# The Forming of a Superconductor Cable During the Winding of a Large Toroidal Field Coil

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The feasible range for the tension force which acts on a superconductor cable during the winding of a large D-shaped TF coil depends strongly on the mechanical properties of the cable, on the geometry of the winding pack, and on the arrangement of the equipment. An upper limit is imposed by the possible damage within the cable. The lower limit is set by the need to assure enough compaction and to overcome the friction forces between the layers. Within this "corridor" an optimal control of elastic prestresses is desirable ... may be chosen regarding the residual stresses and/or the elastic springback after removing of the coil former, respectively .

The paper presents a simplified elastica conductor model (SECM) built by a finite chain of intervals with constant bending moment and curvature. The problem does not allow to linearize the curvature. A bilinear moment-curvature relationship as derived from bending experiments was used to describe the elastoplastic behavior of the cable under different tension forces acting on the "free" end near the supply spool. Due to the geometric and material nonlinearities mentioned no direct solution is possible. The paper describes the discrete model as well as the iterative shooting method which finds the equilibrium shape of the conductor. The distributions of bending moment and shear forces around the D-shaped contour as well as along the conductor are given. They show a pronounced influence of the tension force in the relevant range of 1 to 40 kN. Some inconsistencies due to compromising model simplifications are shown. Desirable improvements are outlined. In particular the possibility of mitigating the stress concentration effect by supporting rollers suitably placed along the "free" conductor near the bobbin is discussed.

N 5/7

# Analyses of Eddy Current Loadings in Fusion Engineering Structures

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SUMMARY

We have investigated both analytically and numerically the eddy current loadings found in fusion engineering structures. The largest loadings of this type arise from plasma disruptions. These loads (in the TFTR device) appear on the time scale of a plasma disruption ($\approx 10^{-3}$s) and may easily reach $3 \times 10^5$ N/m or greater in magnitude. While these loads cause no problems in the primary structure of a fusion engineering device, they may easily become the design load for protective systems and diagnostics.

We first present a simple analytical analysis of the eddy current loading phenomena. Then, we describe the calculation techniques that have been developed to solve numerically for the general behavior of a transient electromagnetic event. These techniques have been used in developing the computer program SPARK, which is now used at the Princeton Plasma Physics Laboratory to solve the general eddy current loading problem. Lastly, some results from SPARK are presented and the manner in which the results from SPARK are translated into a form compatible with the local structural finite element code, NASTRAN, are described.

1. Analytic Analysis

In the following simple problem the qualitative aspects of eddy current behavior will become clear. The more complicated behavior of eddy currents in a real engineering structure must be handled by computer programs such as SPARK. Consider the idealized circuit shown in Figure 1 containing only an self-inductance L and a resistance R. Let the current in the circuit be i and the magnetic flux through the circuit be $\Phi$. The total flux $\Phi_{tot}$ in this circuit is the sum of the flux from an independent, external source $\Phi_{ext}$ and the flux produced by the current in the circuit itself, $\Phi_{self}$. The self-inductance is defined as

$$L = \frac{d\,\Phi_{self}}{di} \tag{1}$$

and since the circuit does not change shape we may say

$$\Phi_{self} = Li \tag{2}$$

giving us

$$\Phi_{tot} = \Phi_{ext} + \Phi_{self} = \Phi_{ext} + Li \tag{3}$$

Using ohm's law we may then write the voltage $\varepsilon$ as

$$\varepsilon = iR = -\frac{d\Phi_{tot}}{dt} = -\left[\dot{\Phi}_{ext} + L\frac{di}{dt}\right] \tag{4}$$

giving us the first order differential equation that is basic to the whole problem:

$$\frac{di}{dt} + \frac{R}{L}i = \frac{-1}{L}\left[\dot{\Phi}_{ext}\right] \tag{5}$$

Now if we identify $\frac{L}{R}$ as the characteristic time of our circuit($\tau$), a wealth of information may be obtained from equation (5). If the external flux undergoes a step change at t=0, i.e.,

$$\Phi_{ext} = \Phi_o\ [1-U(t)]\ ,\quad U(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases} \tag{6}$$

then

$$i(t) = \frac{\Phi_o}{L}\, e^{-t/\tau}\ ,\ t\geq 0 \tag{7}$$

where $\Phi_o/L$ is called the superconducting current.

