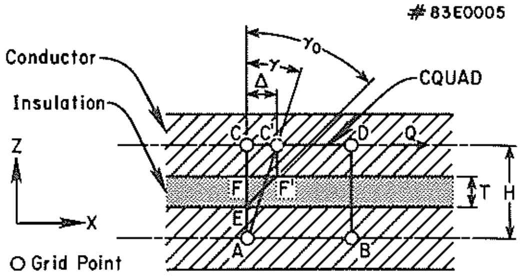

Figure 2 - Schematic Representation of Model 1

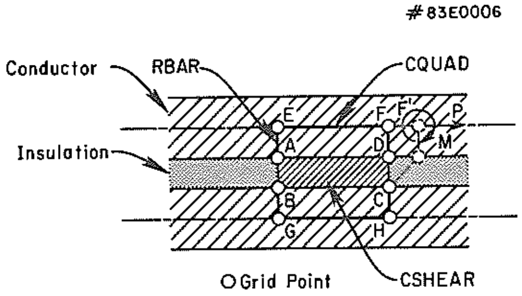

Figure 3 - Schematic Representation of Model 2

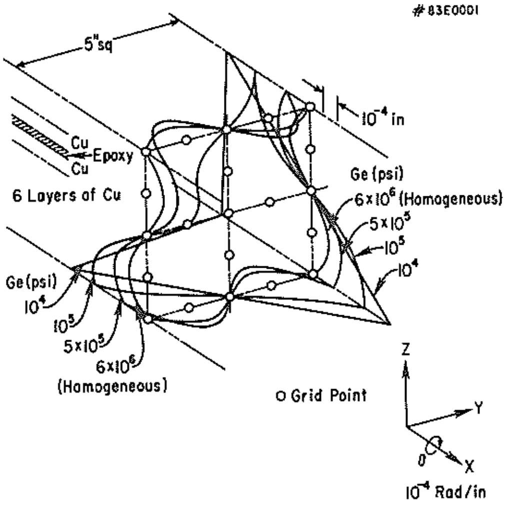

Figure 4 - Effects of the Insulator Equivalent Shear Modulus on Warping (Model 3)

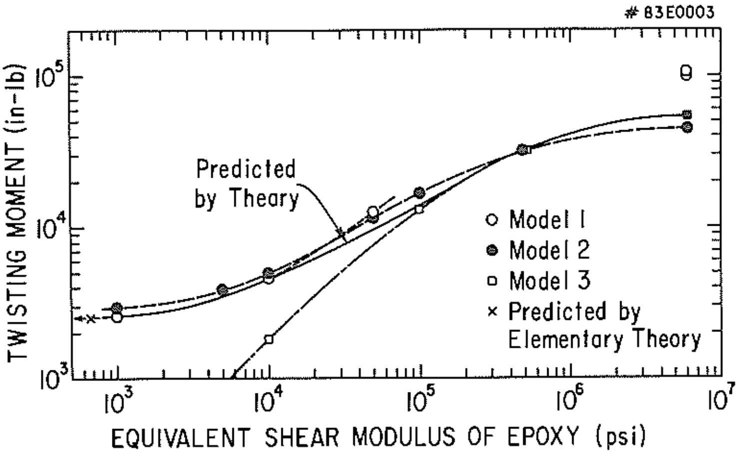

Figure 5 - Analytical Comparison of Twisting Moment vs. Insulator ESM (Rotation Equals 5×10^{-3} Radians)