If the external flux is a ramp function that starts at t=0 and continues for all time, i.e.,

$$\Phi_{ext} = \dot{\Phi}_o\ t\ U(t) \tag{8}$$

then

$$i(t) = -\frac{\dot{\Phi}_o}{R}\left[1-e^{-t/\tau}\right]\ ,\ t\geq 0 \tag{9}$$

where $\dot{\Phi}/R$ is the resistive current.

Equation (9) is exactly the same answer as if we closed the switch in a series circuit containing a battery, resistor, and an inductance. If the external flux decays exponentially in time with time constant $\tau_p$.

$$\Phi_{ext} = \Phi_o \left[1-e^{-t/\tau_p}\, U(t)\right] \tag{10}$$

This gives an induced current

$$i(t) = \frac{\Phi_o}{L}\left[\frac{\tau}{\tau_p-\tau}\right]\left[e^{-t/\tau_p} - e^{-t/\tau}\right] \tag{11}$$

In general, the induced current from any specific external flux change may be evaluated using the following green's function.

$$i(t) = \frac{-1}{L}\int_o^t \Phi_{ext}(\lambda)\left\{U'(t-\lambda) - \frac{1}{\tau}\, e^{-(t-\lambda)/\tau}\right\} d\lambda \tag{12}$$

If the initial flux $\Phi_o$ decays to zero by time $\tau_p$ in a linear fashion we would get

$$\left.\begin{aligned} i(t) &= \frac{\Phi_o}{L}\frac{\tau}{\tau_p}\left[1-e^{-t/\tau}\right], \quad t<\tau_p \\ i(t) &= \frac{\Phi_o}{L}\frac{\tau}{\tau_p}\left[1-e^{-\tau_p/\tau}\right] e^{-(t-\tau_p)/\tau}, \quad t\geq\tau_p \end{aligned}\right\} \tag{13}$$

The constants could be rewritten as

$$\frac{\Phi_o}{L}\frac{\tau}{\tau_p} = \frac{\dot{\Phi}_o}{R} \tag{14}$$

2. Plasma Disruption Loadings in TFTR

The preceding analysis provides some insight into the time dependence of the induced currents and hence forces that are applied to a fusion engineering structure. However, the equations presented are inadequate for analyzing a real tokamak such as TFTR. Real structures have distributed induced currents and have mutual inductances which couple all parts of the structure together.

The changing external flux which produces the induced currents must also be related to actual transient magnetic events in the plasma. The two cases considered in equations (6) and (8) seem to bound this flux swing. An accurate estimate of a plasma disruption is a very difficult problem, plasma disruptions in a tokamak are not fully understood. The problem may be overwhelming or trivial depending upon the choice of a characteristic time for a plasma disruption.

The estimate of plasma behavior now used for engineering design on TFTR is that of a linear decay in plasma current at the rate of one giga-ampere per sec., i.e., $10^9$ ampere/sec. Hence, if the plasma does not move during a disruption the time dependence found in equation

(13) is accurate. However, plasma motion during a disruption may happen and the preceeding equations do not include the effect of a flux change due to a plasma motion. At present, the plasma disruption loadings in TFTR include the cases where the plasma moves inward with constant safety factor (q) and inward motion with constant current density. In all of these cases, detailed numerical calculations are required.

## 3. The SPARK Computer Program

In order to examine the behavior of a real structure a "mesh network model" is required, this approach introduces mutual inductances between meshes in addition to self-inductance terms. Conceptually the same steps are taken but now we have the matrix equations

$$[L]\ \{\dot{I}\} + [R]\ \{I\} = -\ \{\dot{\Phi}\} \tag{15}$$

where [L] and [R] are square matrices of inductance and resistance and {I} and $\{\dot{\Phi}\}$ are column vectors. In the following discussion we are referring to engineering structures that may be modeled as two-dimensional surfaces in a three dimensional world. We do not solve for the most general case of three dimensional current distributions, i.e., we treat thickness as a parameter in the problem much like plate and shell theory.

Background on the techniques used to calculate the L and R matrices may be found in references 1 and 2. In the following text we will conceptually review the approach used in the computer program SPARK.

The first step in analyzing the induced eddy currents in a structure is to describe this surface as a collection of small quadralateral or triangular elements. The distribution and spacing of an adequate set of plate elements in a finite element analysis usually defines an adequate mesh for eddy current analysis. To each quadralateral element we associate four "branches" or sides and to each triangular element we associate three branches. As described in the references, we may now associate an inductance matrix and a resistance to each branch in our network. In SPARK branches may belong to a single element or to at most two elements and the L and R calculations must be modified accordingly. The branch matrixes are then transformed into the mesh matrixes used to formulate equation (15). By addition of the appropriate terms we may now form the mesh matrixes for inductance and resistance. For example, if an element is defined by branches a,b,c, and d then the self inductance of that element is the sum of each of the appropriate branch inductances and the sum of the appropriate mutual inductances among the branches a,b,c, and d. The mutual inductance of one element to another is again a summation calculation. The inductance matrix formulated in this manner is typically very dense. The resistance matrix is formulated in a similar fashion and in this case the paucity of mutual couplings gives up a sparse matrix. The resistance matrix it should be noted is not diagonal.

In order to complete the problem the right hand side of equation (15) must be defined. This is accomplished by defining a physical source for the external flux change, such as an equivalent current distribution for a disrupting plasma, and calculating the forcing voltages that are produced in the mesh equations. In any case, the complex motion of the disrupting plasma is a specified input and it defines the right hand side of equation (15). Now if we assume some initial values for the eddy current distribution, typically all mesh currents set to zero at t=0, we may integrate the equations forward in time.

Before proceeding on it should be noted that some subtle constraints have been passed over. For example, if a plane of symmetry exists in the problem, it may be used to decrease the size of the calculation. Typically only the independent segment of the real problem need be defined for the actual matrix but the formulation of the matrixes must include all the appropriate extra inductive couplings. Another subtle point is that the standard constraint from classical network analysis must be satisfied, i.e.,

$$\ell + n = b + 1 \tag{16}$$

where

b = the number of branches

n = the number of node points (or grid points)

$\ell$ = the number of loops (or meshes)

For example, if a plate had a hole cut in its interior the flux change passing through the hole must be accounted for. This behavior is entered into the matrix equation by the use of "hole-elements". All these points must be handled correctly before the matrix problem statement will make sense.

Let us now assume that equation (15) has been defined and we can solve for the currents by numerical integration. The final step accomplished by SPARK is to translate the transient eddy currents into dynamic forces. The solution of equation (15) gives us the transient currents in the strucure. By defining the background fields that the structure we are analyzing exists in, the Lorentz (JXB) forces may be calculated. SPARK accomplishes this by converting the mesh current back into branch currents, doing (JXB) for each branch and then converting the force distribution along each branch into a pair of end equivalent forces and moments (same as simple beam theory). These loads from each branch are applied to the nodes (or grids) in the problem and summed up. Now a full description of the time dependent forces due to eddy currents exists for our problem and standard techniques may be used to solve for the mechanical transient response.

## 4. Modeling Considerations

The attached Figure 2 illustrates typical results from a SPARK calculation. The structure in this figure is a model of the TFTR generic protective plate. This plate has a single curvature and is located on the inside of the TFTR vacuum vessel. The horizontal line segments on the left and right edges of the plate (viewed straight on) are structural cuts installed to improve eddy current behavior. The solid horizontal line at the center is a symmetry line and is not physically present in the real structure. Directed line segments (arrows) indicate the streamlines of the eddy currents induced by a plasma disruption, approximately 1500 amperes are flowing between each adjacent pair of streamlines in this figure. These current streamlines are produced by a $10^6$ ampere plasma current, starting next to this plate and moving towards the center of the TFTR device as it disrupts.

We have discovered by numerical experiment that the presence of the vacuum vessel is a very small effect for this particular structure. The mutual coupling between the protective plates and the vacuum vessel modify the eddy current pattern by about 5-10%. Since this is a small effect, the SPARK model for this problem only considered the protective plate alone in space.

We have also found that the current distribution in this problem does not vary much in spatial shape as the disruption proceeds and hence the data processing problem may be simplified as follows. If one solves for the resistive current distribution, i.e.,

$$[R]\ \{I\} = -\{\dot{\Phi}\} \quad \text{(independent of time)} \tag{17}$$

one obtains a very close approximation to the spatial distribution of currents obtained from solving equation (15). By taking this resistive solution and scaling it as a function of time, being guided by the actual solution to equation(15), a very good approximation to the true mechanical forces may be made. The reduction in required data processing using this approach is easily a factor of 10.

## 5. Conclusion

The calculation of plasma disruption loads, a very important consideration in devices such as TFTR, has been largely automated by the use of the computer program SPARK. By using the same node and mesh model of the structural problem, for both the eddy current analysis and the structural finite element analysis, great savings in effort have been accomplished. This paper has reviewed the basic equations governing the problem and described the approach used in SPARK to model the more complex problems the electrical/mehcanical analyst must solve in current fusion research devices.

## REFERENCES


/1/ Christensen, U. R., "Time Varying Eddy Currents on a Conducting Surface in 3-D Using a Network Mesh Method," PPPL-1516 (April 1979)

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FIGURE I

# 83E0039

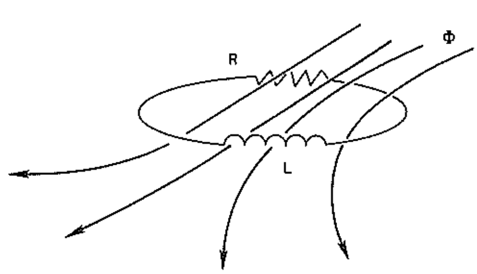
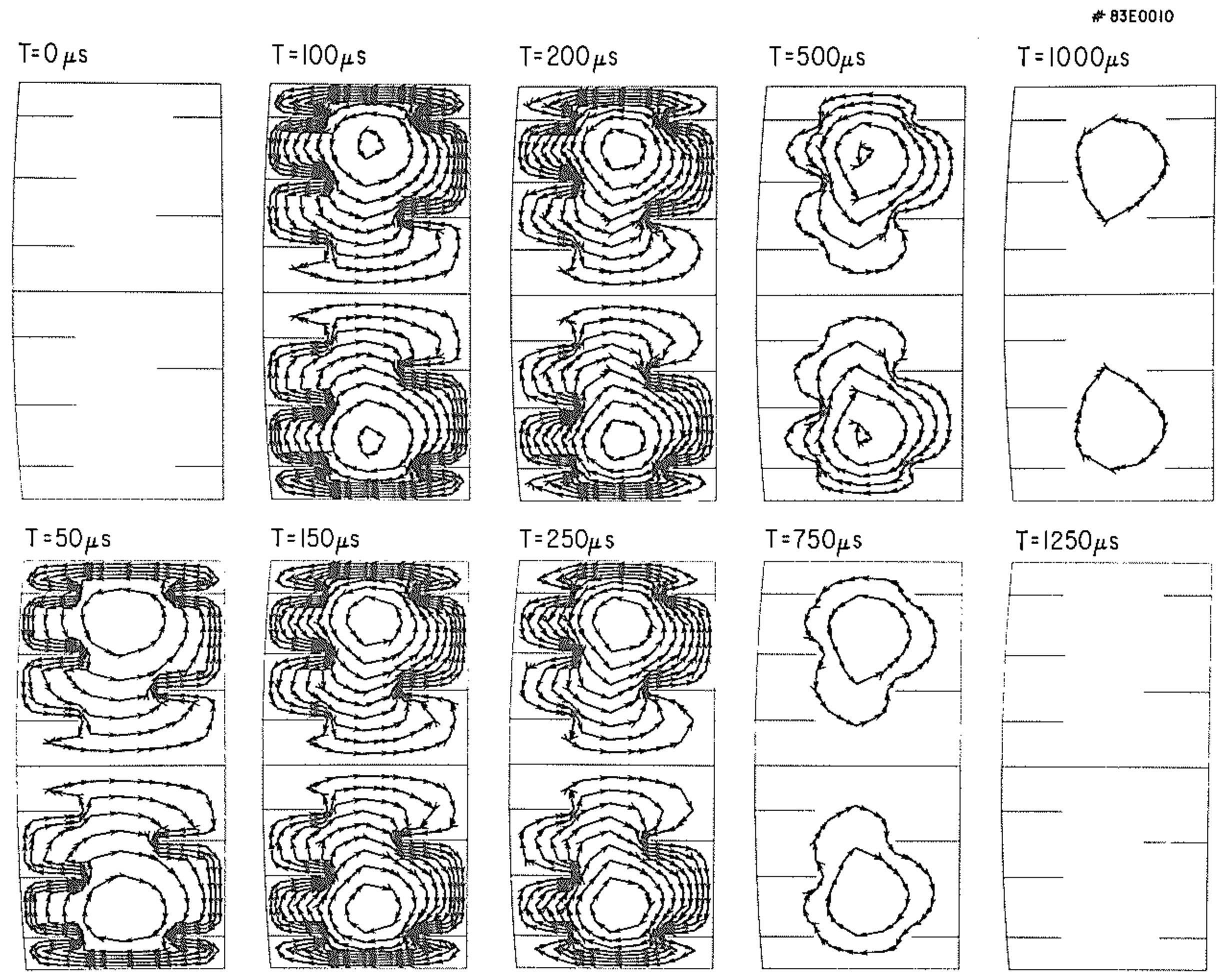


FIGURE 2



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